Quantum statistical enhancement of the collective performance of multiple bosonic engines

GW, Venkatesh *et al*., arXiv:1904.07811 (2019).

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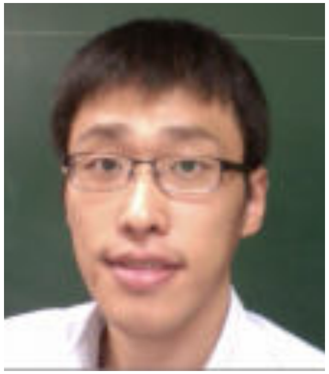

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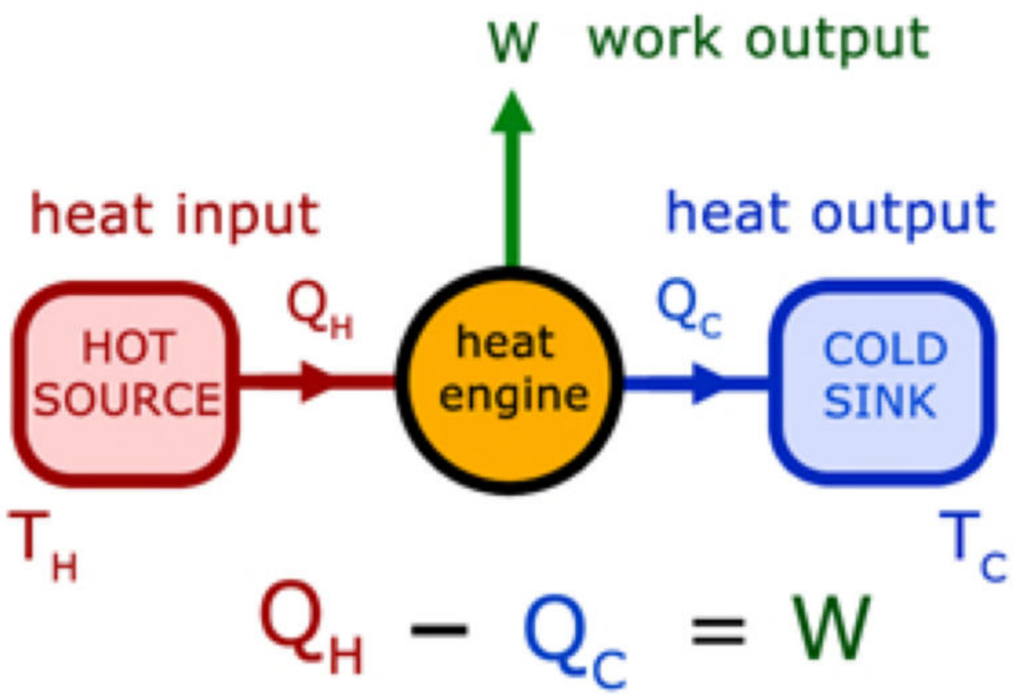
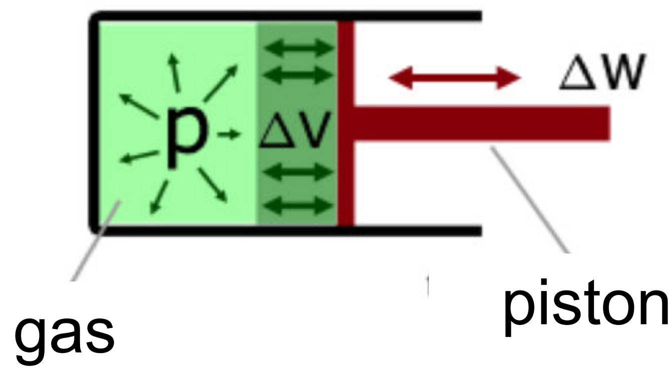
Introduction

Heat engines: important application of thermodynamics

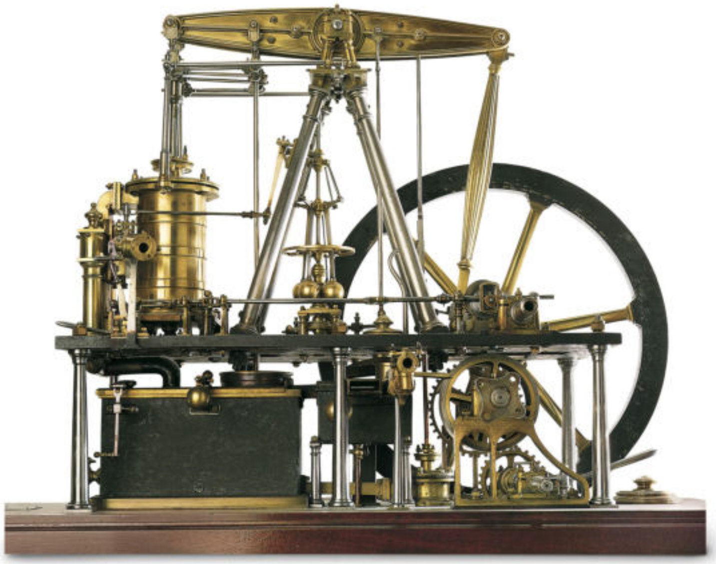
Essential role to understand & build thermodyn.



(From A-level Physics Tutor)

