Nonclassicallity of Two-mode Squeezed Vacuum State of an Oscillating Quantize Scalar field in the FRW Universe

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Abstract

We in our earlier series of papers have used the two-mode formalism of squeezed vacuum state to represent the massive scalar field in the FRW Universe using the semiclassical theory of gravity and successfully explained the phenomenon of particle production and thus the concept of preheating/reheating after inflation. Therefore, it turns out necessary to examine whether the two-mode quantized squeezed vacuum state of the scalar field exhibits classical or non-classical nature in the cosmological context. We in our present article, have made use of the criterion suggested by CT Lee [1], for the existence of nonclassical effects in two-mode states and calculated a cosmological $\mathcal{D}$ parameter with the associated cosmological parameters, to examine the non-classical nature of the two-mode squeezed vacuum state $|\xi_2\rangle$ after inflation during the oscillatory phase of the scalar field.

Introduction

At present, in-spite of the numerous efforts to reconcile General Relativity and Quantum Mechanics, we still do not have a complete conclusive theory of Gravity. Hence, as an approximation to quantum gravity, the gravitational effects of quantum matter fields are often described by a semiclassical theory wherein the gravitational field is given a classical definition (not quantised) in harmony with the axioms of GTR as curvature in the geometry of spacetime and the field which is present in spacetime are treated quantum mechanically in agreement with the notions of QFT. The semiclassical theory is usually formulated as a semiclassical Einstein field equation:

$$G_{\alpha\beta} = \frac{8\pi}{m_p^2}\langle\varphi|T_{\alpha\beta}|\varphi\rangle, \qquad (1)$$

where $G_{\alpha\beta}$, is the Einstein tensor and $T_{\alpha\beta}$ denotes the renormalized expectation value of the energy-momentum tensor of the matter field (acting as the source of gravity) under consideration. Thus, the semiclassical theory of gravity (SG) seems to be a viable method to understand quantum effects and quantum phenomenon in the early universe where quantum gravity effects are considered to be negligible.

SG and Scalar Field Dynamics in $|\xi_2\rangle$ State

In the chaotic inflationary scenario, the source of the gravitational field is the massive scalar field φ defined on a quadratic potential, governed by the time-dependent Schrödinger equation formulated as:

$$i\frac{\partial}{\partial t}|\xi_2(\varphi,t)\rangle = \hat{\mathcal{H}}_m(\varphi,t)|\xi_2(\varphi,t)\rangle, \qquad (2)$$

where $|\xi_2>$ is the matter field quantum state (TMSV) represented as:

$$|\xi_2\rangle = Z_{ab}(r_s,\Phi)|0,0\rangle, \qquad (3)$$

$$Z_{ab}(r_s,\Phi) = \exp\frac{r_s}{2}\left(e^{-i\Phi}\hat{a}\hat{b} - e^{i\Phi}\hat{a}^\dagger\hat{b}^\dagger\right), \qquad (4)$$

here, r_s and Φ are the corresponding squeezing parameter and squeezing angle and $\hat{\mathcal{H}}_m$ is the Hamiltonian for the matter field. As a concrete model, we shall consider a massive inflaton φ_0, the homogeneous and isotropic field corresponding to the space average of the massive scalar field $\varphi(\mathbf{x},\tau)$. Now, the scalar field can be decomposed into the inflaton and fluctuations; the inhomogeneous and anisotropic field as:

$$\varphi(\tau,\mathbf{x}) = \varphi_0(\tau) + \zeta(\tau,\mathbf{x}). \qquad (5)$$

In keeping the analogy with point mechanics and upon quantisation the mode-decomposition yields the corresponding Hamiltonian (a collection of time-dependent harmonic oscillators) for the massive scalar field as obtained as:

$$\mathcal{H}_m(\varphi) = \sum_{\mathbf{k}}\left[\frac{\hat{\pi}_{\mathbf{k}}^2}{2\mathcal{S}^3(\tau)} + \frac{\mathcal{S}^3(\tau)}{2}\left(m^2 + \frac{k^2}{\mathcal{S}^2(\tau)}\right)\hat{\zeta}_{\mathbf{k}}^2\right]. \qquad (6)$$

