Nonclassicallity of Two-mode Squeezed Vacuum ($|\xi_2\rangle$) State of an Oscillating Quantize Scalar field in the FRW Universe

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4. Criterion for nonclassical effects in $|\xi_2\rangle$ state
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Introduction

- In the framework of the chaotic inflationary scenario, (Andrei Linde) where a single massive scalar field drives the early universe into inflation, we study the oscillatory phase of the scalar field after inflation in *Semiclassical Theory of Gravity*.
- *Semiclassical Theory of Gravity*, where the background metric is treated as classical and matter field as *Quantum Mechanical*.
- We make use of the *Non-Classical State Formalisms* (quantum optics two-mode squeezed vacuum $|\xi_2\rangle$ state) to explore chaotic inflation in connection to the semiclassical approximation to gravity.

Methodology

At present, in-spite of the numerous efforts to reconcile General Relativity and Quantum Mechanics, we still do not have a complete conclusive theory of Gravity. Hence, as an approximation to quantum gravity, the gravitational effects of quantum matter fields are often described by a semiclassical theory wherein a suitable renormalized expectation value of the energy-momentum tensor for matter field becomes the source of gravity. The semiclassical Einstein equation is defined as:

$$G_{\alpha\beta} = \frac{8\pi}{m_p^2} \langle\varphi| T_{\alpha\beta} |\varphi\rangle \quad (1)$$

where $G_{\alpha\beta}$, is the Einstein tensor and $T_{\alpha\beta}$ denotes the expectation value of the energy-momentum tensor of the matter field under consideration.
The semiclassical theory of gravity (SG) seems to be a viable method to understand quantum effects and quantum phenomenon in the early universe where quantum gravity effects are considered to be negligible.

SG and Scalar Field Dynamics in $|\xi_2\rangle$ State

In the chaotic inflationary scenario, the source of the gravitational field is the massive scalar field defined on a quadratic potential, governed by the time-dependent Schrödinger equation formulated as:

$$i\frac{\partial}{\partial t}|\xi_2(\varphi, t)\rangle = \hat{\mathcal{H}}_m(\varphi, t)|\xi_2(\varphi, t)\rangle, \tag{2}$$

where $|\xi_2 >$ is the matter field quantum state (TMSV) represented as:

$$|\xi_2\rangle = Z_{ab}(r_s, \Phi)|0, 0\rangle, \tag{3}$$

$$Z_{ab}(r_s, \Phi) = \exp\frac{r_s}{2}\left(e^{-i\Phi}\hat{a}\hat{b} - e^{i\Phi}\hat{a}^\dagger\hat{b}^\dagger\right), \tag{4}$$

here, r_s and Φ are the corresponding squeezing parameter and squeezing angle and $\hat{\mathcal{H}}_m$ is the Hamiltonian for the matter field.

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As a concrete model, we shall consider a massive inflaton φ_0, the homogeneous and isotropic field corresponding to the space average of the massive scalar field $\varphi(\mathbf{x}, \tau)$. Now, the scalar field can be decomposed into the inflaton and fluctuations; the inhomogeneous and anisotropic field as:

$$\varphi(\tau, \mathbf{x}) = \varphi_0(\tau) + \zeta(\tau, \mathbf{x}). \tag{5}$$

In keeping the analogy with point mechanics and upon quantisation the mode-decomposition yields the corresponding Hamiltonian (a collection of time-dependent harmonic oscillators) for the massive scalar field as obtained as:

$$\mathcal{H}_m(\varphi) = \sum_{\mathbf{k}} \left[\frac{\hat{\pi}_{\mathbf{k}}^2}{2\mathcal{S}^3(\tau)} + \frac{\mathcal{S}^3(\tau)}{2} \left(m^2 + \frac{k^2}{\mathcal{S}^2(\tau)} \right) \hat{\zeta}_{\mathbf{k}}^2 \right]. \tag{6}$$

The Friedmann equation in the semiclassical approximation in $|\xi_2\rangle$, takes the following form:

$$\left(\frac{\dot{\mathcal{S}}(\tau)}{\mathcal{S}(\tau)} \right)^2 = \frac{8\pi}{3m_p^2} \frac{1}{\mathcal{S}^3(\tau)} \langle \xi_2 | \hat{\mathcal{H}}_m | \xi_2 \rangle. \tag{7}$$

