QCD

- QCD Lagrangian

$$\underline{\mathcal{L}_{\text{QCD}} = \bar{q}(i\not{D} - m)q - \frac{1}{4}G^{\mu\nu,a}G^{a}_{\mu\nu}}$$

$$\begin{aligned} D_\mu &= \partial^\mu - igT^a A^a_\mu, \\ G^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^a_\mu A^b_\nu \end{aligned}$$

- QCD Beta function : calculated as a negative value

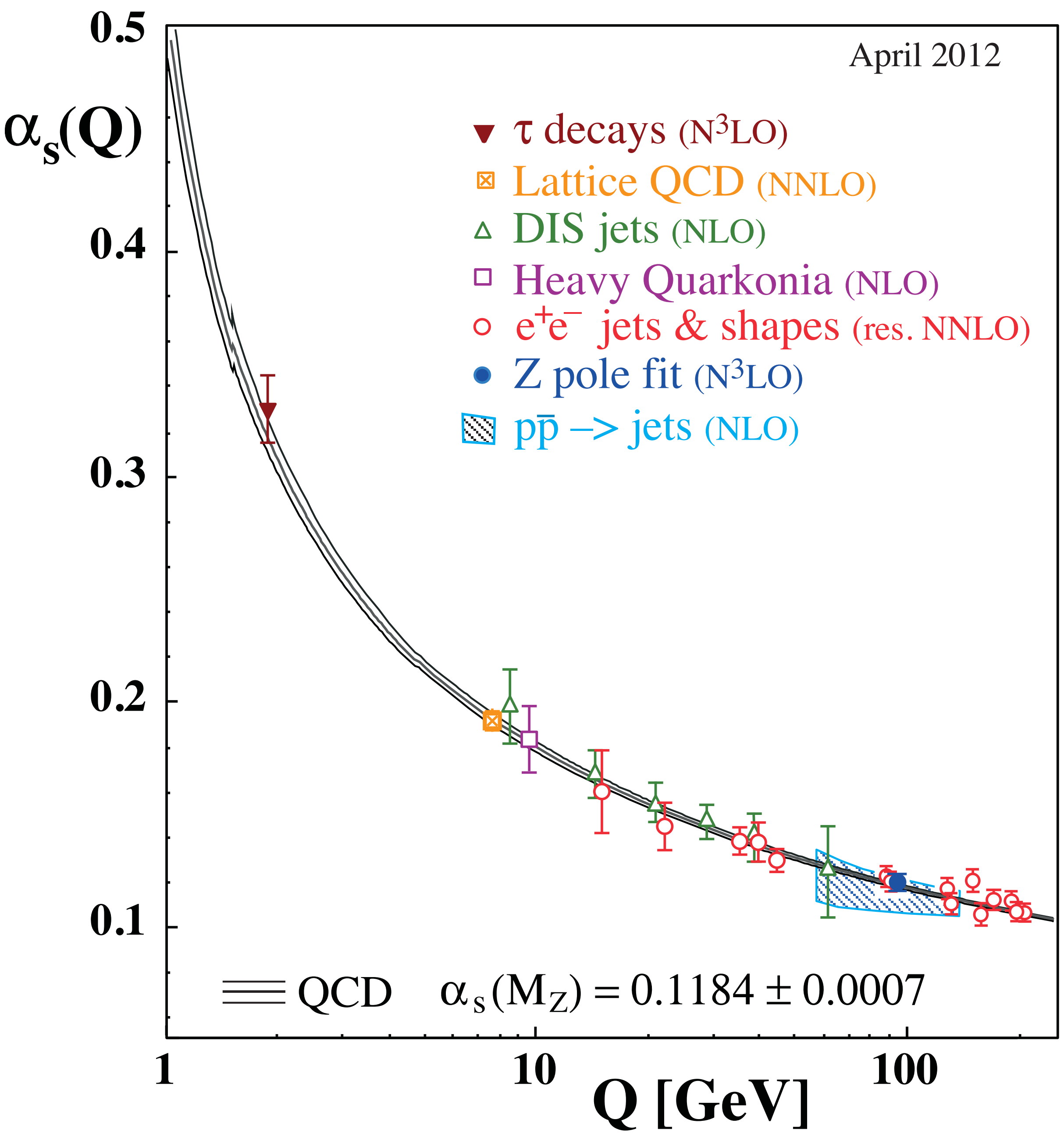
$$\frac{d}{d\ln\mu}g(\mu) = \underline{\beta(g(\mu))}$$

$$\beta(g) = -g\sum_{k=0}\beta_k\left(\frac{\alpha_s}{4\pi}\right)^{k+1} = \underline{-\beta_0\frac{g^3}{16\pi^2} - \beta_1\frac{g^5}{(16\pi^2)^2}} + \cdots$$

$$\beta_0 = \frac{11N_c - 2n_f}{3}, \quad \beta_1 = \frac{34}{3}N_c^2 - \frac{10}{3}N_c n_f - 2C_F n_f$$

Coupling Constant

$$\alpha_s(\mu) = \frac{g^2(\mu)}{4\pi} = \frac{4\pi}{\beta_0 \ln \frac{\mu^2}{\Lambda^2_{QCD}}}\left[1 - \frac{\beta_1 \ln \ln \frac{\mu^2}{\Lambda^2_{QCD}}}{\beta_0^2 \ln \frac{\mu^2}{\Lambda^2_{QCD}}}\right]$$



$$\alpha_s(\mu \to \Lambda_{QCD}) \to \infty$$

Confinement : long distance interactions

$$\alpha_s(\mu \to \infty) \to 0$$

Asymptotically Free : short distance interactions

- Operator Product Expansion (OPE)

$$\langle \mathcal{O}_1(x)\mathcal{O}_2(0)\rangle = \sum_n C_{12}^n(x,\mu)\langle \mathcal{O}_n(\mu)\rangle$$

$C_{12}^n(x,\mu)$: Complex function including Wilson coefficient
Expanded by the short distance *x*

$\langle \mathcal{O}_n(\mu)\rangle$: Includes all the information on the long distance interactions

EX) Hadronic tensor for DIS

$$W_{\mu\nu} = \frac{1}{2\pi}\int d^4z e^{iq\cdot z}\langle N|J_\mu^\dagger(z)J_\nu(0)|N\rangle$$

$z \to 0$: Short distance expansion

- QCD Factorization Theorem
  - Systematically separate the short and long distance interactions

EX) Factorization theorem of DIS structure function

$$F_1(x) = \int_x^1 \frac{dz}{z} H(Q^2, z, \mu) f_{q/p}(\frac{x}{z}, \mu)$$

- Describe the short distance interactions
- Corresponding to Wilson coefficient
- Can be computed by perturbation

- Describe the long distance interactions
- Corresponding to the matrix element of the nonlocal operator
- Cannot be computed, instead fit to experiments

- Structure function has no renormalization scale variance

- Perturbative QCD has a predictive power