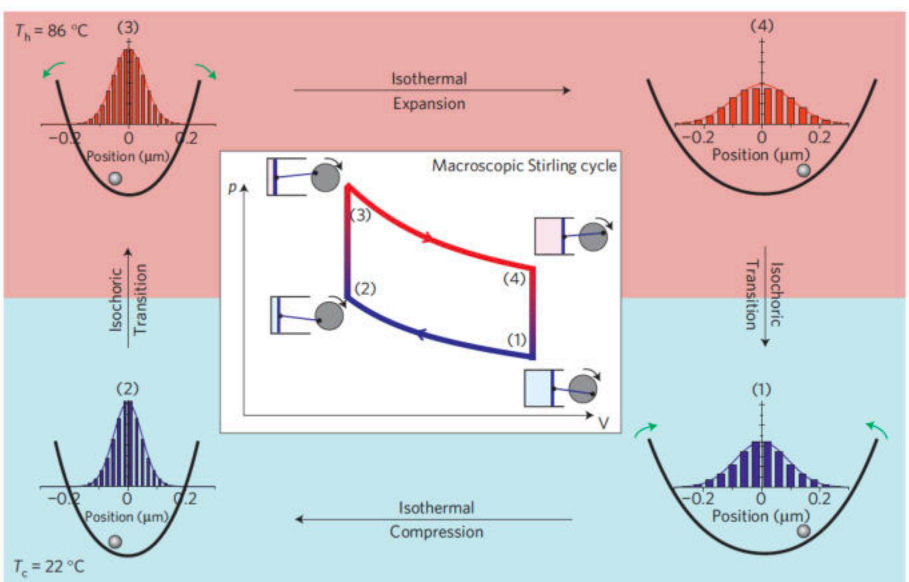


Downsizing the heat engines



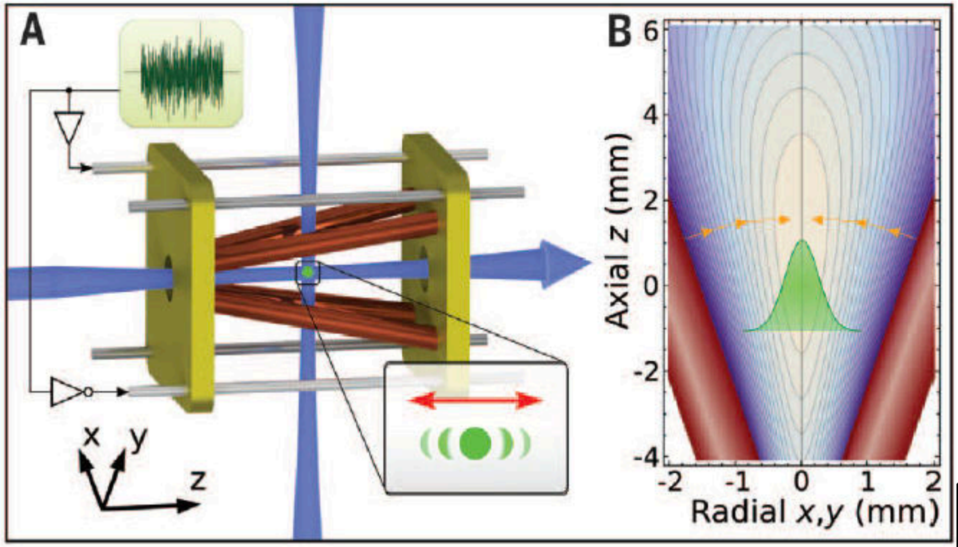
Technological developments
allow us to downsize the heat engines.

A colloidal particle in a trap



Blickle & Bechinger, Nat. Phys. **8**, 143 (2012)

An ion in a trap



Roßnagel *et al.*, Science **352**, 325 (2016)

Quantum heat engines: Incentives

Quantum heat engine (QHE): Engine with working substance made of quantum sys.

<u>Incentives</u>:

- Adiabaticity due to the discrete energy levels
- Improving the performance using quantum resources (e.g., quantum coherence).

e.g., Scully *et al*. Science **299**, 862 (2003)

Kosloff (today's morning lecture)

Old problem, but still intriguing!

Scovil & Schulz-DuBois, PRL **2**, 262 (1959)

3-level maser as a heat engine

Alicki, J. Phys. A **12**, L103 (1979)

Derivation of Carnot bound for an open quantum sys.

Kosloff, J. Chem. Phys. **80**, 1625 (1984)

Bender *et al*., J. Phys. A: Math. Gen. **33** 4427 (2000)

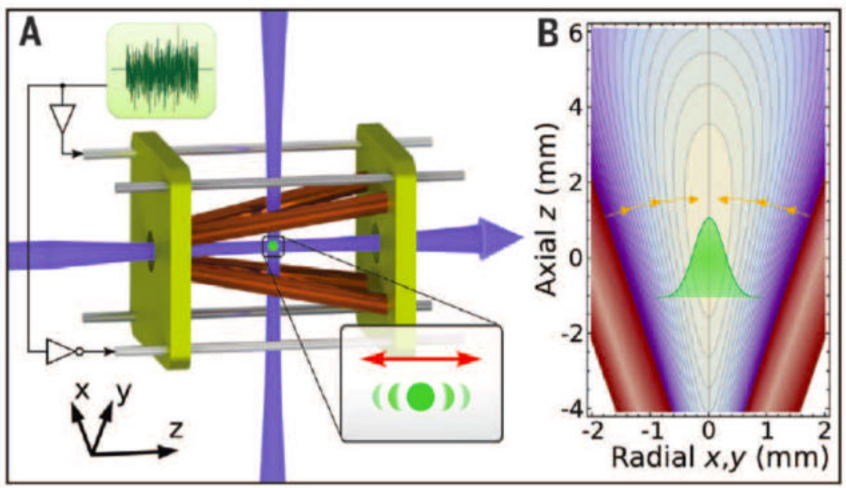
Quan et al., PRE **76**, 031105 (2007)

1. Trapped ions

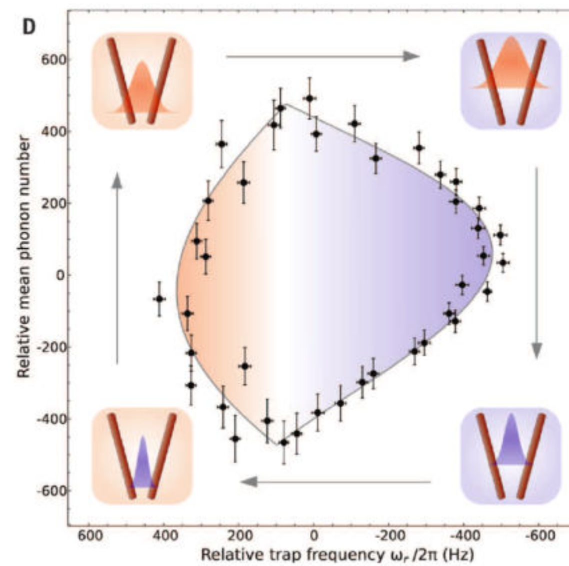
Roßnagel *et al*., Science **352**, 325 (2016)

Maslennikov *et al*., Nat. Commun. **10**, 202 (2019)

von Lindenfels *et al*., PRL **123**, 080602 (2019)



Roßnagel *et al*. (2016)



2. Liquid-state NMR

Petersen *et al*., arXiv:1803.06021 (2018)

3. Diamond NV$^-$ centers

Klatzow *et al*., PRL **122**, 110601 (2019)

Quantum-thermodynamics signatures

Quantum-thermodynamic signatures

Uzdin, Levy & Kosloff, PRX **5**, 031044 (2015)

Quantum effects in thermodynamic quantities

Many-body quantum heat engines

Quantum correlation in the interacting working substance

Jaramillo, Beau, and del Campo, NJP **18**, 075019 (2016)

Hardal, Paternostro, and Müstecaplıoğlu, PRE **97**, 042127 (2018)

Interaction-driven quantum heat engines

Chen, GW, Yu, Guan, and del Campo, npj Quant. Inf. **5**, 88 (2019)

Measurement-driven quantum heat engines

Elouard, Herrera-Marti, Huard, and Auffèves, PRL **118**, 260603 (2017)

Yi, Talkner, and Kim, PRE **96**, 022108 (2017)