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The eigenstates of the Hamiltonian are Fock states [1]

$$\hat{a}_i^\dagger(t)\ \hat{a}_i(t)|n_i,\phi_i,t\rangle = n_i|n_i,\phi_i,t\rangle,\ \ i=1,2 \tag{8}$$

where $\hat{a}_i{}^\dagger$ is the modes creation operators and $\hat{a}_i$ is the associated annihilation operators. These ladder operators for two-mode states can be defined as:

$$\hat{a}(t) = \zeta_1^*(t)\ \hat{\pi} - \mathcal{S}^3\ \dot{\zeta}_1^*(t)\ \hat{\zeta},\quad \hat{a}^\dagger(t) = \zeta_1(t)\ \hat{\pi} - \mathcal{S}^3\ \dot{\zeta}_1(t)\ \hat{\zeta},$$
$$\hat{b}(t) = \zeta_2^*(t)\ \hat{\pi} - \mathcal{S}^3\ \dot{\zeta}_2^*(t)\ \hat{\zeta},\quad \hat{b}^\dagger(t) = \zeta_2(t)\ \hat{\pi} - \mathcal{S}^3\ \dot{\zeta}_2(t)\ \hat{\zeta}, \tag{9}$$

where, ζ_1 and ζ_2 are mode functions of field corresponding to two different modes of the scalar field. It was found that the semiclassical Einstein quantum gravity equation in $|\xi_2\rangle$ state leads to the same power-law expansion $t^{2/3}$ as that of the matter dominated era in an oscillatory phase of the scalar field after inflation [2]. Moreover, the particle created due to the quantum fluctuation of the scalar field in $|\xi_2\rangle$ was obtained as [3]:

$$\boxed{\mathcal{N}_{|\xi_2\rangle}(t,t_0) = (1+2\sinh^2 r)\frac{(t-t_0)^2}{4m^2\tau_0^4} + \sinh^2 r + \frac{\sinh 2r}{4m^2 t_0^4}(t-t_0)^2} \tag{10}$$

[1] S. P. Kim, *Phys. Rev. D* **52**, 3382 (1995); *Phys. Lett. A* **205**, 359 (1995); *J. Korean Phys. Soc.* **28**, S512 (1995). [2] M.Rathore, R.Dhayal and K.K.Venkataratnam, *Int. J. Mod. Phys. D* **29(05)**, 2050035 (2020). [3] M.Rathore et.al., *Int. J. Mod. Phys. D* https://doi.org/10.1142/S0218271820501199

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Since, we use the $|\xi_2\rangle$ state to study particle creation during the oscillatory phase of scalar field, it is important to examine the nature of this quantum optical state in the cosmological context (whether the state exhibit classical or nonclassical feature) with associated cosmological parameters.

- Abstract: We in our present article, have made use of the criterion suggested by CT Lee [4], for the existence of nonclassical effects in two-mode states and calculated a equivalent parameter called as cosmological $\mathcal{D}$ parameter with the associated cosmological parameters, to examine the non-classical nature of the $|\xi_2\rangle$ after inflation during the oscillatory phase of the scalar field.

Criterion for nonclassical effects in $|\xi_2\rangle$ state

We made use of the criterion put forward by Ching Tsung Lee [4] for the existence of nonclassical effects in two-mode states as:

$$\mathcal{D}_{12}^{(2)} = \mathcal{C}_1^{(2)} + \mathcal{C}_2^{(2)} - \mathcal{C}_{12}^{(2)} + (\langle n_1\rangle + \langle n_2\rangle)^2 < 0 \tag{11}$$

where we call the $\mathcal{D}_{12}^{(2)}$ parameter as the cosmological $\mathcal{D}$ parameter. Here, $\mathcal{C}_1^{(2)}$ measures the degree of correlation between two particles from the same mode and is given by following expression:

$$\mathcal{C}_1^{(2)} = \langle \hat{a}^\dagger \hat{a}^\dagger \hat{a}\hat{a}\rangle - \langle \hat{a}^\dagger \hat{a}\rangle^2; \;\; \mathcal{C}_2^{(2)} = \langle \hat{b}^\dagger \hat{b}^\dagger \hat{b}\hat{b}\rangle - \langle \hat{b}^\dagger \hat{b}\rangle^2 \tag{12}$$

and can be rewritten in terms of particle-number moments as

$$\mathcal{C}_i^{(2)} = \langle n_i^{(2)}\rangle - \langle n_i\rangle^2, \quad i = 1, 2 \tag{13}$$

We have $\mathcal{C}_i^{(2)} = 0$, for a coherent state; $\mathcal{C}_i^{(2)} > 0 \Rightarrow$ intramode particle bunching, which is always true for classical states and, in contrast, we have intramode particle antibunching when $\mathcal{C}_i^{(2)} < 0$, which is possible only for nonclassical states.

Analogously, $\mathcal{C}_{12}^{(2)}$ the correlation function between two particles from different modes is defined as:

$$\mathcal{C}_{12}^{(2)} = \langle \hat{a}^\dagger \hat{b}^\dagger \hat{b}\hat{a}\rangle - \langle \hat{a}^\dagger \hat{a}\rangle\ \langle \hat{b}^\dagger \hat{b}\rangle; \; \mathcal{C}_{12}^{(2)} = \langle n_1 n_2\rangle - \langle n_1\rangle\langle n_2\rangle \tag{14}$$

[4] C.T. Lee, Phys. Rev. A **41**, 1721(1990)

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$\mathcal{C}_{12}^{(2)} > 0 \Rightarrow$ intermode particle bunching and $\mathcal{C}_{12}^{(2)} < 0 \Rightarrow$ intermode particle antibunching. Now, in the case where the individual modes are coherent states of equal intensity; so that $\mathcal{C}_{1}^{(2)} = \mathcal{C}_{2}^{(2)} = 0$ and $\langle n_1\rangle = \langle n_2\rangle$. Then, Eq. (11) reduces to:

$$\mathcal{D}_{12}^{(2)} = -2\mathcal{C}_{12}^{(2)}; \tag{15}$$

and the criterion for the existence nonclassical effects becomes $\mathcal{C}_{12}^{(2)} > 0$, which implies intermode particle bunching.

