Power spectrum for scalar and tensor perturbations in Cuscuton bounce

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1. Introduction

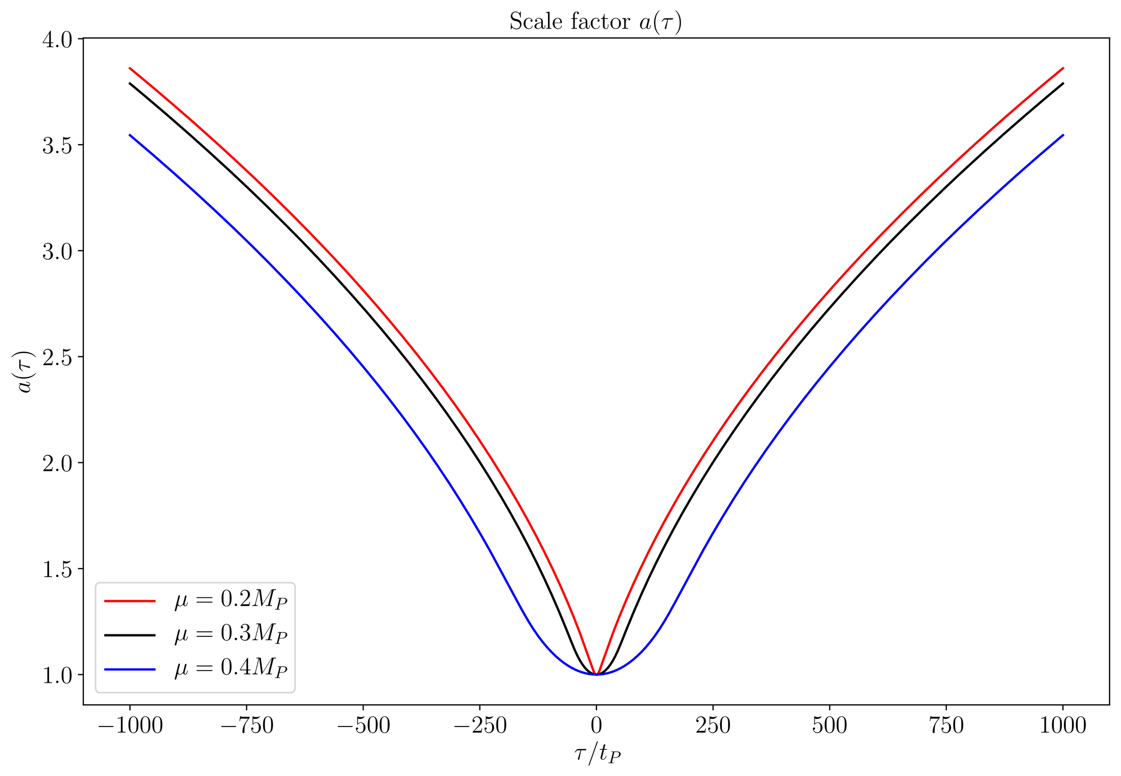
It has been recently shown that a cosmological bounce model based on Cuscuton gravity does not have any ghosts or curvature instabilities [1, 2]. We explore whether Cuscuton bounce can provide an alternative to inflation for generating near scale-invariant scalar perturbations. While a single field Cuscuton bounce generically produces a strongly blue power spectrum, we demonstrate that scale-invariant entropy modes can be generated with a spectator field. This work is based on [3], which also shows that the perturbations are stable, and considers different initial conditions.

