Fractal Dimension: Scale of homogeneity

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Abstract: Fractal dimensions of the large scale matter distribution of the Universe can be used to theorize the scale of homogeneity. The scale of transition to homogeneity is defined as the scale above which the fractal dimension of underlying point distribution is equal to the ambient dimension of the space in which points are distributed. A decade ago Yadav et al. (2010) have defined the scale of homogeneity to be the scale above which the deviation of fractal dimension from the ambient dimension becomes smaller than the statistical dispersion of this deviation. In our work, we use LCDM Gadget2 simulations to present dependency of this definition on the epoch. We use the connection between the fractal dimensions and the correlation function to quantify the statistical dispersion in the weak clustering limit.

Simulation Setup: We use a boxsize of 1024 Mpc and 512^3 particles.We randomly pick 10 subsamples of 128^3 particles from it.

Formalism: We use the correlation dimension (D_2) to find the scale of homogeneity.

$$D_2(r) = \frac{\partial log C_2(r)}{\partial log(r)} \quad \text{(Equation 1)}$$

$$C_2(r) = \frac{1}{NM}\sum_{i=1}^{M} n_i(r)$$

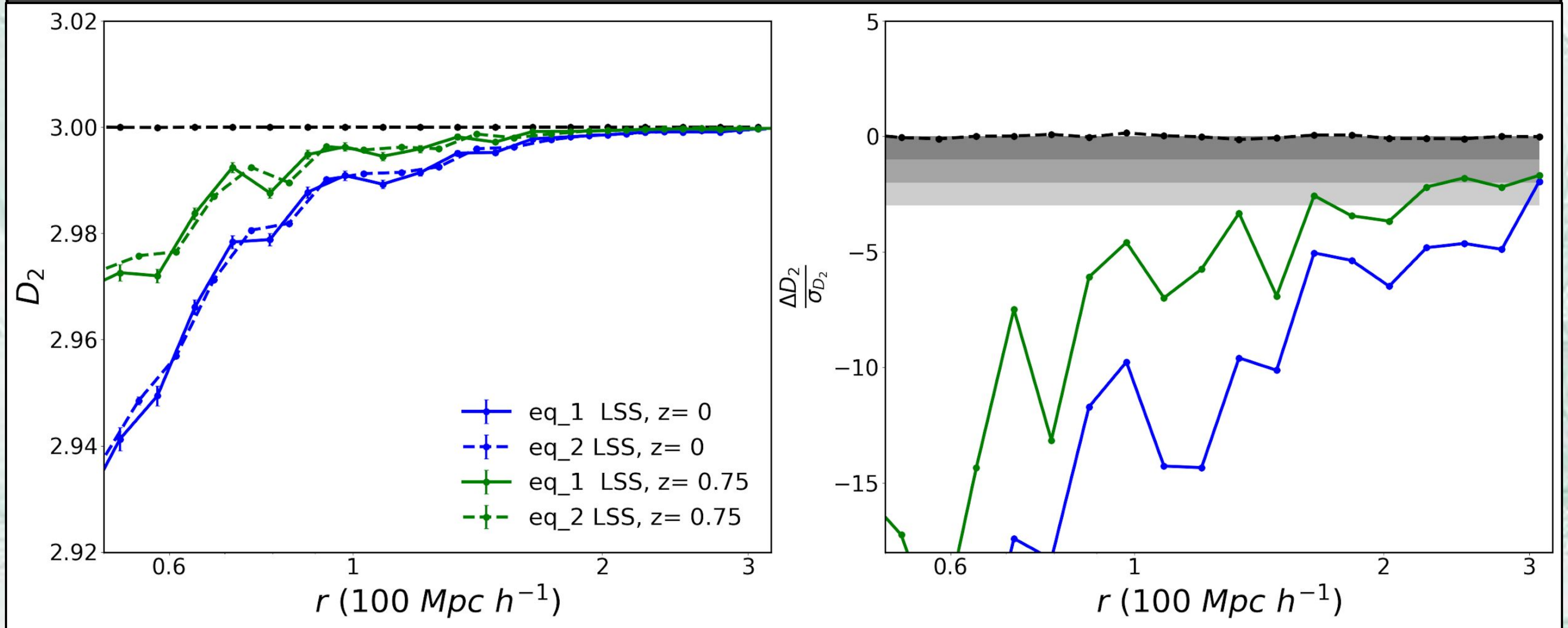
Where: N is the number of points in the distribution, M is the number of centres of a sphere of radius r and n_i is the number of particles within that sphere.

Since our distribution has a finite average density, this allows us to relate correlation integral and correlation functions. In weak clustering limit Bagla et al 2008 showed that

$$D_2(r) \simeq D[1-(\bar{\xi}(r)-\xi(r))] \quad \text{(Equation 2)}$$

References:
Bagla et al. 2008, MNRAS, 390
Yadav et al. 2010, MNRAS, 405
Sinha et al. 2020, MNRAS, 491

D2: Scale of homogeneity (Redshift Dependence)



Left Panel: Shows that with the increase in length scale fractal dimension approaches the ambient dimension (D = 3). The black curve shows the result of considering a homogenous distribution with the same setup. The error bars are calculated by cross-correlating (Sinha et al. 2020) subsamples. This shows there exists a scale of homogeneity which is independent of redshift.

Right Panel: We plot the ratio of deviation in D_2 from D with respect to the standard deviation in D_2. The gray regions show the 1σ, 2σ and 3σ cutoffs. *We find that scale of homogeneity (within 2σ) is* ***about 300 MPC for z = 0 and 0.75 in our simulation.***