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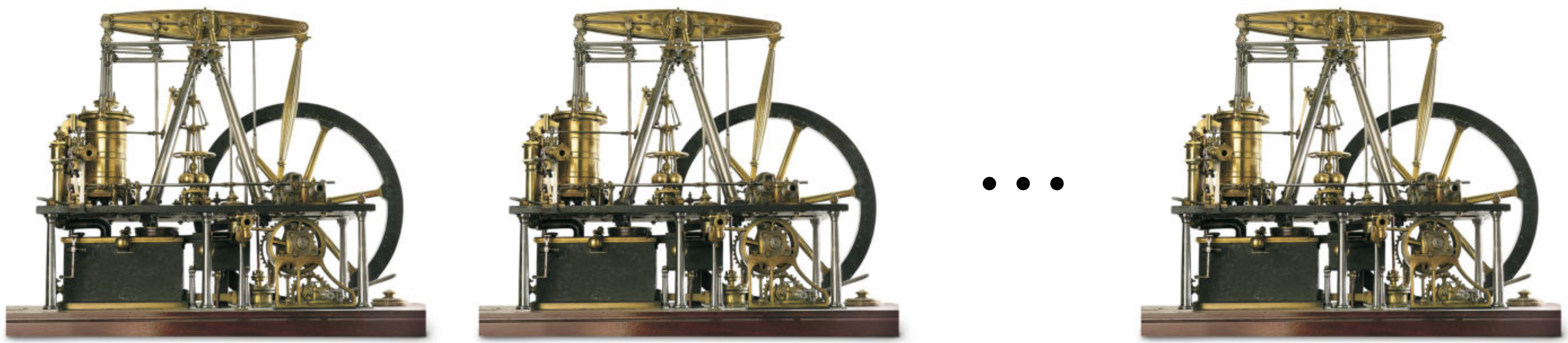
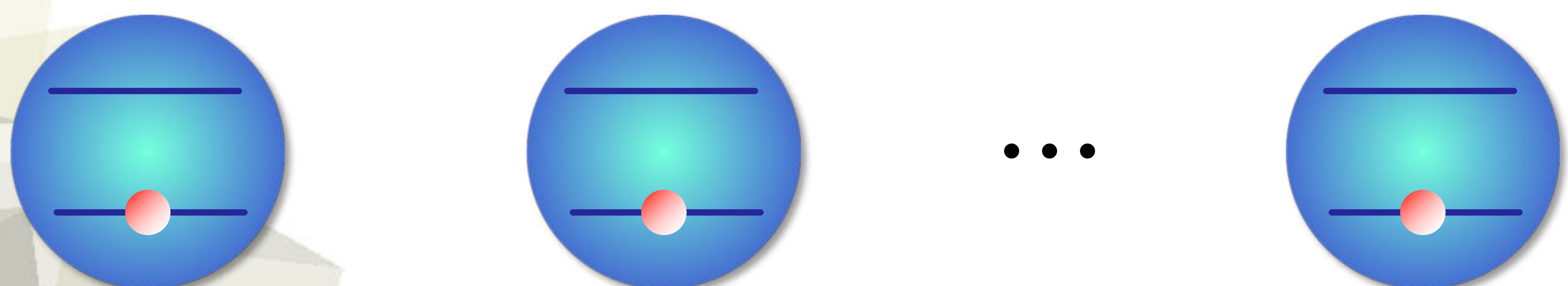
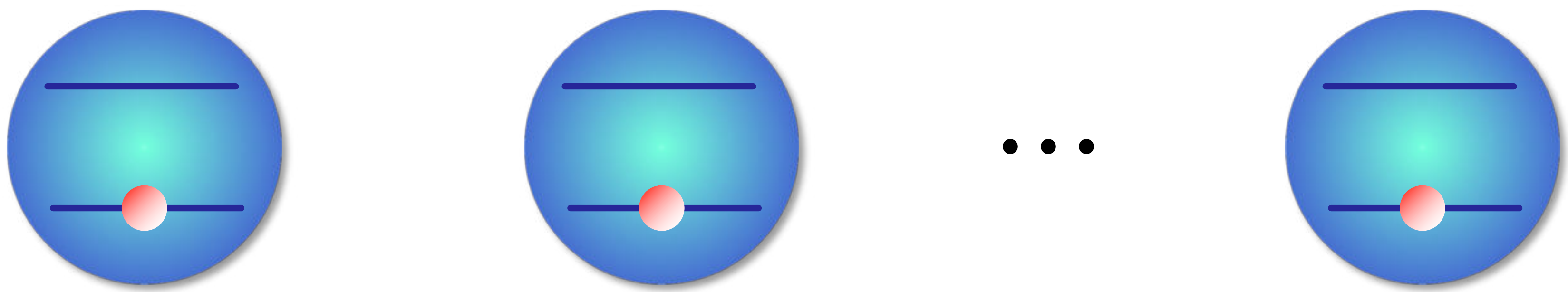
An ensemble of multiple identical heat engines



What if engines are indistinguishable?



Take-home message



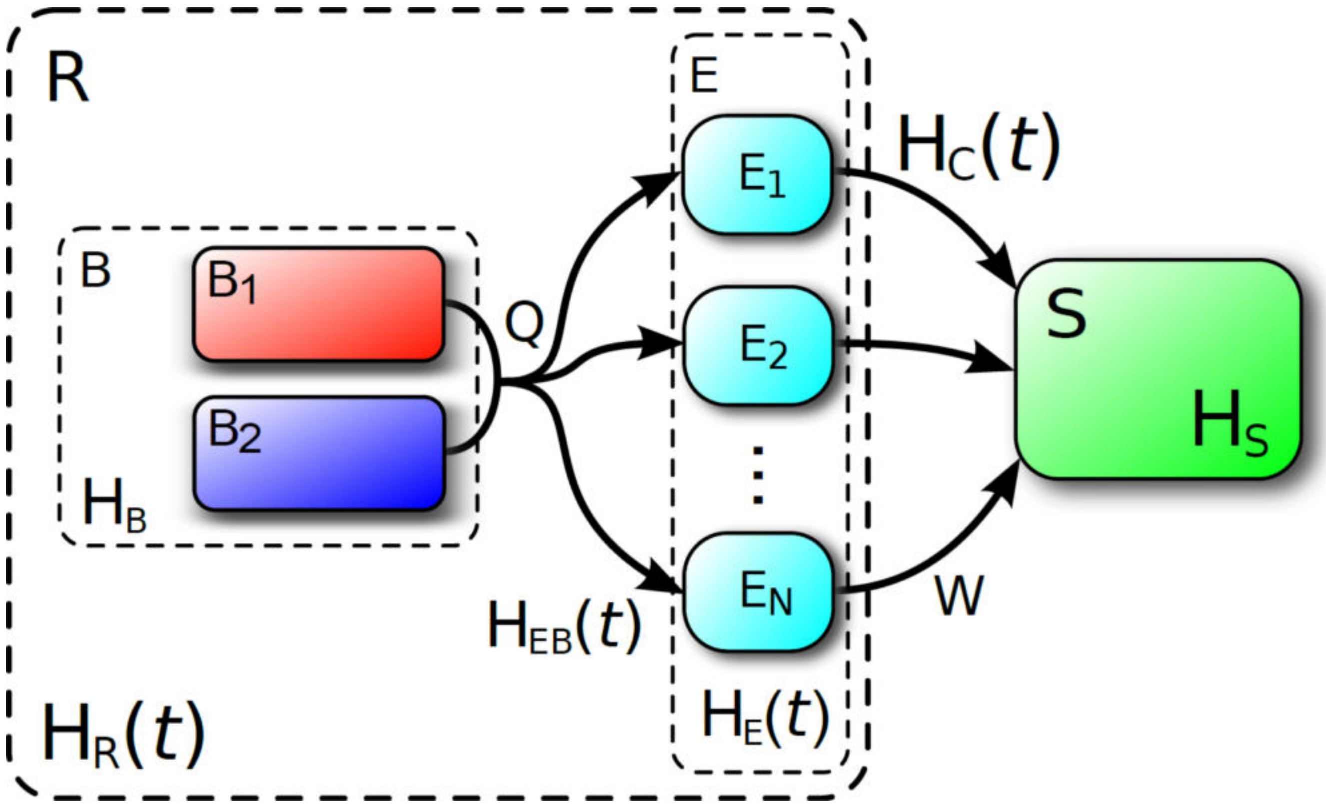
An ensemble of indistinguishable bosonic identical engines