Using the above criterion, we examined the $|\xi_2\rangle$ state and obtain the expression for correlations functions in the oscillatory phase of the scalar field as: (using $|\xi_2\rangle$ state definition, identities refer Eq. (3), (4) and (9) along with the approximation ansatz with x=mt)

$$\begin{aligned}\mathcal{C}_{1}^{(2)} = \mathcal{C}_{2}^{(2)} = \Big[& 32x^4x_0^4\sinh^4(r) + 2x^2x_0^2\,(x-x_0)^2(-4\cosh(2r)+3\cosh(4r)+3) \\ & - \left(4x^2x_0^2\sinh^2(r) + (x-x_0)^2\cosh(2r)\right)^2 + 3\,(x-x_0)^4\cosh^2(2r)\Big]\frac{1}{16x_0^8} > 0;\end{aligned} \tag{16}$$

$$\mathcal{C}_{12}^{(2)} = \frac{\left(x^4 + \left(2x^4+1\right)x_0^4 - 4x_0x^3 + 6x_0^2x^2 - 4x_0^3x\right)\sinh^2(2r)}{8x_0^8} > 0 \tag{17}$$

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and on substituting in the criterion Eq.(11), we obtain:

$$\mathcal{D}_{12}^{(2)} = \frac{1}{4x_0^8}\left[2x_0^2x^2\left(x^2(\cosh(4r) - \cosh(2r)) + x^2 + 3\right) - 4x_0^3x(x^2(\cosh(4r) - \cosh(2r)) + x^2 + 1) + x_0^4(2x^2\cosh(4r) - 2\left(2x^4 + x^2\right)\cosh(2r) + 4x^4 + 2x^2 + 1) + x^4 - 4x_0x^3\right] \leq 0 \tag{18}$$

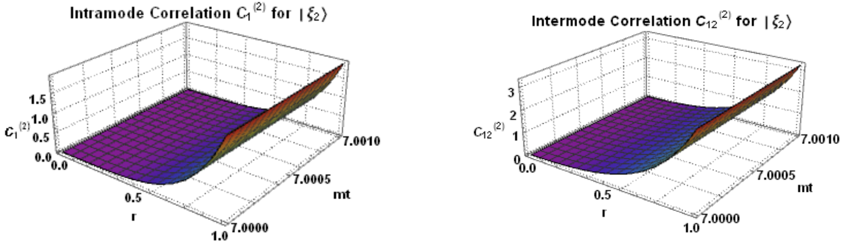

Fig 1,2: Plots shows variation of $\mathcal{C}_1^{(2)}$ and $\mathcal{C}_{12}^{(2)}$ with r and mt for $|\xi_2\rangle$ state.

Discussion and Conclusions

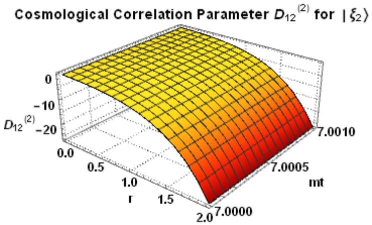

Fig 3: Plot shows variation of $\mathcal{D}_{12}^{(2)}$ with r and mt.

From Eq. (18) we see that the $|\xi_2\rangle$ state are always nonclassical ($\mathcal{D}_{12}^{(2)} < 0$) , except when $r = 0$. But from Eq. (16) ($\mathcal{C}_1^{(2)} = \mathcal{C}_2^{(2)} > 0$), we see that each mode by itself is definitely a classical state. Therefore the correlation between particles from two different modes plays the exclusive role in overcoming the classical effects in individual modes and rendering the two-mode squeezed vacuum states nonclassical. Furthermore, from Eq. (17) ($\mathcal{C}_{12}^{(2)} > 0$), we see clearly that the intermode correlation signifies particle bunching. Thus, we conclude that, the analysis done with cosmological $\mathcal{D}$ parameter shows that the $|\xi_2\rangle$ is consistent with its nonclassical nature with the associated cosmological parameters during the oscillatory phase of the scalar field in the semiclassical theory of gravity.

REFERENCES

1. S. P. KIM, *Phys. Rev. D* **52**, 3382 (1995); *Phys. Lett. A* **205**, 359 (1995); *J. Korean Phys. Soc.* **28**, S512 (1995).
2. M. RATHORE ET.AL, *Int. J. Mod. Phys. D* **29** (05), 2050035 (2020).
3. M. RATHORE ET.AL, *Int. J. Mod. Phys. D* HTTPS://DOI.ORG/10.1142/S0218271820501199.
4. C.T. LEE, *Phys. Rev. A* **41**, *1721(1990)*

Thank You So Much..