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Exact Solution Approach to Warm Inflation

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Warm Inflation in Modified Cosmologies

- The paradigm of Warm Inflation has achieved well deserved interest in the Cosmo Community.
- Warm Inflation has interesting features that properly differentiates it from the usual cold inflation scenario, like no need of reheating and it's also consistent, even for single field cases, with UV completion criterion from String Theory.
- Naturally, there has been a lot of work on studying Warm Inflation in various general cosmological models.
- In particular, some interesting cases are Braneworld from String Theory, Loop Quantum Cosmology and various other modified gravity theories.

A way to tackle many scenarios at once !

- We consider a generalized Friedmann equation of the form $F(H) = \frac{8\pi}{3m_p^2}\rho$ as the basis of our investigation.
- The Friedmann Equation, complemented by the basics of Warm Inflation ,

$$\ddot{\phi} + (3H + \Gamma)\dot{\phi} + V'(\phi) = 0$$

$$\dot{\rho}_r + 4H\rho_r = 3HQ\dot{\phi}^2$$

- Also, as is usual for Inflationary Scenarios, the potential dominates over the kinetic part of the energy density and we have,

$$F(H) \approx \frac{8\pi}{3m_p^2}V(\phi)$$

Getting the key Inflationary Quantities !

- Using the fundamental equations, we have the field velocity as,

$$\dot{\phi} = -\frac{m_p^2}{8\pi} \frac{F_{,H} H'}{H(1+Q)}$$

- This allows us to further express

$$\rho_r = \frac{3Q}{4} \left(\frac{m_p^2}{8\pi} \frac{F_{,H} H'}{H(1+Q)} \right)^2 \qquad T = \left(\frac{3Q}{4\alpha} \right)^{1/4} \left(\frac{m_p^2}{8\pi} \frac{F_{,H} H'}{H(1+Q)} \right)^{1/2}$$

$$V(\phi) = \rho_\phi \left(1 - \frac{m_p^2}{24\pi} \left(\frac{F_{,H} H'}{H(1+Q)} \right)^2 \right) \qquad N = \int_{\phi_e}^{\phi} \frac{8\pi}{m_p^2} \frac{H^2(1+Q)}{F_{,H} H'} d\phi$$

The slow roll and other inflationary parameters

- The usual ε and η parameters are given by

$$\epsilon = \frac{m_{p^2}}{8\pi}\frac{F_{,H}}{H(1+Q)}\left(\frac{H'}{H}\right)^2 \qquad \eta = \frac{m_{p^2}}{8\pi}\frac{F_{,H}}{H(1+Q)}\frac{H''}{H}$$

- Warm Inflation needs some parameters which take into account the field decay rate,

$$\beta = \frac{m_p^2}{8\pi}\frac{F_{,H}}{H}\frac{\Gamma' H'}{\Gamma H}\frac{1}{1+Q} \qquad \delta = \frac{m_p^2}{8\pi}\left(\frac{F_{,H}}{(1+Q)H}\right)^2\frac{\Gamma'' H'^2}{\Gamma H^2}$$

- And some more useful parameters,

$$\sigma = \frac{m_p^2}{8\pi}\frac{1}{1+Q}\frac{F,H}{H}\frac{H'''}{H'} \qquad \psi = \frac{m_p^2}{8\pi}\frac{F_{,HHH}}{(1+Q)}\frac{H'^2}{H}$$

Perturbation Spectra Analysis

- We can find important perturbation parameters using the usual definitions for both the scalar and tensor density perturbations.
- The Scalar Density perturbation amplitude squared is given by ,

$$P_s(k)^2 = \frac{4}{25}\left(\frac{H}{|\dot{\phi}|}\right)^2 d\phi^2$$

Where, $d\phi^2 = \frac{k_F T}{2\pi}$

- The definition of the freeze out number $k_F = \sqrt{\Gamma H}$ remains as usual.

Key perturbation parameters

- The scalar spectral index is given by,

$$n_s = 1 + \frac{d\ln P_s(k)^2}{d\ln k} = 1 + \frac{9}{4}\beta + \frac{9}{4}\epsilon - \frac{3}{2}\gamma - \frac{3}{2}\eta$$

- The running of the scalar spectral index is hence given as ,

$$\alpha_s = \frac{15}{4}(\beta\gamma + \eta\beta + \gamma\epsilon + \eta\epsilon) - \frac{9}{2}(\epsilon\beta + \epsilon^2 + \beta^2 + \eta\gamma) - \frac{3}{2}(\psi\epsilon) + \frac{9}{4}\delta$$

- And the tensor-to-scalar ratio is,

$$r = \frac{2F}{m_p^2}\left(\frac{4\alpha}{3}\right)^{\frac{1}{4}}\left(\Gamma^{\frac{-3}{2}} H'^{\frac{-3}{2}} \epsilon^3 H^3\right)^{\frac{1}{2}}$$

This completes the outline of the approach!

- The Exact Solution Approach hence allows one to study Warm Inflation in a large class of modified cosmological scenarios.
- The method requires only three pieces of information :

1) The form of the Cosmology one wants to deal with
2) An ansatz for the Hubble Parameter in terms of the field parameter
3) An ansatz for the Dissipation Coefficient in terms of the Field parameter, Temperature or both of them

Exact Solution Approach to Warm Inflation

- For Q=0 , Del Campo's approach
- For $F(H) = H^2$, Warm HJ Approach
- For Q=0 and $F(H) = H^2$, Cold HJ Approach

Warm Inflation in a Tsallis Entropy modified Cosmology

- Tsallis Cosmology is defined by the Friedmann equation of the form,

$$F(H) = H^{2(2-\kappa)}$$

- We use the ansatz the completely field dependent ansatz,

$$H(\phi) = H_o\phi^n \qquad \Gamma(\phi) = \Gamma_o\phi^m$$

Choosing a completely field dependent form of the dissipation coefficient can implicitly take into account temperature dependence.