→ **Collective enhancement of work output!**

Genuine quantum effect

Setup of the problem

N QHEs **outcoupled** to an external system



$$H(t) = H_R(t) + H_C(t) + H_S$$

Work resources (engines + baths): $H_R(t) = H_E(t) + H_B + H_{EB}(t)$

Coupling btwn resources & ext. sys.: $H_C(t) = g_C(t)\, V_R \otimes V_S$

Bath & ext. sys. are time-independent.

Engine models

N two-level atoms, each atom working as a QHE.

Indistinguishable case

Engine Hamiltonian: $H_E(t) = 2\Omega(t)\, S_z + 2\Delta\, S_x$

$$S_x \equiv (a^\dagger b + b^\dagger a)/2 \qquad S_z \equiv (a^\dagger a - b^\dagger b)/2$$

(a: annihilation op. of g. st. atoms)

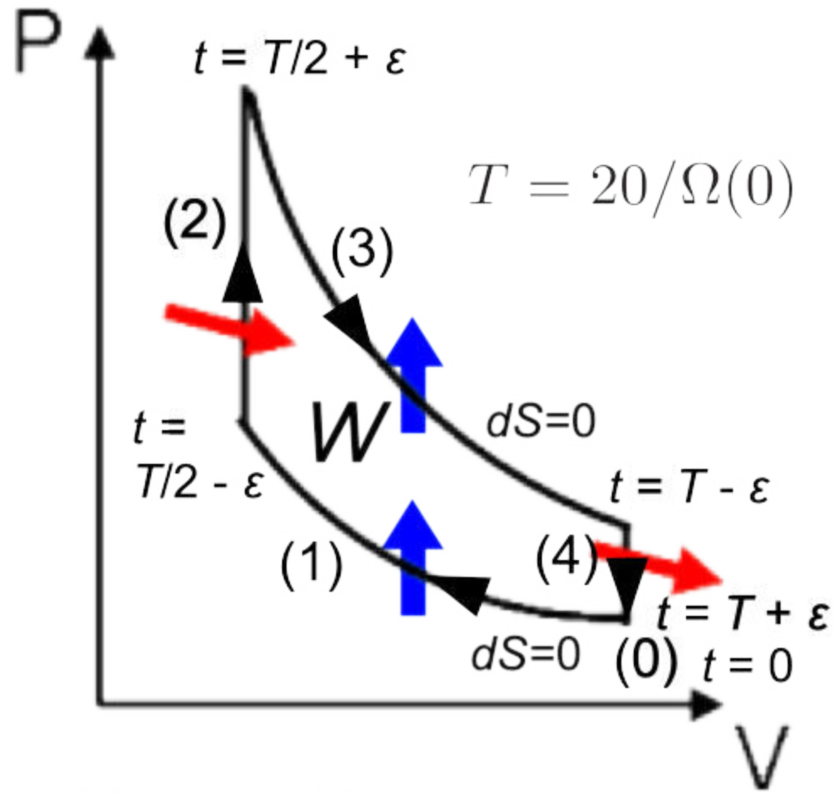
Coupling Hamiltonian: $H_C(t) = g_C(t)\, 2S_x \otimes V_S$

Distinguishable case

Engine Hamiltonian: $H_E(t) = 2\Omega(t) \sum_{j=1}^{N} \frac{\sigma_{j,\,z}}{2} + 2\Delta \sum_{j=1}^{N} \frac{\sigma_{j,\,x}}{2}$

Coupling Hamiltonian: $H_C(t) = g_C(t)\, 2 \sum_{j=1}^{N} \frac{\sigma_{j,\,x}}{2} \otimes V_S$

Quantum Otto cycle protocol



$$H_E(t) = 2\Omega(t)\, S_z + 2\Delta\, S_x$$

(0) Initial st.: $\rho_0^E = Z_{\beta_c}^{-1}(0)\exp\left[-\beta_c H_E(0)\right]$

(1), (3) Adiabatic compress./exp.: linear sweep

$$\Omega(t) = \begin{cases} \Omega(0) + vt & (0 \le t \le T/2) \\ \Omega(0) + v\,(T - t) & (T/2 \le t \le T) \end{cases} \qquad \begin{matrix} v = -0.1\Omega(0)^2 \\ \Omega(0) = 1 \end{matrix}$$

(2), (4) Hot/cold isochores: Set $g_C = 0$ and thermalize at β_h or β_c.

