Universe in a Black Hole from Spin and Torsion

Nikodem Popławski

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Universe in a black hole

• The conservation law for total angular momentum in curved spacetime, consistent with Dirac equation, requires that the affine connection has antisymmetric part: torsion. In the simplest theory with torsion, Einstein-Cartan gravity, the torsion tensor is generated by spin of fermions.

• Gravitational collapse of a spherically symmetric sphere of a spin fluid creates an event horizon. The matter within the horizon collapses to extremely high densities, at which torsion acts like gravitational repulsion.

• Without shear, torsion prevents a singularity and replaces it with a nonsingular bounce. With shear, torsion prevents a singularity if the number of fermions increases during contraction via quantum particle production.

• Particle production during expansion produces enormous amounts of matter and can generate a finite period of inflation. The resulting closed universe on the other side of the event horizon may have several bounces. Such a universe is oscillatory, with each cycle larger in size than the previous cycle, until it reaches the cosmological size and expands indefinitely.
Einstein-Cartan-Sciama-Kibble gravity

- Action variation with respect to metric and torsion. $S^k_{ij} = \Gamma^k_{[i \ j]}$

- Covariant derivative of metric is zero. Lagrangian density is proportional to Ricci scalar (as in GR).

- Cartan equations:
  **Torsion** is proportional to **spin** density of fermions. ECSK differs significantly from GR at densities $> 10^{45} \text{ kg/m}^3$; passes all tests.

  \[ S_{jik} - S_{igjk} + S_{kgji} = -\frac{1}{2}\kappa s_{ikj} \]


- Einstein equations: torsion terms moved to RHS. **Curvature** is proportional to **energy and momentum** density.
Gravitational collapse of spin fluid sphere

Dirac particles can be averaged macroscopically as a spin fluid.

\[ s^{\mu\nu\rho} = s^{\mu\nu} u^\rho \quad s^{\mu\nu} u_\nu = 0 \quad s^2 = s^{\mu\nu} s_{\mu\nu} / 2 \]

Collapse can be parametrized by the closed FLRW metric. Einstein-Cartan equations become Friedmann equations for scale factor \( a \).

\[ \frac{\dot{a}^2}{c^2} + 1 = \frac{1}{3} \kappa \left( \epsilon - \frac{1}{4} \kappa s^2 \right) a^2 \]
\[ \frac{\dot{a}^2 + 2a\ddot{a}}{c^2} + 1 = -\kappa \left( p - \frac{1}{4} \kappa s^2 \right) a^2 \]

Spin and torsion modify the energy density and pressure with a negative term proportional to the square of the fermion number density \( n \), which acts like repulsive gravity.

Torsion generating nonsingular bounce

For relativistic matter, Friedmann equations can be written in terms of temperature: $\epsilon \approx 3\rho \sim T^4$, $n \sim T^3$, and put in nondimensional form with temperature $x$ and scale factor $y$:

\[
\frac{\dot{a}^2}{c^2} + 1 = \frac{1}{3} \kappa (h_* T^4 - \alpha n T^6) a^2
\]
\[
\frac{\dot{a}}{a} + \frac{\dot{T}}{T} = 0 \quad \alpha = \kappa (\hbar c)^2 / 32
\]
\[
\dot{y}^2 + 1 = (3x^4 - 2x^6)y^2
\]
\[
x y = C > 0
\]

Two turning points ($\dot{y} = 0$) for a closed Universe with torsion exist if $C > (8/9)^{1/2}$. They are positive – **no cosmological singularity**!

Particle production generating inflation

Near a bounce, particle production enters through a term \( \sim H^4 \), with \( \beta \) as a production parameter.

\[
\frac{\dot{a}}{a} \left[ 1 - \frac{3\beta}{c^3 h n_1 T^3 \left( \frac{\dot{a}}{a} \right)^3} \right] = -\frac{\dot{T}}{T}
\]

To avoid eternal inflation: the \( \beta \) term < 1, so \( \beta < \beta_{cr} \approx 1/929 \).

For \( \beta \approx \beta_{cr} \) and during an expansion phase, when \( H = \dot{a}/a \) reaches a maximum, the \( \beta \) term is slightly lesser than 1 and:

\( T \sim \text{const}, \quad H \sim \text{const} \).

**Exponential expansion** lasts about \( t_{\text{Planck}} \) then \( H \) and \( T \) decrease. Inflation ends when torsion weakens. No scalar fields needed. Dynamics similar to plateau-like inflation & consistent with CMB. S. Desai & NP, PLB 755, 183 (2016).
Torsion & particle production: opposing shear and generating matter & entropy

Shear opposes torsion in Raychaudhuri equation. Shear and torsion terms grow with decreasing scale factor according to $a^{-6}$. To avoid singularity, fermion number density must grow faster than $a^{-3}$. This condition during a contracting phase can happen because of particle production.

If quantum effects in the gravitational field near a bounce do not produce enough matter, then the closed Universe reaches the maximum size and then contracts to another bounce, beginning the new cycle. Because of matter production, a new cycle reaches larger size and last longer than the previous cycle.

<table>
<thead>
<tr>
<th>$\beta/\beta_{cr}$</th>
<th>Number of bounces</th>
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<tr>
<td>0.996</td>
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</tr>
<tr>
<td>0.984</td>
<td>2</td>
</tr>
<tr>
<td>0.965</td>
<td>3</td>
</tr>
<tr>
<td>0.914</td>
<td>5</td>
</tr>
<tr>
<td>0.757</td>
<td>10</td>
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</tbody>
</table>

When the Universe reaches a size at which the cosmological constant is dominating, then it avoids another contraction and starts expanding to infinity.