- After working through developed procedure for the exact solution approach, one can finally get the,

Field Parameter

$$\phi^{m+2-n(4-2\kappa)}(t)=\frac{(4-2\kappa)n-m-2}{n}\phi_e^{m+2-n(3-2\kappa)}H_o(t-t_o)+\left(\phi_e^{m+2-n(3-2\kappa)}\left(1+\frac{(m+2-n(3-2\kappa))N}{n}\right)\right)^{\frac{(m+2-n(4-2\kappa))}{m+2-n(3-2\kappa)}}$$

Hubble Parameter

$$H(t)=H_o\left[\frac{(4-2\kappa)n-m-2}{n}\phi_e^{m+2-n(3-2\kappa)}H_o(t-t_o)+\left(\phi_e^{m+2-n(3-2\kappa)}\left(1+\frac{(m+2-n(3-2\kappa))N}{n}\right)\right)^{\frac{m+2-n(4-2\kappa)}{m+2-n(3-2\kappa)}}\right]^{\frac{n}{(m+2-n(4-2\kappa))}}$$

Dissipation Coefficient

$$\Gamma(t)=\Gamma_o\left[\frac{(4-2\kappa)n-m-2}{n}\phi_e^{m+2-n(3-2\kappa)}H_o(t-t_o)+\left(\phi_e^{m+2-n(3-2\kappa)}\left(1+\frac{(m+2-n(3-2\kappa))N}{n}\right)\right)^{\frac{m+2-n(4-2\kappa)}{m+2-n(3-2\kappa)}}\right]^{\frac{m}{(m+2-n(4-2\kappa))}}$$

Inflationary Potential

$$V(\phi)=\frac{3m_p^2H_o^{4-2\kappa}\phi^{4-2\kappa}}{8\pi}-\frac{1}{2}\left(\frac{3m_p^2 2(2-\kappa)H_o^{4-2\kappa}n\phi^{(3-2\kappa)n-m-1}}{8\pi\Gamma_o}\right)^2$$

Relation between energy densities

$$\rho_r(\phi)=\frac{m_p^2(2-\kappa)\phi_e^{m+2-n(3-2\kappa)}\rho_\phi}{16\pi\phi^{m+2-n(3-2\kappa)}}$$

Number of e-folds and particular field values

- The Number of e-folds is given by,

$$N = \int_{\phi_e}^{\phi} \frac{8\pi}{3m_p^2} \frac{H_o \Gamma_o \phi^n \phi^m}{2(2-\kappa) H_o^{3-2\kappa} \phi^{n(3-2\kappa)} n H_o \phi^{n-1}}$$

- Field value at the time Inflation ends is,

$$\frac{3m_p^2 2(2-\kappa) n^2 H_o^{3-2\kappa}}{8\pi \Gamma_o} = \phi_e^{m+2-n(3-2\kappa)}$$

- This finally gives the field value at Horizon exit as,

$$\phi^{m+2-n(3-2\kappa)} = \phi_e^{m+2-n(3-2\kappa)} \left(1 + \frac{(m+2-n(3-2\kappa))N}{n}\right)$$

Slow roll and other parameters at Horizon exit

- All the needed parameters at horizon exit can be written in terms of ε as,

$$\epsilon = \frac{n}{n + (m + 2 - n(3 - 2\kappa))N}$$

$$\beta = \frac{m}{n}\epsilon \qquad \gamma = (3 - 2\kappa)\epsilon \qquad \eta = \frac{n-1}{n}\epsilon$$

$$\sigma = \frac{(n-1)(n-2)}{n}\epsilon \quad \delta = \frac{(m)(m-1)}{n^2}\epsilon^2 \quad \psi = (3 - 2\kappa)(2 - 2\kappa)\epsilon$$

Observational match up !

- This gives us the scalar spectral index and it's running as,

$$n_s = 1 + \left[\frac{9m}{4n} + \frac{9}{4} + 3\kappa - \frac{9}{2} + \frac{3}{2}\left(\frac{n-1}{n}\right)\right]\frac{n}{(m+2-n(2-2\kappa))N}$$

$$\alpha_s = \left[\frac{15}{4}\left(\frac{m}{n}(3-2\kappa) + \frac{(n-1)m}{n^2} + 3 - 2\kappa + \frac{n-1}{n}\right) + \frac{9}{4}\frac{m(m-1)}{n^2} - \right.$$
$$\left.\frac{9}{2}\left(\frac{m}{n} + 1 + \left(\frac{m}{n}\right)^2 + \frac{n-1}{n}(3-2\kappa)\right) - 3(3-2\kappa)(1-\kappa)\right]\left[\frac{n}{n+(m+2-n(3-2\kappa))N}\right]^2$$

- For N=60, we can now match up the values of the free parameters of the model with observational constraints on the above two parameters. An appropriate tuning is $(m, n, \kappa) = (3, -5, 1.4)$, which gives us the pretty accurate values,

$$n_s \approx 0.964912 \qquad \alpha_s \approx -0.003$$

Final Comments and conclusions

- The Approach is quite a general way to understand Warm Inflation in a variety of cosmological scenarios.
- The approach works well only if the inflaton field follows the usual Klein-Gordon Form.
- Hence the method only works well for theories where the background metric alongside the perturbations themselves are not modified.
- This method is, to the best knowledge of the author, the most general form of the Hamilton Jacobi Approach to Inflation.