Average of work by outcoupled QHEs

Assumptions

- Initial st.: $\rho_0 = \rho_0^R \otimes |0\rangle_{S\,S}\langle 0|$ (Ext. sys. *S* is in the ground st.)
- $g_C(t) = 0$ at $t = 0, T/2, T$ ➡ $[H_S, H(t)] = 0$ @ these moments

Average of work by perturbation theory

$$\langle w_N \rangle = \sum_{i\neq 0} \epsilon_i^S p_i$$ with probability $$p_i = \mathrm{Tr}_R \left[{}_S\langle i| U^{(I)}(T,0)\, \rho_0\, U^{(I)\dagger}(T,0) |i\rangle_S \right]$$

Propagator for one cycle in int. pict.: $$U^{(I)}(T,0) \approx I - i\int_0^T dt\, g_C(t) V_R^{(I)}(t) \otimes V_S^{(I)}(t)$$

$$\boxed{p_i \simeq \int_0^T dt \int_0^T dt'\, g_C(t)\, g_C(t')\ {}_S\langle i| V_S^{(I)}(t)|0\rangle_S\, {}_S\langle 0|V_S^{(I)}(t') i\rangle_S\ \langle V_R^{(I)}(t')\, V_R^{(I)}(t)\rangle_{\rho_0^R}}$$

Coupling ops. (int. pict.): $V_R^{(I)}(t) \equiv U_R^\dagger(t,0)\, V_R\, U_R(t,0)$ $\quad V_S^{(I)}(t) \equiv e^{iH_S t}\, V_S\, e^{-iH_S t}$

$U_R(t,0) \equiv \mathcal{T} \exp\left[-i\int_0^t H_R(t')\, dt'\right]$ $\quad \langle \cdots \rangle_{\rho_0^R} \equiv \mathrm{Tr}_R[\cdots \rho_0^R]$

Quantum statistical enhancement of outcoupled work

$$p_i \simeq \int_0^T dt \int_0^T dt'\, g_C(t)\, g_C(t')\ {}_S\langle i|V_S^{(I)}(t)|0\rangle_S\, {}_S\langle 0|V_S^{(I)}(t')|i\rangle_S\ \langle V_R^{(I)}(t')\, V_R^{(I)}(t)\rangle_{\rho_0^R}$$

Impulse-type coupling $\quad g_C(t) = g\,\delta(t - t_1) \qquad (0 \le t_1 \le T/2)$

$$\langle w_N \rangle \simeq g^2 \left\langle [V_R^{(I)}(t_1)]^2 \right\rangle_{\rho_0^R} \sum_{i \neq 0} \epsilon_i^S \left| {}_S\langle i|V_S^{(I)}(t_1)|0\rangle_S \right|^2$$

Within adiabatic approx. for the engines:

$$\left\langle [V_R^{(I)}(t_1)]^2 \right\rangle_{\rho_0^R}^{\text{indist}} \ge \left\langle [V_R^{(I)}(t_1)]^2 \right\rangle_{\rho_0^R}^{\text{dist}}$$

for any parameter values at $N \ge 1$.

$$\mathcal{E} \equiv \frac{\langle w_N \rangle^{\text{indist}}}{\langle w_N \rangle^{\text{dist}}} \ge 1$$

Quantum statistical enhancement!

Quantum statistical enhancement of outcoupled work

$$\mathcal{E} \equiv \frac{\langle w_N \rangle^{\mathrm{indist}}}{\langle w_N \rangle^{\mathrm{dist}}} \geq 1$$

Quantum statistical enhancement!

Statistical indistinguishability leads to restrictions on the allowed states.

$$\begin{cases} \text{Distinguishable: } 2^N \text{ states} \\ \text{Indistinguishable: } N + 1 \text{ symmetrized states} \end{cases}$$

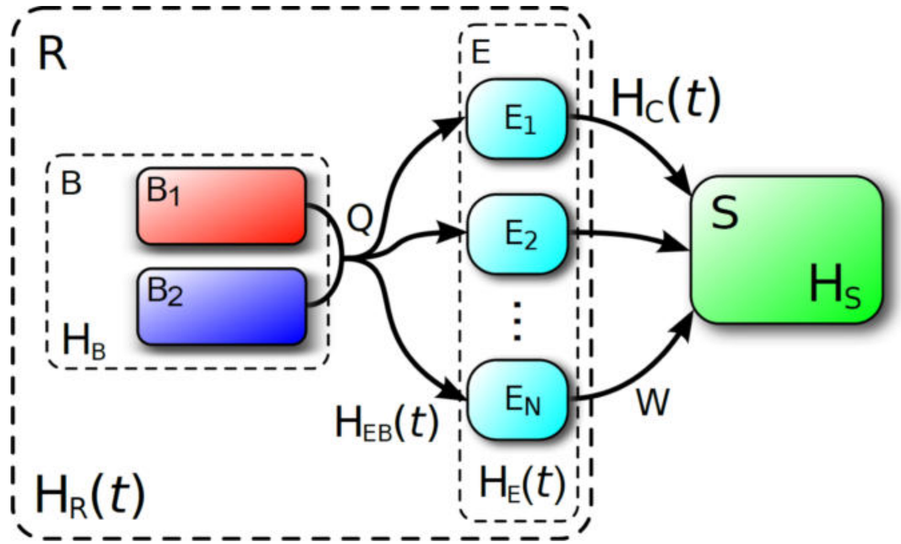
→ Collective nature in the indistinguishable case

Quantum statistical enhancement of outcoupled work

Demonstration

External sys. (S): $H_S = \omega c^\dagger c$

Coupling op.: $V_S = c^\dagger + c$ (H_C : dipole coupling)



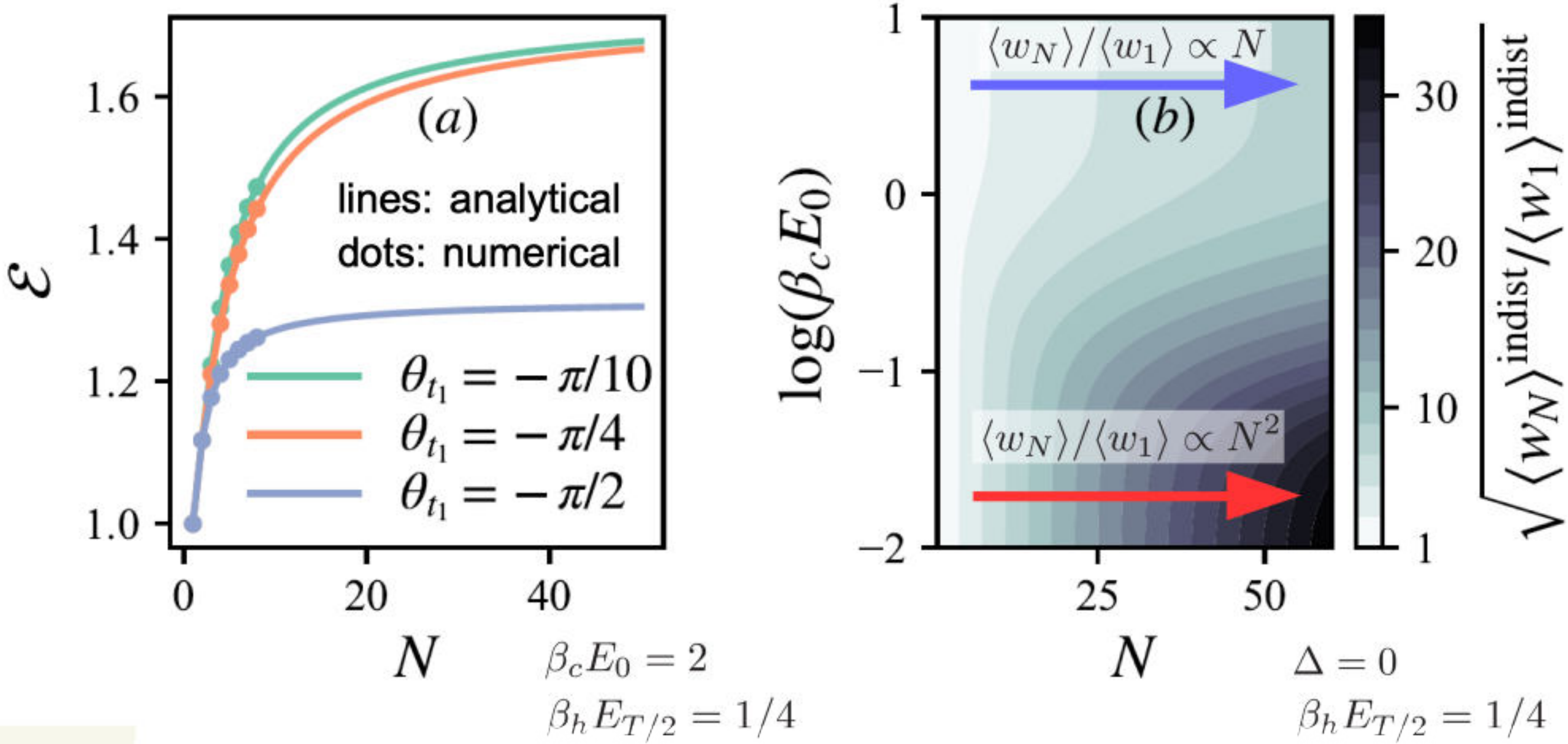
Engine Hamiltonian: $H_E(t) = 2\Omega(t)\, S_z + 2\Delta\, S_x$ $\tan\theta_t = -\Omega(t)/\Delta$