The Friedmann equation in the semiclassical approximation in $|\xi_2\rangle$, takes the following form:

$$\left(\frac{\dot{\mathcal{S}}(\tau)}{\mathcal{S}(\tau)}\right)^2 = \frac{8\pi}{3m_p^2}\frac{1}{\mathcal{S}^3(\tau)}\langle\xi_2|\hat{\mathcal{H}}_m|\xi_2\rangle. \qquad (7)$$

The eigenstates of the Hamiltonian are Fock states

$$\hat{a}_i^\dagger(t)\ \hat{a}_i(t)|n_i,\phi_i,t\rangle = n_i|n_i,\phi_i,t\rangle,\ i=1,2 \qquad (8)$$

where $\hat{a}_i{}^\dagger$ is the modes creation operators and $\hat{a}_i$ is the associated annihilation operators. These ladder operators for two-mode states can be defined as:

$$\hat{a}(t) = \zeta_1^*(t)\ \hat{\pi} - \mathcal{S}^3\ \dot{\zeta}_1^*(t)\ \hat{\zeta},\quad \hat{a}^\dagger(t) = \zeta_1(t)\ \hat{\pi} - \mathcal{S}^3\ \dot{\zeta}_1(t)\ \hat{\zeta},$$
$$\hat{b}(t) = \zeta_2^*(t)\ \hat{\pi} - \mathcal{S}^3\ \dot{\zeta}_2^*(t)\ \hat{\zeta},\quad \hat{b}^\dagger(t) = \zeta_2(t)\ \hat{\pi} - \mathcal{S}^3\ \dot{\zeta}_2(t)\ \hat{\zeta}, \qquad (9)$$

where, ζ_1 and ζ_2 are mode functions of field corresponding to two different modes of the scalar field. It was found that the semiclassical Einstein quantum gravity equation in $|\xi_2\rangle$ state leads to the same power-law expansion $t^{2/3}$ as that of the matter dominated era in an oscillatory phase of the scalar field after inflation [2]. Moreover, the particle created due to the quantum fluctuation of the scalar field in $|\xi_2\rangle$ was obtained as [3]:

$$\mathcal{N}_{|\xi_2\rangle}(t,t_0) = (1+2\sinh^2 r)\frac{(t-t_0)^2}{4m^2\tau_0^4} + \sinh^2 r + \frac{\sinh 2r}{4m^2t_0^4}(t-t_0)^2 \qquad (10)$$

Since, we used the $|\xi_2\rangle$ state to study particle creation during the oscillatory phase of scalar field, it is important to examine the nature of this quantum optical state in the cosmological context (whether the state exhibit classical or nonclassical feature) with associated cosmological parameters.

Criterion for nonclassical effects in $|\xi_2\rangle$ state

We made use of the criterion put forward by Ching Tsung Lee [1] for the existence of nonclassical effects in two-mode states given as:

$$\mathcal{D}_{12}^{(2)} = \mathcal{C}_1^{(2)} + \mathcal{C}_2^{(2)} - \mathcal{C}_{12}^{(2)} + (\langle n_1\rangle + \langle n_2\rangle)^2 < 0 \qquad (11)$$

where we call the $\mathcal{D}$ parameter as the cosmological $\mathcal{D}$ parameter. Here, $\mathcal{C}_1^{(2)}$ measures the degree of correlation between two particles from the same mode and is given by following expression for the two modes as:

$$\mathcal{C}_1^{(2)} = \langle\hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a}\rangle - \langle\hat{a}^\dagger\hat{a}\rangle^2;\ \ \mathcal{C}_2^{(2)} = \langle\hat{b}^\dagger\hat{b}^\dagger\hat{b}\hat{b}\rangle - \langle\hat{b}^\dagger\hat{b}\rangle^2, \qquad (12)$$

and can be rewritten in terms of particle-number moments as:

$$\mathcal{C}_i^{(2)} = \langle n_i^{(2)}\rangle - \langle n_i\rangle^2,\quad i=1,2 \qquad (13)$$

