# Generating Catalogues of Lensed Galaxies to Test Weak Lensing Pipelines

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## Lensing Formalism

Weak lensing is a broad category of gravitational lensing interactions, in which a source object (often a galaxy, located in the 'source plane') is distorted by some source of gravity, transforming its coordinates between some 'source plane' and 'lensing plane,' with a gravitational source existing between the two.' This coordinate transformation for our source between its source coordinates, $\beta$ and its lensing coordinates, $\theta$, is

$$\beta - (\theta - \alpha(\theta)) = 0 \tag{1}$$

Where $\theta$ is a two dimensional positional vector, and $\alpha(\theta)$ is the deflection a light ray experiences passing through the source plane. The deflection can be expressed as a first order derivative of the lensing potential $\alpha(\theta) = \nabla\psi(\theta)$. This lensing potential is given by the Poisson equation,

$$\nabla^2\psi(\theta) = 2\kappa(\theta) \tag{2}$$

Where $\kappa$ is the surface mass density in the lensing plane, also called the convergence. The deflection is a simple, first order effect that describes how light rays change coordinates; when dealing with objects of resolvable angular size, however, it becomes interesting to consider higher order effects. Taylor expanding Eq(1), we can write

$$\beta_i = A_{ij}\theta_j + \frac{1}{2}D_{ijk}\theta_j\theta_k \tag{3}$$

Where $A_{ij}$ and $D_{ijk}$ are built out of second and third order derivatives of the lensing potential, respectively. Eq(2) shows that the convergence will be included in $A_{ij}$, along with the shear, which is written as,

$$\gamma = \gamma_1\hat{i} + \gamma_2\hat{j} = \left(\frac{\psi_{,11} - \psi_{,22}}{2}\right)\hat{i} + \left(\psi_{,12}\right)\hat{j} \tag{4}$$

where the comma notation above denotes partial differentiation with respect to $\theta_i$ (ie, $\psi_{,i} = \frac{\partial\psi}{\partial\theta_i}$). Third order derivatives of the lensing potential are known as the flexion. The first flexion is expressed below as derivatives of the shear,

$$\mathcal{F} = \mathcal{F}_1\hat{i} + \mathcal{F}_2\hat{j} = \left(\gamma_{1,1} + \gamma_{2,2}\right)\hat{i} + \left(\gamma_{2,1} - \gamma_{1,2}\right)\hat{j} \tag{5}$$

Each of these components describes a different kind of distortion that weak lensing introduces to objects. Of particular interest to us is the shear, which measures a stretching effect, and the first flexion, which measures an arc-like distortion. We can see these effects more clearly in Figure 1 below.