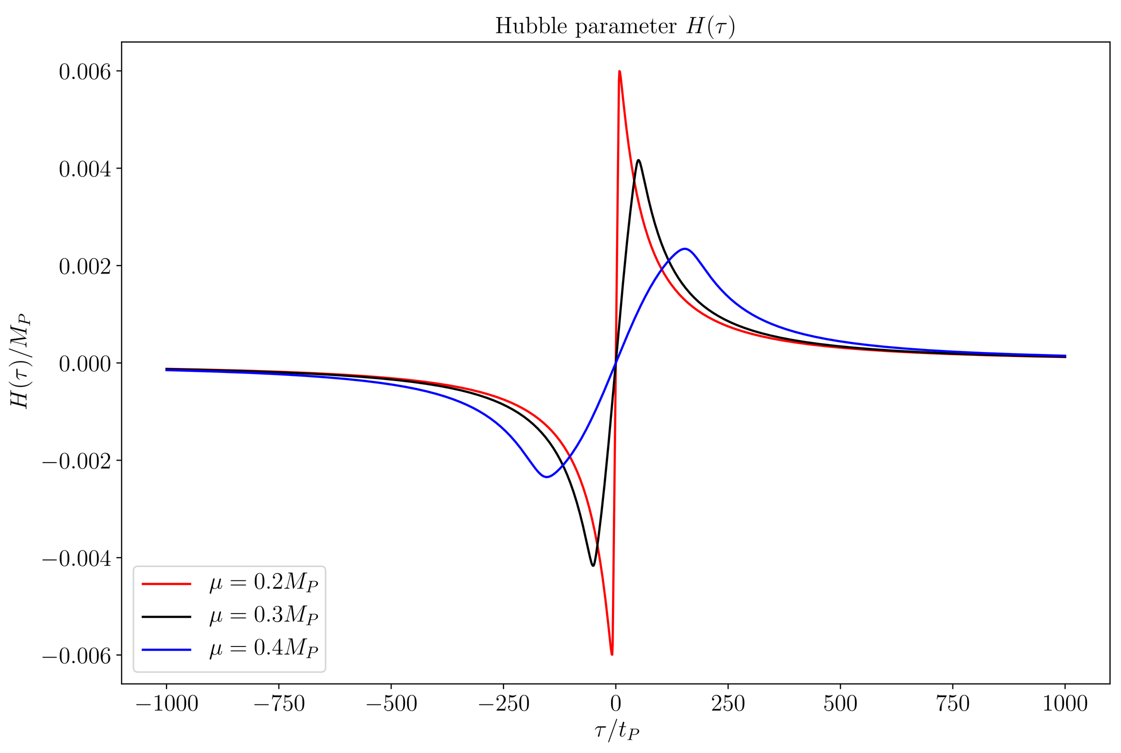
2. Background dynamics

The potential for Cuscuton is:

$$V(\varphi) = m^2(\varphi^2 - \varphi_\infty^2) - m^4\left[e^{(\varphi^2-\varphi_\infty^2)/m^2} - 1\right],$$

which generates the background:





3. Single field Cuscuton bounce

For a single field Cuscuton bounce, with Cuscuton field φ, matter field π, the action is given by

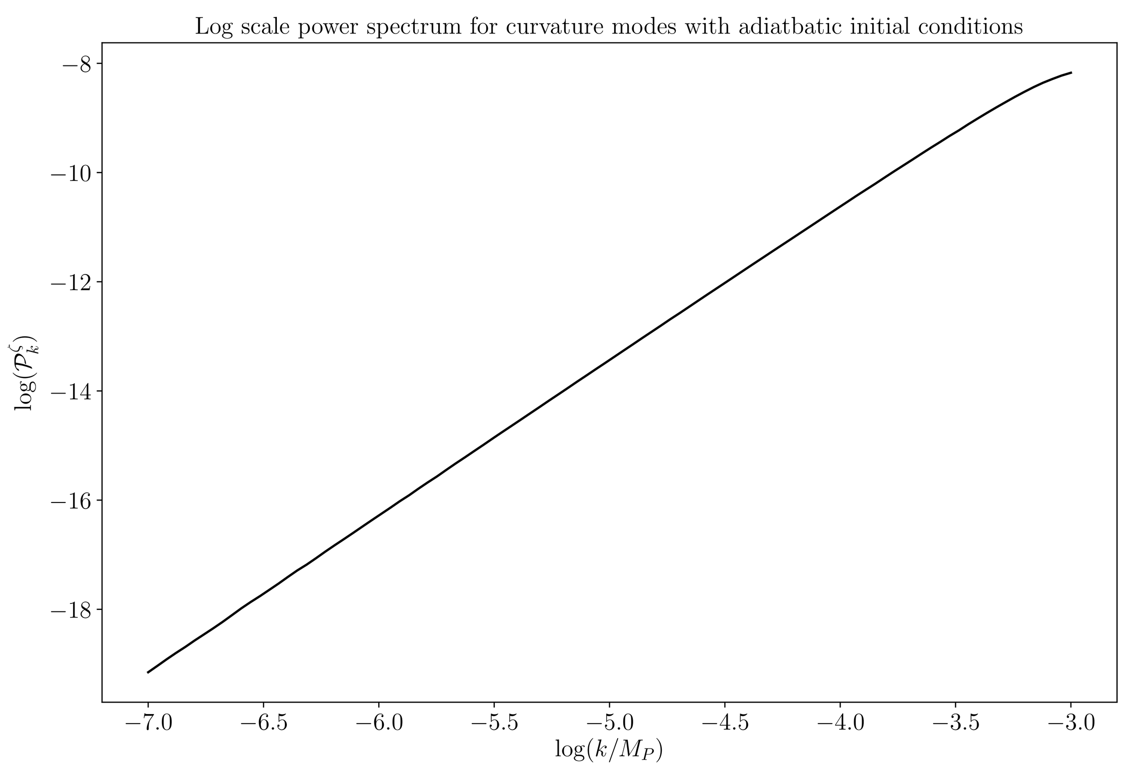
$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - \frac{1}{2}D_\mu\pi D^\mu\pi - \mu^2\sqrt{-D_\mu\varphi D^\mu\varphi} - V(\varphi)\right],$$