Coupling Hamiltonian: $H_C(t) = g_C(t)\, 2S_x \otimes V_S$

Indistinguishable: $S_{x,y,z}$ (Schwinger rep.)

Distinguishable: $S_{x,y,z} = \sum_{j=1}^{N} \frac{\sigma_{j,(x,y,z)}}{2}$

Quantum statistical enhancement of outcoupled work



Parameters $\tan\theta_t = -\Omega(t)/\Delta \qquad E_t \equiv \sqrt{\Omega(t)^2 + \Delta^2}$

$\omega = 2\pi \times 0.05/T \qquad g = 0.01 \qquad t_1 = 0.35T/2$

$\theta_{t_1} = \{-\pi/10, -\pi/4, -\pi/2\} \Leftrightarrow \Delta = \{4.2\Omega(0), 1.4\Omega(0), 0\}$

- Good agreement btwn analytical & numerical results.
- Quantum stat. enhancement ($\mathcal{E} > 1$) for any parameters.

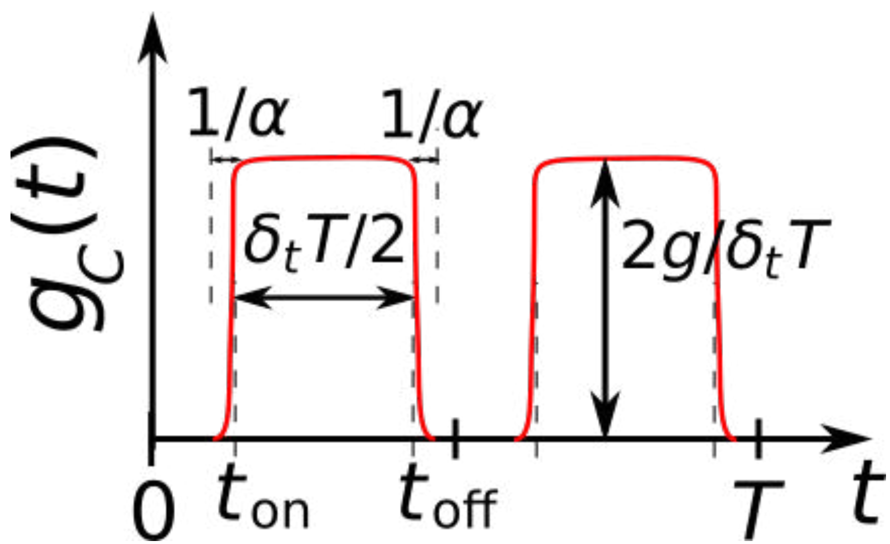
Quantum statistical enhancement of outcoupled work

Continuous coupling

$$g_C(t) = \frac{g}{\delta_t T} \sum_{n=0}^{1} \left\{ \tanh\left[\alpha\left(t - t_{\text{on}} - \tfrac{nT}{2}\right)\right] - \tanh\left[\alpha\left(t - t_{\text{off}} - \tfrac{nT}{2}\right)\right] \right\}$$

α : switching rate

$\delta_t T/2$: duration of the coupling



T is sufficiently large so that engine dynamics is adiabatic.

$\alpha T \gg 1$ & $\delta_t \approx 1$ so that coupling is almost always on.

