WORKSHOP AND SCHOOL ON GEOMETRIC ANALYSIS AND DISCRETE GEOMETRY

Date: February 10–13 ,2020 Place: Room 8101, KIAS

	10th (Mon.)	11st (Tue.)	12nd (Weds.)
10:00-11:00	Mason Pember	Mason Pember	Christian Müller
11:00-11:30		*Tea time*	
11:30-12:00	Yoshiki Jikumaru	Kosuke Naokawa	Hyeongki Park
12:00-14:30		*Lunch*	
14:30-15:20	Miyuki Koiso	Daehwan Kim	Shintaro Akamine
15:20-16:00		*Tea time*	
16:00-16:50	Joseph Cho	Juncheol Pyo	Jui-En Chang
16:50-17:00		*Break time*	
17:00-17:50	Sungmin Yoo	Jinwoo Shin	

	13rd (Thu.)
10:00-10:30	Denis Polly
10:30-11:00	*Tea time*
11:00-12:00	Christian Müller
	Lunch

1. School on Discrete Geometry

Feb. 10th (Mon.)

Discrete Omega surfaces

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Omega surfaces are a large class of surfaces that constitute an integrable system. Discovered by Demoulin in 1911, they are characterised by the existence of an enveloping isothermic sphere congruence. Examples include linear Weingarten surfaces and isothermic surfaces. In this talk, I shall present the smooth and discrete theories of these surfaces, utilising Laguerre geometry.

Stability problem of equilibrium discrete curves and surfaces modelled by anisotropic energy

Yoshiki Jikumaru, Kyushu University
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An equilibrium surface for anisotropic energy gives a mathematical model of soap bubbles and crystals and it is characterized by a constant anisotropic mean curvature (CAMC) surface. In this talk, we formulate its discretization for curves and surfaces from variational viewpoint inspired by the paper by Pothier and Rossman (J. reine angew. Math., 2002), and discuss its stability problem. We show that the regular polygons are the only equilibrium discrete curves for the length functional and show the instability of non-convex regular polygons. We also show the conservation law for discrete CAMC curves and a criterion for unstable discrete CAMC curves, and give a stability result for discrete CAMC surfaces which is a generalization of the result by Polthier and Rossman.

Feb. 11st (Tue.)

Discrete Weierstrass type representations

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The Weierstrass-Enneper representation has been used extensively to create interesting examples of minimal surfaces in Euclidean 3-space from holomorphic functions. It is well known that this mechanism can be interpreted as the Christoffel transform of a map into the unit sphere. Bobenko and Pinkall (1996) used this interpretation to obtain a Weierstrass representation for discrete minimal nets in Euclidean 3-space.

There are many other Weierstrass type representations for surfaces (smooth and discrete) with certain curvature in 3-dimensional space forms. In this talk we shall see how these can be unified using the theory of Omega surfaces.

Topologies and singularities of discrete developable surfaces

Kosuke Naokawa, Hiroshima Institute of Technology
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A ruled surface, which is generated by a continuous motion of a line in Euclidean 3-space $\mathbb{R}^3$, is called developable if its Gaussian curvature vanishes identically. Developable surfaces have the property of ruled surfaces locally isometric to $\mathbb{R}^2$ with the standard metric. Planes, cones, cylinders and tangential surfaces are typical examples. This property gives a natural idea for discretizing developable surfaces. In fact, a 'discrete' motion of a line, that is, a sequence of lines in $\mathbb{R}^3$ is called a dicrete developable surface if any adjacent two lines of the sequence lie in a plane in $\mathbb{R}^3$, as in the following figures:

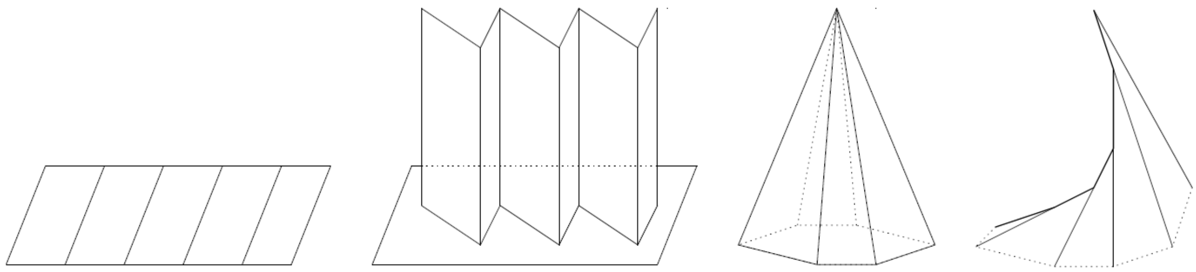

FIGURE 1. Discrete versions of a plane, cylindrical surface, cone and tangential surface, respectively.