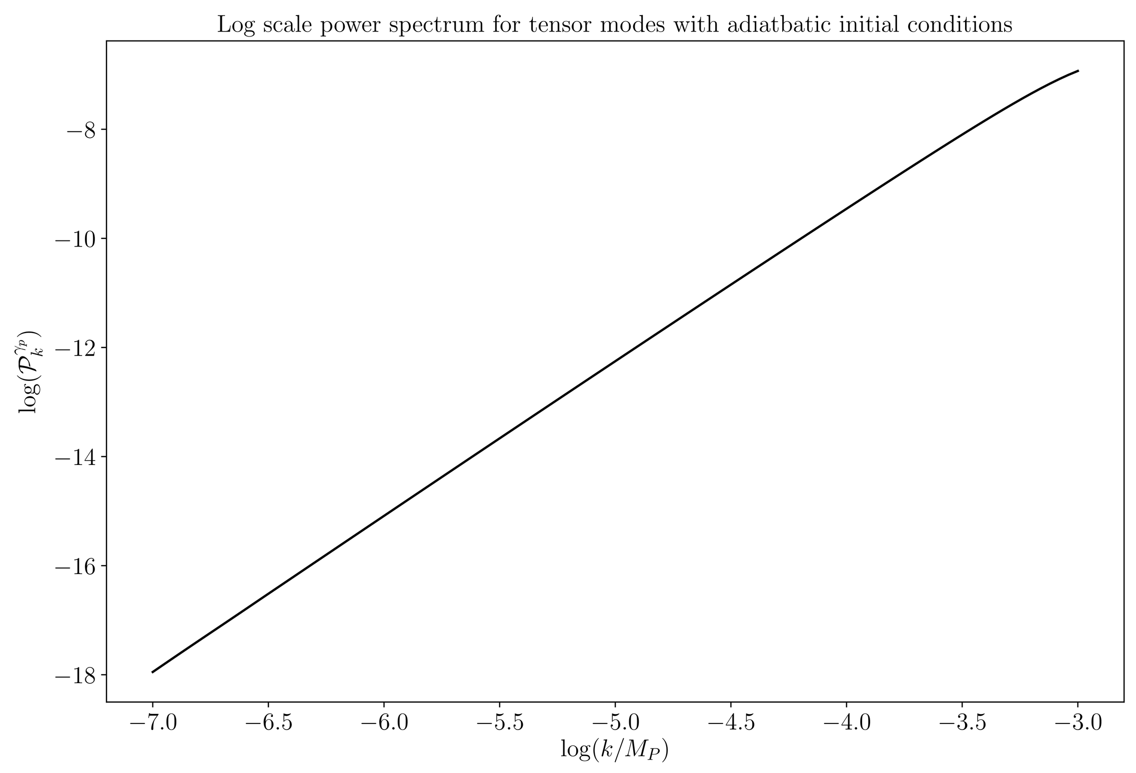
We consider scalar and tensor perturbations to the metric and perturbations to the Cuscuton field $\varphi = \varphi_0 + \delta\varphi$ while working in the unitary gauge with $\delta\pi = 0$. For curvature perturbations ζ and tensor perturbations γ_{ij}, recasting into Mukhanov-Sasaki variables in Fourier space, with $v_k = M_P z(\tau,k)\zeta_k$ and $v_p = a(\tau)M_P\gamma_p/2$ for each polarization mode, the equation of motions are given by

$$v_k'' + \left(c_s^2k^2 - \frac{z''}{z}\right)v_k = 0, \quad v_p'' + \left(k^2 - \frac{a''}{a}\right)v_p = 0.$$