We have $\mathcal{C}_i^{(2)} = 0$, for a coherent state; $\mathcal{C}_i^{(2)} > 0 \Rightarrow$ intramode particle bunching, which is always true for classical states and, in contrast, we have intramode particle antibunching when $\mathcal{C}_i^{(2)} < 0$, which is possible only for nonclassical states. Analogously, $\mathcal{C}_{12}^{(2)}$ the correlation function between two particles from different modes is defined as:

$$\mathcal{C}_{12}^{(2)} = \langle\hat{a}^\dagger\hat{b}^\dagger\hat{b}\hat{a}\rangle - \langle\hat{a}^\dagger\hat{a}\rangle\ \langle\hat{b}^\dagger\hat{b}\rangle;\ \mathcal{C}_{12}^{(2)} = \langle n_1n_2\rangle - \langle n_1\rangle\langle n_2\rangle \qquad (14)$$

$\mathcal{C}_{12}^{(2)} > 0 \Rightarrow$ intermode particle bunching and $\mathcal{C}_{12}^{(2)} < 0 \Rightarrow$ intermode particle antibunching. Now, in the case where the individual modes are coherent states of equal intensity; so that $\mathcal{C}_1^{(2)} = \mathcal{C}_2^{(2)} = 0$ and $\langle n_1\rangle = \langle n_2\rangle$. Then, Eq. (11) reduces to:

$$\mathcal{D}_{12}^{(2)} = -2\mathcal{C}_{12}^{(2)}; \qquad (15)$$

and the criterion for the existence nonclassical effects becomes $\mathcal{C}_{12}^{(2)} > 0$, which implies intermode particle bunching.

Using the above criterion, we examined the $|\xi_2\rangle$ state and obtained the expression for correlations functions in the oscillatory phase of the scalar field as: (using $|\xi_2\rangle$ state definition, identities refer Eq. (3), (4) and (9) along with the approximation ansatz with x=mt)

$$\mathcal{C}_1^{(2)} = \mathcal{C}_2^{(2)} = \Big[32x^4x_0^4\sinh^4(r) + 2x^2x_0^2(x-x_0)^2(-4\cosh(2r) + 3\cosh(4r) + 3)$$
$$-\left(4x^2x_0^2\sinh^2(r) + (x-x_0)^2\cosh(2r)\right)^2 + 3(x-x_0)^4\cosh^2(2r)\Big]\frac{1}{16x_0^8} > 0; \qquad (16)$$

$$\mathcal{C}_{12}^{(2)} = \frac{(x^4 + (2x^4+1)x_0^4 - 4x_0x^3 + 6x_0^2x^2 - 4x_0^3x)\sinh^2(2r)}{8x_0^8} > 0 \qquad (17)$$

and on substituting in the criterion Eq.(11), we obtain:

$$\mathcal{D}_{12}^{(2)} = \frac{1}{4x_0^8}\Big[2x_0^2x^2\left(x^2(\cosh(4r) - \cosh(2r)) + x^2 + 3\right) - 4x_0^3x(x^2(\cosh(4r)$$
$$-\cosh(2r)) + x^2 + 1) + x_0^4(2x^2\cosh(4r) - 2\left(2x^4 + x^2\right)\cosh(2r) + 4x^4$$
$$+ 2x^2 + 1) + x^4 - 4x_0x^3\Big] \leq 0 \qquad (18)$$