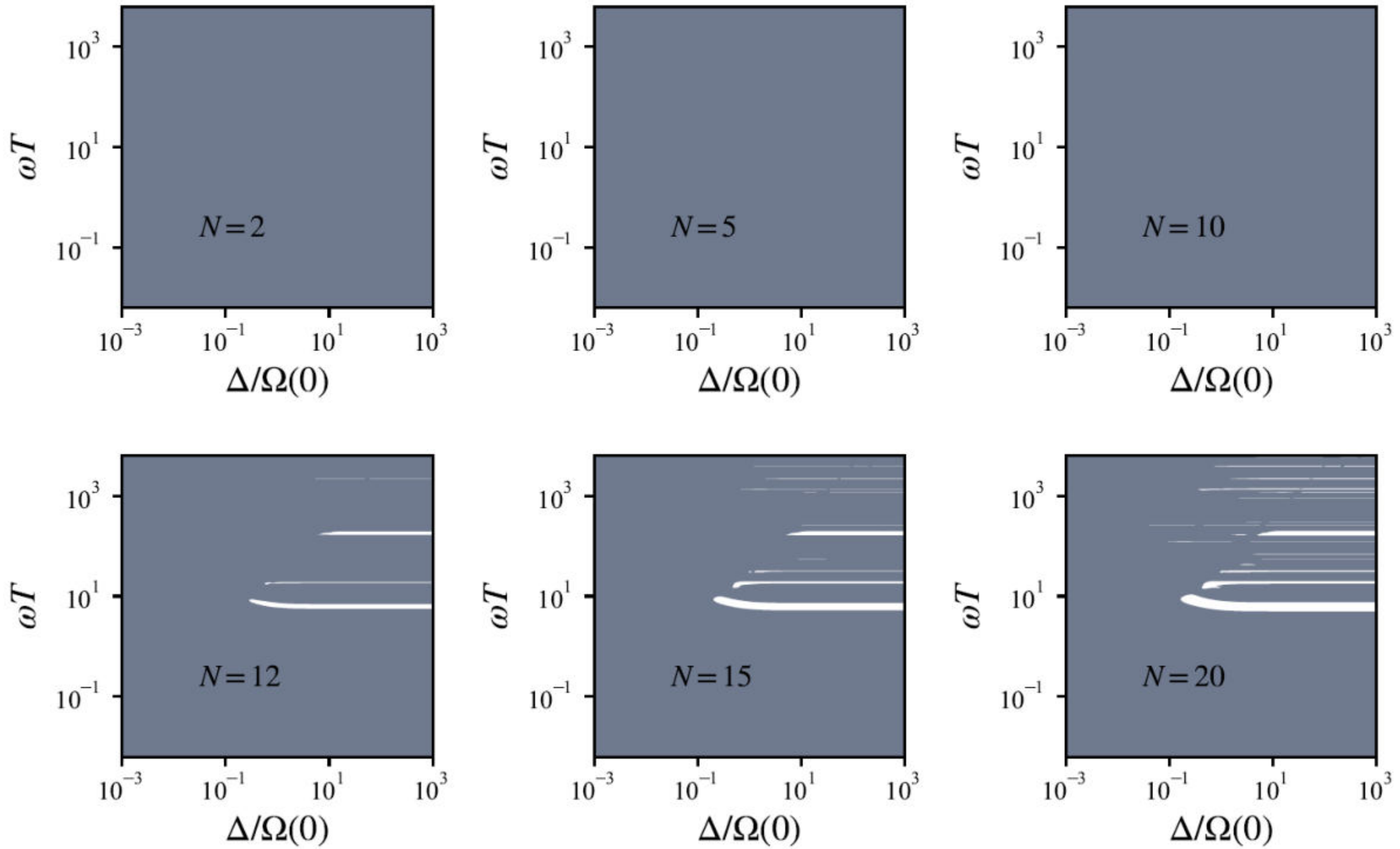
Quantum statistical enhancement of outcoupled work

grey: $\mathcal{E} > 1$ (enhanced)

white: $\mathcal{E} < 1$

$g = 0.01 \quad \alpha = 2142/T \quad \delta_t = 0.9$

$t_{\mathrm{on}} = (1-\delta_t)T/4 \quad t_{\mathrm{off}} = t_{\mathrm{on}} + \delta_t T/2$



- Enhancement is present at small N, indep. of other parameters.
- Regions with no enhancement vanish as $\Delta/\Omega(0) \to 0$.

Ergotropy of the external system

Ergotropy: Maximum extractable energy by a cyclic unitary.

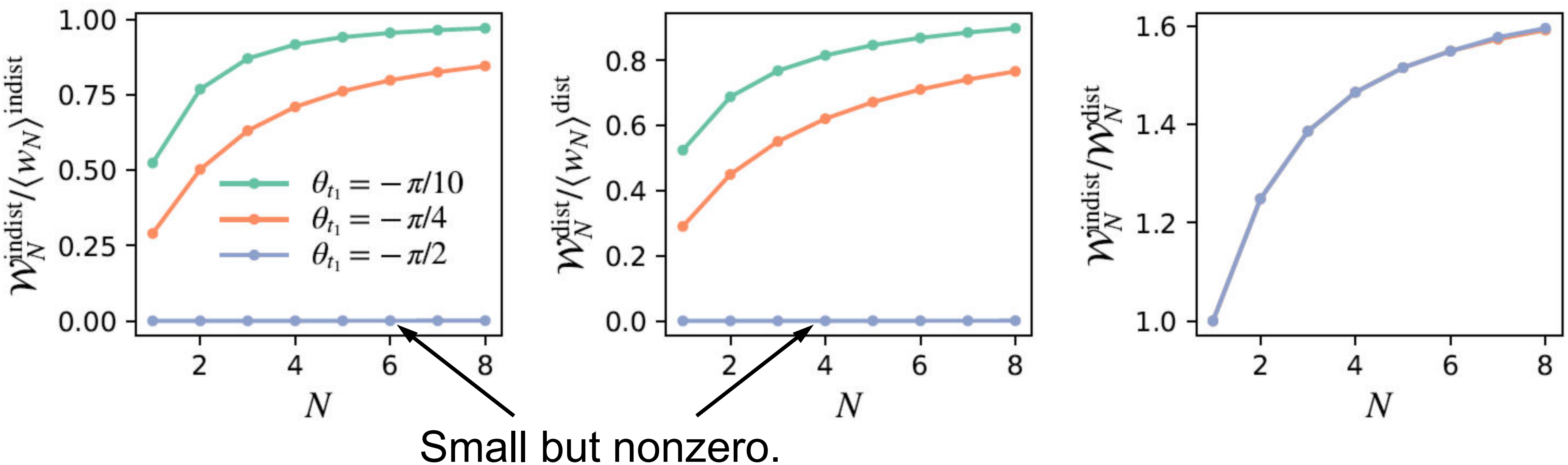
Lenard, J. Stat. Phys. (1978); Pusz & Woronowicz, Commun. Math. Phys. (1978).

$$\mathcal{W}_N \equiv \mathrm{Tr}_S[\rho_T^S H_S] - \min_{U_S} \mathrm{Tr}_S[U_S \rho_T^S U_S^\dagger H_S]$$

Measure of the “quality” of the energy stored for later use.

Passive st. (e.g., thermal st.): $\mathcal{W}_N = 0$

Ergotropy of the external system



· Ergotropy of the final st. of the external sys. *S* is nonzero unlike heat transfer.

➡ Engines indeed do work on the external sys. at quantum enhanced amount!

· Ergotropy for the indistinguishable bosonic engines is enhanced compared to the distinguishable case.

Performance of an ensemble of indistinguishable QHEs

1. Quantum enhancement of outcoupled work due to the statistical indistinguishability of bosonic work resources.

Enhancement: $\mathcal{E} \equiv \langle w_N \rangle^{\mathrm{indist}} / \langle w_N \rangle^{\mathrm{dist}} > 1$

2. $\mathcal{E} > 1$ is guaranteed for sufficiently small *N*.

GW, Venkatesh *et al*., arXiv:1904.07811 (2019).