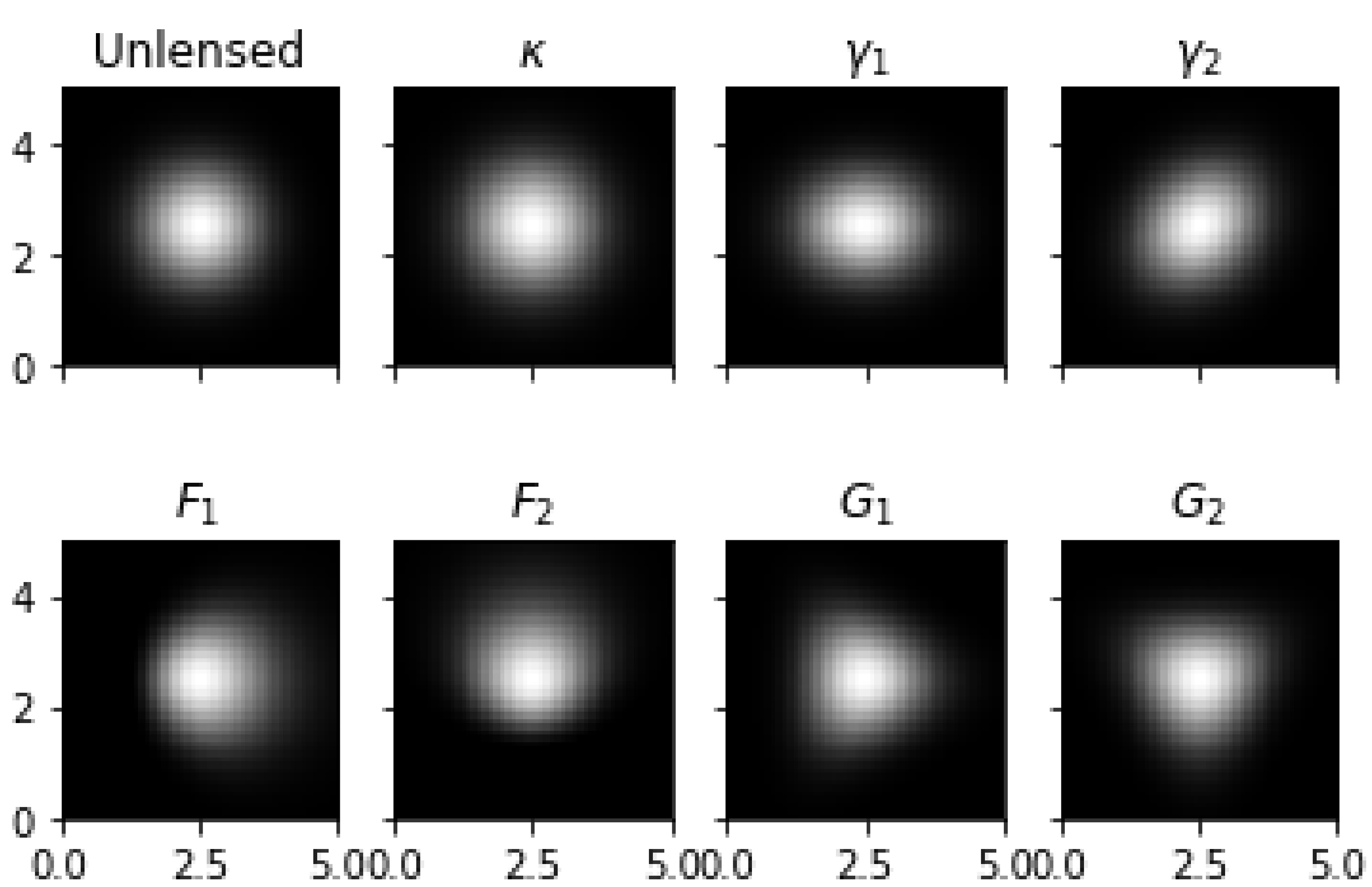



Fig. 1: The effects of weak lensing distortions. Here I draw an unlensed Gaussian galaxy with a radius of 1 arcsec, then lens it with a 10 % convergence/shear, and 0.28 arcsec$^{-1}$ flexion. Postage stamp is 5x5 arcseconds in size.

## Abstract

Gravitational lensing is a particularly powerful tool to study the mass distribution of galaxy clusters; sensitive to small scale structure in a way that few other probes are. Shear mapping is a common tool that uses first order lensing effects to describe mass distributions of lenses, an effort that may be significantly aided by future measurements of flexion, a second order lensing effect. To act as a test for weak lensing pipelines that incorporate flexion, we present a software tool for producing images of lensed galaxies with both first and second order lensing effects present. Realistic lensing systems are constructed from Multidark halo catalogues, treating dark matter halos as NFW profiles and constructing a 2D lensing potential, from which shear and flexion fields are constructed. Source galaxies are drawn as a Sersic profile with a randomly chosen intrinsic ellipticity, and are then convolved with a gaussian point spread function (PSF) to simulate atmospheric distortions. The resulting images form a mock catalogue of lensed images to be read by lensing pipelines, where tests can be run for a wide range of input flexion, PSFs, and galaxy models. We examine early tests of the `Lenser` pipeline, running a test of 100 images that shows agreement between inputted and extracted flexions, with residuals in the components of the first flexion of $\sigma_{RMS} = 0.0136, 0.0077$.