In this talk, we give several results related to topologies and 'singularities' of discrete developable surfaces. This project is based on a joint work with Chirstian Müller (TU-Wien).

Feb. 12nd (Wed.)

Curvature for polyhedral surfaces

Christian Müller, Technische Universität Wien
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We explore discrete curvature notions in particular for polyhedral surfaces but also for special polygonal surfaces. We will lay our focus on vanishing and constant discrete and semi-discrete mean curvature notions which discretize minimal and cmc surfaces.

A hinged linkage mechanism that follows from discrete integrable equations

Hyeongki Park, Kyushu University
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We consider a family of linkage mechanisms called the Kaleidocycles, which consist n-copies of a rigid body joined together by hinges to form a ring. We formulate Kaleidocycles as discrete closed space curves, and present particular paths in the configuration space of them, which are governed by the semi-discrete mKdV and sine-Gordon equations.

Feb. 13rd (Thur.)

Discrete channel linear Weingarten surfaces

Denis Polly, Technische Universität Wien
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Channel surfaces are characterized by a number of equivalent properties. Recently, a discretisation preserving these properties has been suggested. We investigate discrete channel surfaces with a special linear relationship between their discrete principal curvatures.

Discrete Koenigs nets and applications

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Koenigs nets constitute a special subclass of conjugate nets with their characterizing properties being invariant under projective transformations. We will discuss structure preserving discretizations and how they appear in applications.

2. Workshop on Geometric Analysis

Feb. 10th (Mon.)

Stable anisotropic capillary hypersurfaces in a wedge

Miyuki Koiso, Kyushu University
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We study surfaces with constant anisotropic mean curvature in the domain bounded by a wedge in the three-dimensional euclidean space, which are critical points of "the anisotropic surface energy and the wetting energy" for variations preserving the enclosed volume. We show a uniqueness result for local minimizers of the total energy. The result is generalized to hypersurfaces in higher dimensional spaces. Moreover, the result is applied to the uniqueness problem for stable solutions of partially-crystalline variational problems for piecewise smooth hypersurfaces.

Timelike Thomsen surfaces

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We classify all timelike minimal surfaces with planar curvature lines. Then we characterize these surfaces in terms of their generating null curves. Finally, we examine the relationship between these surfaces and timelike Thomsen surfaces.

Relative Kähler-Ricci flow on a holomorphic family of strongly pseudoconvex domains

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In 2012, Schumacher proved that the variation of Kähler-Einstein metrics on a holomorphic family of canonically polarized compact Kähler manifolds is positive definite on the total space. In his paper, he showed that the geodesic curvature, which measures the positivity of the horizontal direction, satisfies a certain elliptic PDE. Applying the maximum principle to this PDE, he obtained the positivity. In 2013, Berman proved a parabolic version of the Schumahcer's result. More precisely, he proved that the geodesic curvature of a holomorphic family of canonically polarized compact Kähler manifolds satisfies a parabolic equation. A parabolic maximum principle implies that the positivity of the geodesic curvature is preserved along the Kähler-Ricci flow. In this talk, we will briefly introduce the results of Schumacher and Berman and show how to apply Berman's method to a holomorphic family of strongly pseudoconvex domains, which are noncompact complete Kähler manifolds. This is joint work with Young-Jun Choi.

Feb. 11st (Tue.)

Properties of the solitons for the inverse mean curvature flow

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The inverse mean curvature flow has been extensively studied not only as a type of geometric flows, but also for its applications to geometric inequalities. Analyzing special solutions of geometric flow is a natural way to understand, and so does the inverse mean curvature flow. In this talk, we focus on the homothetic and translating solitons for the inverse mean curvature flow that are self-similar solutions deformed by only homothety and translation under the flow, respectively. To be specific, we introduce several examples of the solitons and the incompleteness for the solitons are observed from several examples and then, the incompleteness of any translating soliton and the homothetic solitons with restrict homothetic ratio, namely, $C < \frac{1}{n}$, can be proved by applying maximum principle. Their area growths are obtained.