Indistinguishable case

Engine Hamiltonian: $H_E(t) = 2\Omega(t)\, S_z + 2\Delta\, S_x$

$$S_x \equiv (a^\dagger b + b^\dagger a)/2 \qquad S_z \equiv (a^\dagger a - b^\dagger b)/2$$

linear sweep:
$$\Omega(t) = \begin{cases} \Omega(0) + vt & (0 \le t \le T/2) \\ \Omega(0) + v\,(T-t) & (T/2 \le t \le T) \end{cases}$$

Dipolar coupling: $H_C(t) = g_C(t)\, 2S_x \otimes V_S$

Adiabatic approximation:

$$U_R(t, t_0) \approx \sum_{m=-N/2}^{N/2} |m, \theta_t\rangle_{E\,E}\langle m, \theta_{t_0}|\, e^{-im\phi(t,t_0)}$$

$$\phi(t, t_0) = \int_{t_0}^{t} dt'\, 2E_{t'}$$

$$E_t \equiv \sqrt{\Omega(t)^2 + \Delta^2}$$

$$\tan\theta_t = -\Omega(t)/\Delta$$

$$v = -0.1\Omega(0)^2$$
$$\Omega(0) = 1$$
$$T = 20/\Omega(0)$$

Distinguishable case

Engine Hamiltonian: $$H_E(t) = 2\Omega(t)\sum_{j=1}^{N}\frac{\sigma_{j,\,z}}{2} + 2\Delta\sum_{j=1}^{N}\frac{\sigma_{j,\,x}}{2}$$

Dipolar coupling: $$H_C(t) = g_C(t)\,2\sum_{j=1}^{N}\frac{\sigma_{j,\,x}}{2}\otimes V_S$$

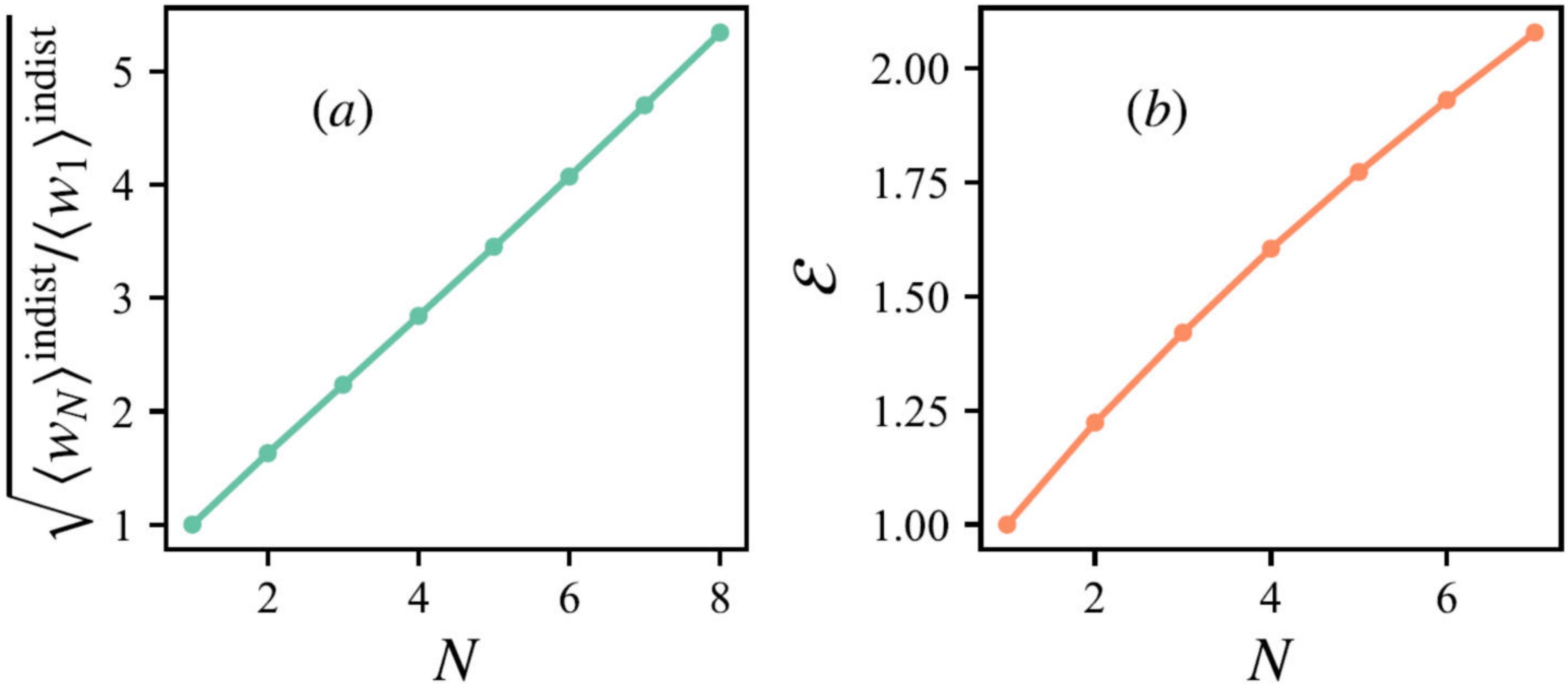
Adiabatic approximation:

$$U_R(t,t_0) \approx \bigotimes_{j=1}^{N}\sum_{m=-1/2}^{1/2}|m,t\rangle_{j\,j}\langle m,t_0|e^{-im\phi(t,t_0)}$$

Continuous non-perturbative coupling

For $\Delta = 0$

$g = 0.5$



p_i for a general coupling

$$p_i^{\text{indist}} = \sum_{t_0=0,T/2} 4|d_i(t_0)|^2 \langle m^2\rangle_{t_0} + |\tilde{c}_i^+(t_0)|^2 \left[\frac{N}{2}\left(\frac{N}{2}+1\right) - F_+(N, \beta_{t_0} E_{t_0})\right] + |\tilde{c}_i^-(t_0)|^2 \left[\frac{N}{2}\left(\frac{N}{2}+1\right) - F_-(N, \beta_{t_0} E_{t_0})\right]$$
$$+ 8\Re[d_i(0) d_i^*(T/2)] \langle m\rangle_0 \langle m\rangle_{T/2}, \tag{S23}$$

$$p_i^{\text{dist}} = \sum_{t_0=0,T/2} |d_i(t_0)|^2 \left[N + N(N-1)\tanh^2(\beta_{t_0} E_{t_0})\right] + |\tilde{c}_i^+(t_0)|^2 \left[\frac{N}{2}(1+\tanh(\beta_{t_0} E_{t_0}))\right] + |\tilde{c}_i^-(t_0)|^2 \left[\frac{N}{2}(1-\tanh(\beta_{t_0} E_{t_0}))\right]$$
$$+ 2\Re[d_i(0) d_i^*(T/2)] N^2 \tanh(\beta_c E_0) \tanh(\beta_h E_{T/2}). \tag{S24}$$

$$\tilde{c}_i^{\pm}(t_0) = \int_{t_0}^{t_0+T/2} dt\, g_C(t) \sin\theta_t\, {}_S\langle i|V_S^I(t)|0\rangle_S\, e^{\pm i\phi(t,t_0)},$$
$$d_i(t_0) = -\int_{t_0}^{t_0+T/2} dt\, g_C(t) \cos\theta_t\, {}_S\langle i|V_S^I(t)|0\rangle_S,$$