# Duel of cosmological screening lengths

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  - novel expression for the gravitational potential
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- Conclusion

# Introduction

**Weak field limit of general relativity** → **Newtonian gravitational interaction** between nonrelativistic massive bodies at **sub-horizon cosmological scales** (perturbed FLRW spacetime)

How is it modified at **large distances**?

**Yukawa-type screening** of gravity proposed within the scheme of **discrete cosmology & relativistic perturbation theory**

Can both concepts coexist?
**YES!**

What is the appropriate analytical expression for the gravitational potential?

How does the interaction range compare to the size of the largest known cosmic structures?

# Screening in discrete cosmology

Perturbations in **discrete cosmology** (for pure **ΛCDM** model - negligible radiation):

Nonrelativistic matter presented as separate point-like particles $\rightarrow \rho = \sum_n m_n \delta(\boldsymbol{r} - \boldsymbol{r}_n)$

$\rho$: mass density

Unperturbed FLRW metric:

$$ds^2 = a^2\left(d\eta^2 - \delta_{\alpha\beta} dx^\alpha dx^\beta\right)$$

$$\frac{3\mathcal{H}^2}{a^2} = \kappa\bar{\varepsilon} + \Lambda$$

$$\frac{2\mathcal{H}' + \mathcal{H}^2}{a^2} = \Lambda$$

- $\eta$: conformal time
- $a(\eta)$ : scale factor
- $x^\alpha$: comoving coordinates ; $\alpha, \beta = 1,2,3$
- $\mathcal{H} \equiv a'/a$ ; $' \equiv d/d\eta$
- $\kappa \equiv 8\pi G_N/c^4$ ; $G_N$: Newtonian gravitational constant
- $\bar{\varepsilon}$: average energy density of nonrelativistic matter

Perturbed metric for the inhomogeneous Universe:

**Weak gravitational field limit** } Metric corrections are considered as 1st order quantities.

$$ds^2 = a^2\left[(1 + 2\Phi)d\eta^2 + 2B_\alpha dx^\alpha d\eta - (1 - 2\Phi)\delta_{\alpha\beta} dx^\alpha dx^\beta\right]$$

- $\Phi(\eta, \boldsymbol{r})$: scalar perturbation
- $\mathbf{B}(\eta, \boldsymbol{r})$: vector perturbation

# Screening in discrete cosmology

Einstein equations yield:

$$\Delta\Phi - \frac{3\kappa\bar{\rho}c^2}{2a}\Phi = \frac{\kappa c^2}{2a}\delta\rho - \frac{3\kappa c^2\mathcal{H}}{2a}\Xi \longrightarrow \Phi = \frac{1}{3} - \frac{\kappa c^2}{8\pi a}\sum_n \frac{m_n}{|\boldsymbol{r}-\boldsymbol{r}_n|}\exp(-q_n) + \frac{3\kappa c^2\mathcal{H}}{8\pi a}\sum_n \frac{m_n[\tilde{\boldsymbol{v}}_n(\boldsymbol{r}-\boldsymbol{r}_n)]}{|\boldsymbol{r}-\boldsymbol{r}_n|}\frac{1-(1+q_n)\exp(-q_n)}{q_n^2}$$

$$\Delta\mathbf{B} - \frac{2\kappa\bar{\rho}c^2}{a}\mathbf{B} = -\frac{2\kappa c^2}{a}\left(\sum_n \rho_n\tilde{\boldsymbol{v}}_n - \nabla\Xi\right) \longrightarrow \mathbf{B} = \frac{\kappa c^2}{8\pi a}\sum_n\left[\frac{m_n\tilde{\boldsymbol{v}}_n}{|\boldsymbol{r}-\boldsymbol{r}_n|}\frac{\left(3+2\sqrt{3}q_n+4q_n^2\right)\exp(-2q_n/\sqrt{3})-3}{q_n^2} + \frac{m_n[\tilde{\boldsymbol{v}}_n(\boldsymbol{r}-\boldsymbol{r}_n)]}{|\boldsymbol{r}-\boldsymbol{r}_n|^3}(\boldsymbol{r}-\boldsymbol{r}_n)\frac{9-\left(9+6\sqrt{3}q_n+4q_n^2\right)\exp(-2q_n/\sqrt{3})}{q_n^2}\right]$$