## Pipeline

1. The lensing potential $\psi$ of a given cluster, chosen from a Multidark halo catalogue, is calculated in the lensing plane. Clusters are treated as NFW profiles, with a lensing potential of

$$\begin{aligned} \psi(\theta) &= 4\kappa_s h(x) \\ \kappa_s &= \frac{\rho_s r_s}{\Sigma_{\text{crit}}} \\ h(x) &= \ln\left(\frac{x}{2}\right) + \begin{cases} 2\arctan^2\left(\sqrt{\frac{x-1}{x+1}}\right), & x > 1 \\ -2^2\left(\sqrt{\frac{1-x}{1+x}}\right), & x < 1 \\ 0, & x = 1 \end{cases} \\ x &= \frac{\theta}{r_s} \end{aligned} \tag{6}$$

2. Higher order parameters are computed through numerical differentiation of $\psi$.
3. Source galaxies are modeled as Sersic profiles with a random initial intrinsic ellipticity.
4. Source galaxies are transformed by the lensing parameters at their coordinates, iteratively solving for their coordinates in the lensing plane by Eq(3).
5. After lensing, galaxies are convolved with a gaussian PSF, then have a background and poisson noise added to simulate optical effects. The image data is then stored as FITS files, along with noise maps. These files make up a catalogue of lensed images that can be exported to lensing tools.
6. A key is produced with the lensing parameters for each galaxy, along with the mass distribution in the lensing plane.

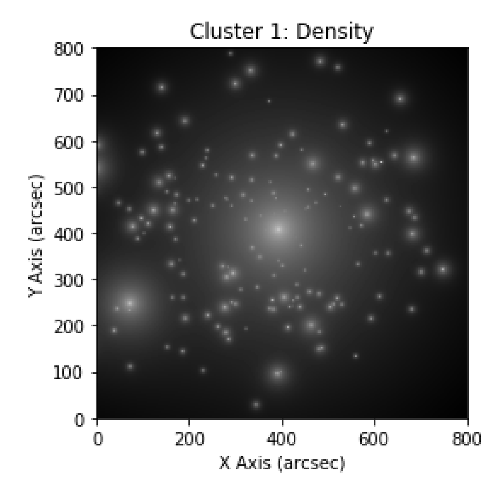



Fig. 2: The density plot of a selected MultiDark cluster, with $M = 6.64 \times 10^{14} M_\odot$ and 162 distinct halos.