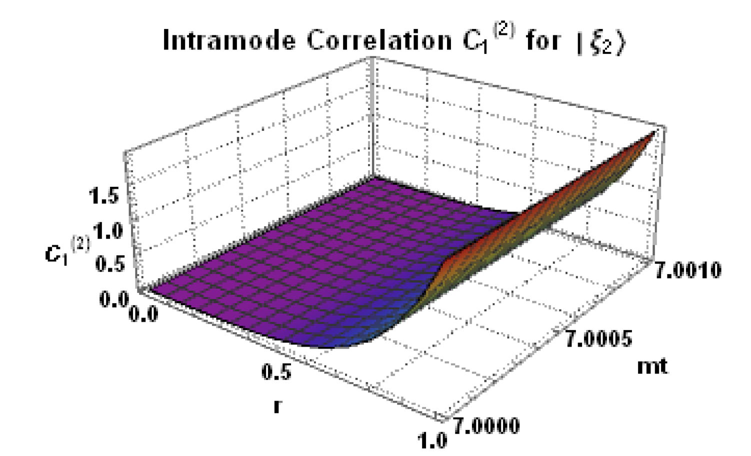

Figure 1: Variation of $\mathcal{C}_1^{(2)}$ with r and mt.

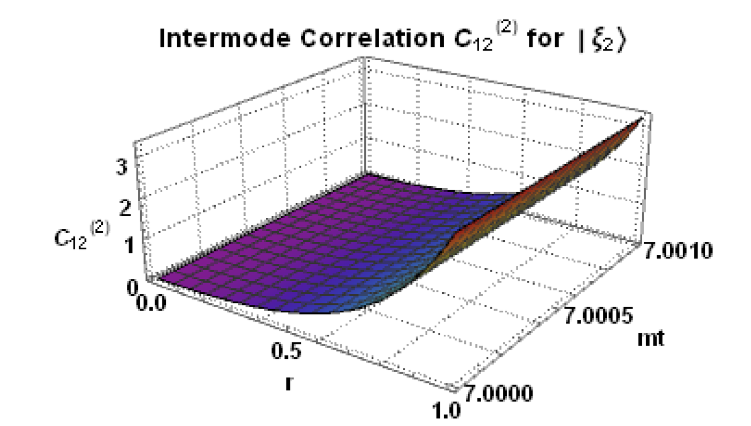

Figure 2: Variation of $\mathcal{C}_{12}^{(2)}$ with r and mt.

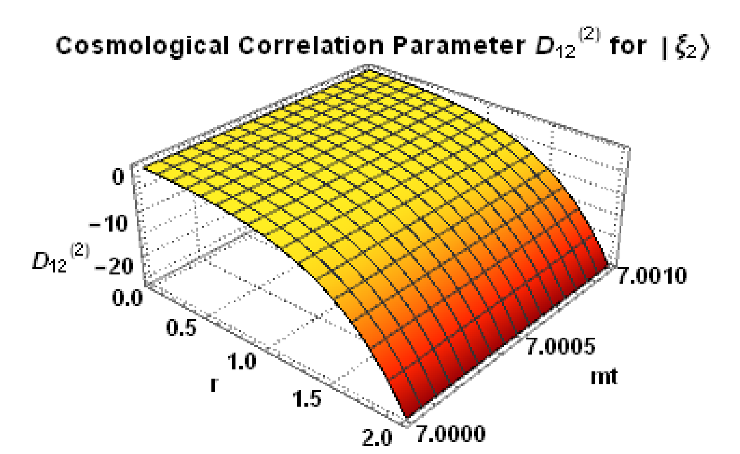

Figure 3: Variation of $\mathcal{D}_{12}^{(2)}$ with r and mt.

Discussion

From Eq. (18) we see that the $|\xi_2\rangle$ state are always nonclassical, except when $r=0$. But from Eq. (16), we see that each mode by itself is definitely a classical state. Therefore, the correlation between particles from two different modes plays the exclusive role in overcoming the classical effects in individual modes and rendering the two-mode squeezed vacuum states nonclassical. Furthermore, from Eq. (17), we see clearly that the intermode correlation signifies particle bunching. Thus we conclude that, the analysis done with cosmological $\mathcal{D}$ parameter shows that the $|\xi_2\rangle$ is consistent with its nonclassical nature with the associated cosmological parameters during the oscillatory phase of the scalar field in the semiclassical theory of gravity.

References

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