- $\Delta \equiv \delta^{\alpha\beta}\partial^2/(\partial x^\alpha \partial x^\beta)$
- $\bar{\rho}$: average mass density ($\bar{\varepsilon} = \bar{\rho}c^2/a^3$)
- $\delta\rho(\eta, \boldsymbol{r}) \equiv \rho - \bar{\rho}$
- $\tilde{\boldsymbol{v}}_n$: peculiar velocity of the n$^{\text{th}}$ particle

$$\Delta\Xi = \nabla\sum_n \rho_n\tilde{\boldsymbol{v}}_n \rightarrow \Xi = \frac{1}{4\pi}\sum_n m_n\frac{\tilde{\boldsymbol{v}}_n(\boldsymbol{r}-\boldsymbol{r}_n)}{|\boldsymbol{r}-\boldsymbol{r}_n|^3}$$

# Screening in discrete cosmology

- $\Phi, \mathbf{B}$ are valid for all scales & conform with

  - Minkowski background limit

  - (sub-horizon) Newtonian cosmological approximation

- Screening of gravity hinted by $\mathbf{\Phi}$

$$\boldsymbol{q}_n(\eta, \boldsymbol{r}) \equiv \sqrt{\frac{3\kappa\bar{\rho}c^2}{2a}}(\boldsymbol{r} - \boldsymbol{r}_n) = \frac{a(\boldsymbol{r} - \boldsymbol{r}_n)}{\lambda} \quad , \qquad \lambda \equiv \sqrt{\frac{2a^3}{3\kappa\bar{\rho}c^2}}$$

$\lambda$ → Screening length defining the interaction range

M. Eingorn, ApJ 825, 84 (2016); arXiv:1509.03835

# Velocity-dependent contribution to $\boldsymbol{\Phi}$

Equation of motion of an arbitrary particle of the system:

$$(a\tilde{\boldsymbol{v}}_k)' = -a\left(\nabla\Phi|_{\boldsymbol{r}=\boldsymbol{r}_k} + \mathcal{H}\mathbf{B}|_{\boldsymbol{r}=\boldsymbol{r}_k}\right) = \sum_{n\neq k} \boldsymbol{f}_n(\eta, \boldsymbol{r}_k)$$

$\boldsymbol{f}_n(\eta, \boldsymbol{r})$: force per unit mass induced by the n$^{th}$ particle

$$\boldsymbol{f}_n = -\frac{\kappa c^2}{8\pi}\left[\frac{m_n(\boldsymbol{r}-\boldsymbol{r}_n)}{|\boldsymbol{r}-\boldsymbol{r}_n|^3}(1+q_n)\exp(-q_n) + \mathcal{H}\frac{m_n[\tilde{\boldsymbol{v}}_n(\boldsymbol{r}-\boldsymbol{r}_n)]}{|\boldsymbol{r}-\boldsymbol{r}_n|^3}(\boldsymbol{r}-\boldsymbol{r}_n)\right.$$

$$\times\frac{9(1+q_n+q_n^2/3)\exp(-q_n) - \left(9+6\sqrt{3}q_n+4q_n^2\right)\exp(-2q_n/\sqrt{3})}{q_n^2}$$

$$\left. + \mathcal{H}\frac{m_n\tilde{\boldsymbol{v}}_n}{|\boldsymbol{r}-\boldsymbol{r}_n|}\frac{\left(3+2\sqrt{3}q_n+4q_n^2\right)\exp(-2q_n/\sqrt{3}) - 3(1+q_n)\exp(-q_n)}{q_n^2}\right]$$

# Velocity-dependent contribution to $\Phi$

$$(a\tilde{\boldsymbol{v}}_k)' = -a\left(\nabla\Phi|_{\boldsymbol{r}=\boldsymbol{r}_k} + \mathcal{H}\mathbf{B}|_{\boldsymbol{r}=\boldsymbol{r}_k}\right) = \sum_{n\neq k} \boldsymbol{f}_n(\eta, \boldsymbol{r}_k)$$

overall velocity-dependent part in $\boldsymbol{f}_n$ for various $q_n$:

$3H\lambda a v_n/c^2 \sim 2 \div 4 \times 10^{-3}$

- $H = c\mathcal{H}/a$ ;
  $H_0 \approx 70 \text{ km s}^{-1}\text{Mpc}^{-1}$
- $av_n = c\tilde{v}_n$ ;
  $(av_n)_0 \sim 250 \div 500 \text{ km s}^{-1}$
- $\lambda_0 \approx 3.7 \text{ Gpc}$

velocity-dependent part in $\Phi$ only:

$1 \div 2 \times 10^{-2}$ for $q_n \leq 3$

$$(a\tilde{\boldsymbol{v}}_k)' = -a\left(\nabla\tilde{\Phi}|_{\boldsymbol{r}=\boldsymbol{r}_k}\right)$$

No source containing $\Xi$!

$\tilde{\Phi}$: $\tilde{\boldsymbol{v}}_n$-free part of $\Phi$

$$\Delta\tilde{\Phi} - \frac{3\kappa\bar{\rho}c^2}{2a}\tilde{\Phi} = \frac{\kappa c^2}{2a}\delta\rho \;\longrightarrow\; \tilde{\Phi} = \frac{1}{3} - \frac{\kappa c^2}{8\pi a}\sum_n \frac{m_n}{|\boldsymbol{r} - \boldsymbol{r}_n|}\exp(-q_n)$$

# Velocity-dependent contribution to $\Phi$

Gravitational field generated by

- point-like mass → Velocity-dependent contribution seems inessential for cosmological simulation purposes.
- continuous distribution of mass (example: ball of comoving radius $r_b$ and uniform mass density $\rho_b > \bar{\rho}$):

$$\widetilde{\Phi}_b = -\frac{\kappa c^2 \lambda^3}{2a^4}\frac{\rho_b - \bar{\rho}}{r}\left[\frac{ar_b}{\lambda}\cosh\left(\frac{ar_b}{\lambda}\right) - \sinh\left(\frac{ar_b}{\lambda}\right)\right]\exp\left(-\frac{ar}{\lambda}\right) \quad , \qquad \Delta\widetilde{\Phi} - \frac{3\kappa\bar{\rho}c^2}{2a}\widetilde{\Phi} = \frac{\kappa c^2}{2a}\delta\rho$$

$$\Phi_{vb} = -\frac{3\kappa c^2 \mathcal{H}\lambda^5}{2a^6}\frac{\rho_b \tilde{v}_b}{r^2}$$
$$\times\left\{-\frac{1}{3}\left(\frac{ar_b}{\lambda}\right)^3 + \left(1 + \frac{ar}{\lambda}\right)\left[\frac{ar_b}{\lambda}\cosh\left(\frac{ar_b}{\lambda}\right) - \sinh\left(\frac{ar_b}{\lambda}\right)\right]\exp\left(-\frac{ar}{\lambda}\right)\right\} \quad , \qquad \Delta\Phi_v - \frac{3\kappa\bar{\rho}c^2}{2a}\Phi_v = -\frac{3\kappa c^2\mathcal{H}}{2a}\Xi$$

$\Phi_v$: velocity-dependent part of $\Phi$

$$\Phi_v = \frac{3\kappa c^2\mathcal{H}}{8\pi a}\sum_n \frac{m_n[\tilde{\boldsymbol{v}}_n(\boldsymbol{r} - \boldsymbol{r}_n)]}{|\boldsymbol{r} - \boldsymbol{r}_n|}\frac{1 - (1 + q_n)\exp(-q_n)}{q_n^2}$$

$$\frac{\Phi_{vb}}{\widetilde{\Phi}_b} \propto \frac{3\mathcal{H}\lambda\tilde{v}_b}{a}\frac{\rho_b}{\rho_b - \bar{\rho}}$$

# Screening in linear perturbation theory

Relativistic perturbation theory applicable to large scales $\delta\varepsilon(\eta,\boldsymbol{r}) \ll \bar{\varepsilon}$

$$\Phi = \frac{D_1}{a}\phi$$

$D_1(\eta)$ : linear growth factor

$\phi(\boldsymbol{r})$

O. Hahn, A. Paranjape, Phys. Rev. D 94, 083511 (2016); arXiv:1602.07699

$$\Delta\Phi - 3\mathcal{H}(\Phi' + \mathcal{H}\Phi) = \frac{1}{2}\kappa a^2 \delta\varepsilon$$