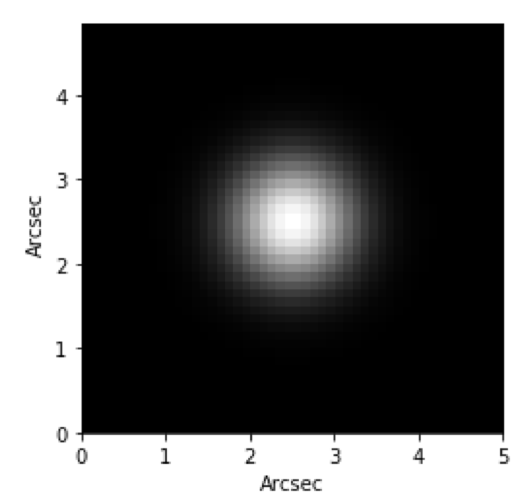
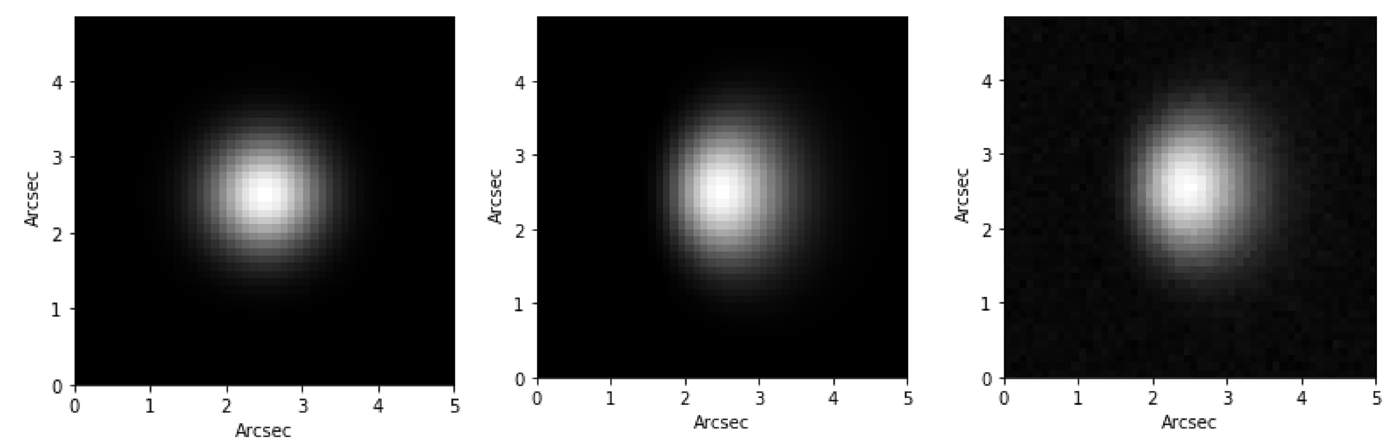



Fig. 3: A sample source galaxy, with a characteristic size of 1 arcsec

## Pipeline (Visualized)




A single galaxy, followed through the pipeline. Left image: A source galaxy with some amount of initial eccentricity. Middle: The galaxy transformed into the lensing plane. Right: The galaxy after being convolved with a PSF and with noise added.

## Testing

In developing this tool, I ran a number of tests with the software tool `Lenser`, in which I produced some number of lensed galaxies and handed them off for analysis by `Lenser`. I compare the input flexions in my tool to the values that `Lenser` returns. In the early stages of the project, this was a way to check that this tool was generating images correctly. As my project became more refined, these tests also served as a verification that `Lenser` was measuring flexion correctly. The most recent test, with 100 galaxies lensed by an SIS cluster without substructure, is presented here.

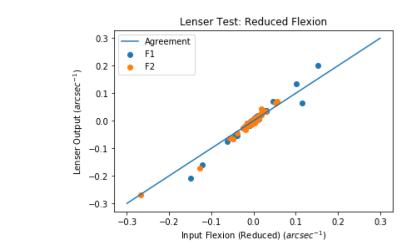



Fig. 7: We find agreement between the input and `Lenser` measured flexions.

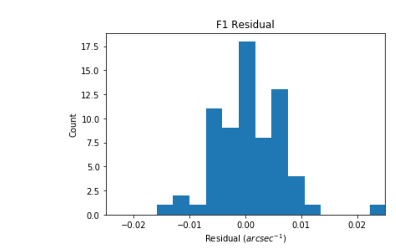



Fig. 8: The residuals of F1, with RMS value of 0.0136.

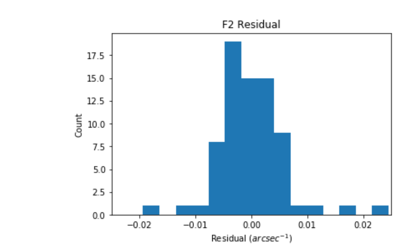



Fig. 9: The residuals of F2, with RMS value of 0.0077.

There is clearly a non-negligible bias present in these residuals, which seems to indicate the presence of some other factor/variable than the input flexion that is influencing the results we get out of `Lenser`. That said, the residuals are of order $10^{-2}$, which still indicates reasonable agreement here. As the pipeline becomes more developed it will be possible to create a wider range of tests for more weak lensing tools.

## References

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The `Lenser` code: https://github.com/DrexelLenser/Lenser

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