Solitons of the mean curvature flow

Juncheol Pyo, Pusan National University
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Translating solitons and self shrinkers are solitons of the mean curvature flow. They are not only special solutions of the MCF but blow-up models of singularities of MCF. In this talk, we firstly introduce a half-space type theorem of translating solitons. More precisely, we prove that complete translating solitons can lie on the upper part of a hyperplane and cannot lie on the lower part of it. Secondly, we introduce a rigidity of self-shrinker with free boundary in a ball. We prove that any graphical self-shrinker with free boundary in a ball is flat disk passing through the center of the ball.

The second Yamabe invariant

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The Yamabe invariant is a crucial factor in solving the Yamabe problem. In this talk, we introduce the generalization of the Yamabe invariant. We define the second Yamabe invariant as the infimum of the second eigenvalue of the Yamabe operator over the metrics conformal to g and of volume 1. We discuss when it is attained and its properties.

Feb. 12nd (Wed.)

Duality of boundary value problems for minimal and maximal surfaces

Shintaro Akamine, Nagoya University
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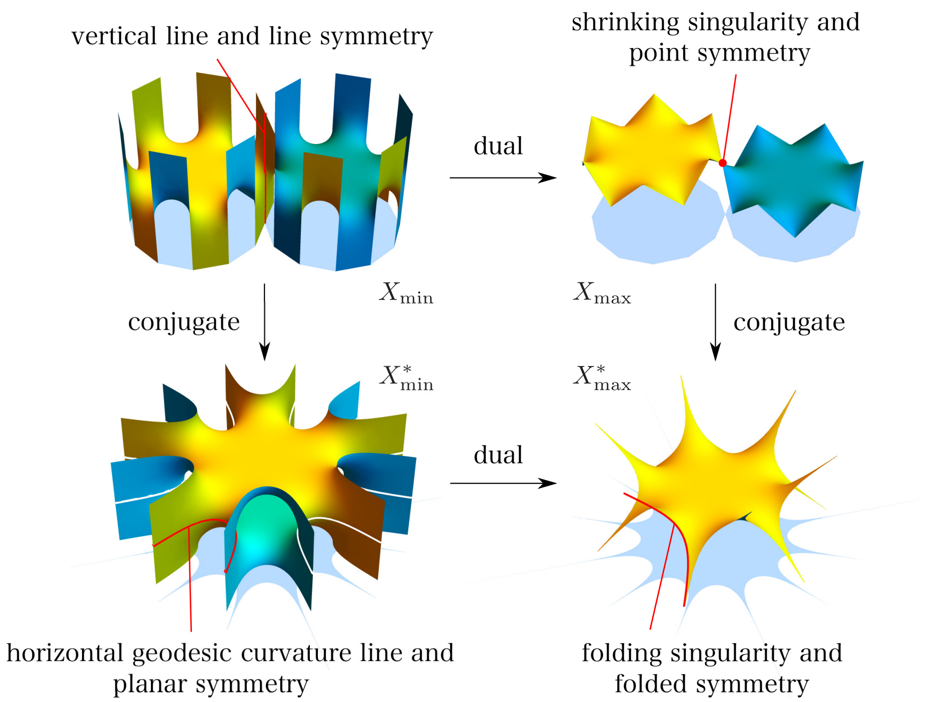
In 1966, Jenkins and Serrin gave existence and uniqueness results for infinite boundary value problems of minimal surfaces in the Euclidean space, and after that such solutions have been studied by using the univalent harmonic mapping theory. In this talk, we show that there exists a one-to-one correspondence between solutions of infinite boundary value problems for minimal surfaces and those of lightlike line boundary problems for maximal surfaces in the Lorentz-Minkowski space. We also investigate some symmetry relations associated with the above correspondence together with their conjugations as below. This talk is based on the preprint arXiv:1909.00975, which is the joint work with Hiroki Fujino (Nagoya University).



The uniqueness of self-shrinking networks with 2 enclosed regions

Jui-En Chang, National Taiwan University
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The network flow is a generalization of the curve shortening flow from curves to planar networks. To study the formation of singularities, solutions which move by homothety scaling plays an important role. Such network is called a regular shrinker. Up to now, only the shrinker with less than 2 triple junctions or 1 enclosed region is classified. In this talk, I'll present our recent result about the classification of regular shrinker with 2 enclosed regions. This is a joint work with Dr. Yang-Kai Lue.