Solutions to these equations were found to be stable , and the power spectra for each of these are given below. Assuming adiabatic vacuum initial conditions, both spectra are strongly blue.

$$\mathcal{P}_k^{\zeta_k}(\tau_f) = \frac{k^3}{2\pi^2}|\zeta_k(\tau_f)|^2 = \frac{k^3}{2\pi^2M_p^2}\frac{|v_k(\tau_f)|^2}{z^2(\tau_f)}, \quad \mathcal{P}_k^{\gamma_p}(\tau_f) = \frac{k^3}{2\pi^2}|\gamma_p(k,\tau_f)|^2 = \frac{k^3}{2\pi^2}\frac{|2v_p(k,\tau_f)|^2}{M_P^2\, a^2(\tau_f)}$$





4. Two field Cuscuton bounce

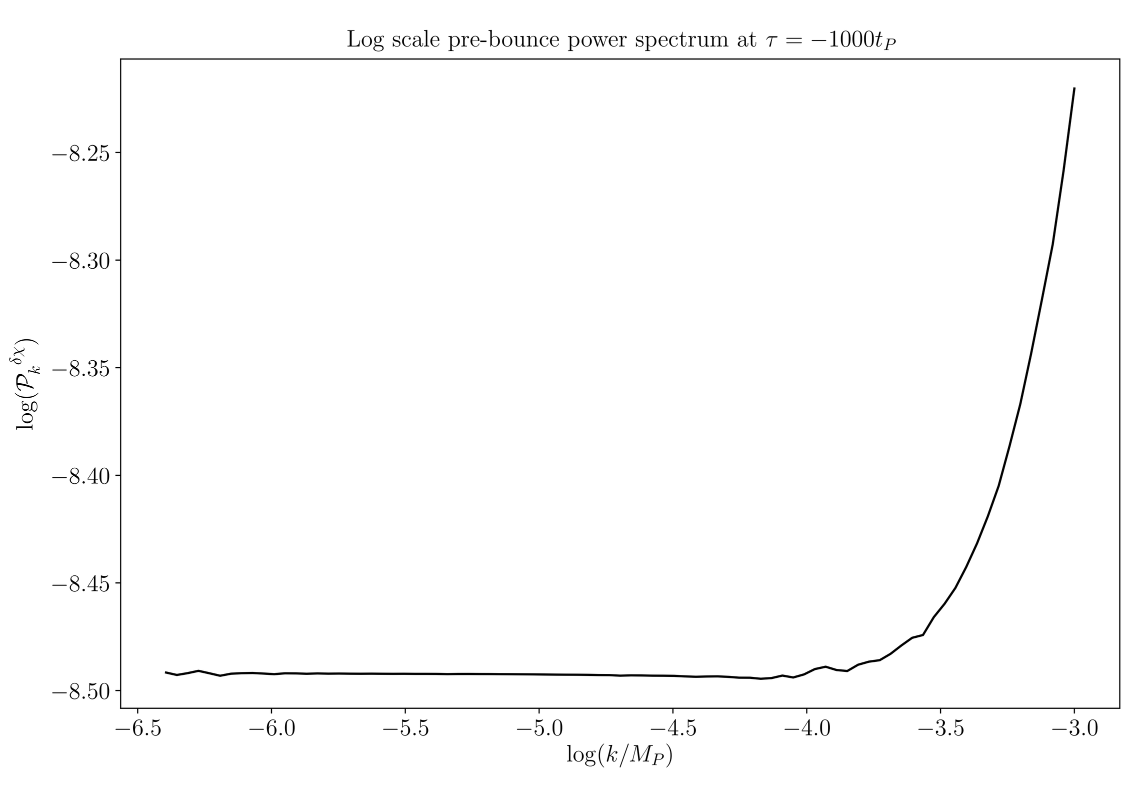
We modify the single field bounce by adding a spectator field χ which non-minimally couples to the matter field. This is the entropic mechanism. used to produce scale invariance for entropy perturbations in models like Ekpyrosis [4, 5]. The action (new terms in red) is then:

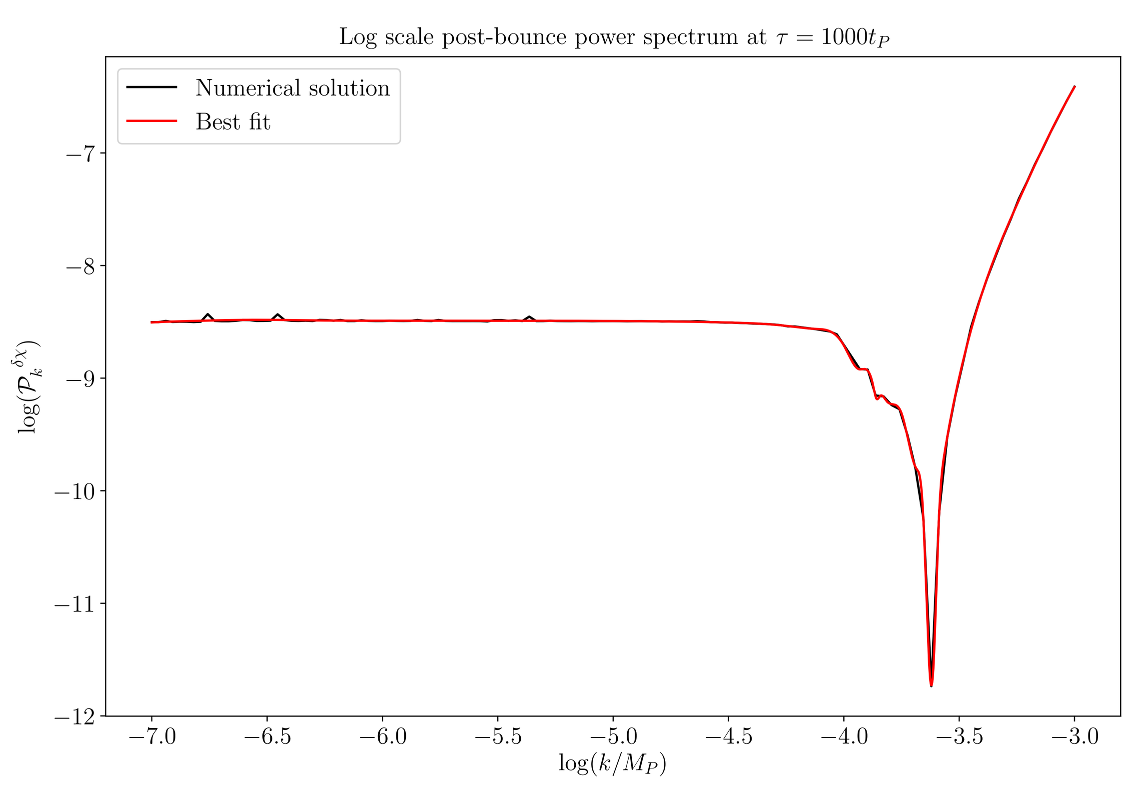
$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - \frac{1}{2}D_\mu\pi D^\mu\pi - \frac{1}{2}F(\dot\pi, \nabla_i\pi, \ldots)D_\mu\chi D^\mu\chi - \mu^2\sqrt{-D_\mu\varphi D^\mu\varphi} - V(\varphi)\right],$$

The coupling function is taken to be $F(\dot\pi) = \rho_m/(\Lambda M_P^2)$ where Λ is a free parameter. Similar to the previous section, we can get the equation of motion and power spectrum for the perturbations $\delta\chi$ in Mukahnov-Sasaki form $u_k = M_P q(\tau,k)\delta\chi_k$ along with its power spectrum:

$$u_k'' + \left(k^2 - \frac{q''}{q}\right)u_k = 0, \quad \mathcal{P}_k^{\delta\chi}(\tau_f) = \frac{k^3}{2\pi^2}|\delta\chi_k(\tau_f)|^2 = \frac{k^3}{2\pi^2}\frac{|u_k(\tau_f)|^2}{M_p^2 q(\tau_f)^2}.$$

This results in a scale-invariant power spectra both before and after the bounce.





5. Conclusion

We have investigated the power spectrum for Cuscuton bounce.

- Two field Cuscuton bounce can produce scale-invariant power spectra.
- One field Cuscuton bounce is generically unable to acquire scale-invariance
- These entropy perturbations can be converted into adiabatic perturbations, which can potentially match the observed tilt.

6. References

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[5] E. I. Buchbinder, J. Khoury and B. A. Ovrut, *New Ekpyrotic cosmology*, *Phys. Rev. D* **76** (2007) 123503 [hep-th/0702154].