$$\Phi' + \mathcal{H}\Phi = -\frac{1}{2}\kappa a^2 \bar{\varepsilon} v$$

$v(\eta,\boldsymbol{r})$ : velocity potential

$$\Phi'' + 3\mathcal{H}\Phi' + (2\mathcal{H}' + \mathcal{H}^2)\Phi = 0$$

$$\Delta\Phi - 3\mathcal{H}\frac{D_1'}{D_1}\Phi = \frac{1}{2}\kappa a^2 \delta\varepsilon$$

$$\Phi' + \mathcal{H}\Phi = \frac{D_1'}{D_1}\Phi$$

$$D_1'' + \mathcal{H}D_1' + (\mathcal{H}' - \mathcal{H}^2)D_1 = 0$$

$$D_1^{(+)} \propto H \int \frac{da}{(aH)^3} \propto \frac{\mathcal{H}}{a}\int \frac{da}{\mathcal{H}^3}$$

$$D_1^{(-)} \propto \frac{\mathcal{H}}{a}$$

F. Bernardeau, S. Colombi, E. Gaztañaga, R. Scoccimarro, Phys. Rept. 367, 1 (2002); arXiv:astro-ph/0112551

Comoving screening length $l$:

$$l \equiv \frac{1}{\sqrt{3\mathcal{H}^2 f}}, \quad f \equiv \frac{d\ln D_1}{d\ln a}$$

# Helmholtz equations compared

From linear perturbation theory:

$$\Delta\Phi - 3\mathcal{H}\frac{D_1'}{D_1}\Phi = \frac{1}{2}\kappa a^2 \delta\varepsilon$$

$\Phi$ does not explicitly contribute to the energy density fluctuation

$$\frac{D_1'}{D_1}\Phi = -\frac{1}{2}\kappa a^2 \bar{\varepsilon}\nu \longrightarrow l(\eta)$$

From discrete cosmology:

$$\Delta\Phi - \frac{3\kappa\bar{\rho}c^2}{2a}\Phi = \frac{\kappa c^2}{2a}\delta\rho - \frac{3\kappa c^2\mathcal{H}}{2a}\Xi$$

$\delta\rho \rightarrow \tilde{\Phi}$, $\Xi \rightarrow \Phi_v$

$$\delta\varepsilon = \frac{c^2}{a^3}\delta\rho + \frac{3\bar{\rho}c^2}{a^3}\Phi$$

# Structure growth & velocity-dependent contribution to $\Phi$

Large-scale spatial regions: $\Xi = \bar{\rho}\nu$ $\longrightarrow$ $\Delta\Phi - \frac{3\kappa\bar{\rho}c^2}{2a}\Phi = \frac{\kappa c^2}{2a}\delta\rho - \frac{3\kappa\bar{\rho}c^2\mathcal{H}}{2a}\nu$

$$\delta\rho = \frac{2D_1}{\kappa c^2}\Delta\phi - \left(\frac{6\mathcal{H}D_1'}{\kappa c^2} + \frac{3\bar{\rho}D_1}{a}\right)\phi \quad \xrightarrow{\text{Fourier Transform}} \quad \widehat{\delta\rho} = -\frac{2}{\kappa c^2}\left(D_1 k^2 + 3\mathcal{H}D_1' + \frac{3\kappa\bar{\rho}c^2 D_1}{2a}\right)\hat{\phi}$$

M. Eingorn, R. Brilenkov, Phys. Dark Univ. 17, 63 (2017); arXiv:1509.08181

Can the term $\propto \nu$ be safely ignored?

**No!**

$$\widehat{\delta\rho}'' + \mathcal{H}\frac{1+2\chi}{1+\chi}\widehat{\delta\rho}' - \frac{1}{1+\chi}\frac{\kappa\bar{\rho}c^2}{2a}\widehat{\delta\rho} = 0$$

$$\chi \equiv a^2/(k^2\lambda^2)$$

matter-dominated evolution stage: $\mathcal{H}^2 = \kappa\bar{\rho}c^2/(3a)$

growing mode : $D_1^{(+)} \propto a$

$$\widehat{\delta\rho} \propto k^2\hat{\phi}\left(a + \frac{5\kappa\bar{\rho}c^2}{2k^2}\right) \quad ✓$$

$$\widehat{\widehat{\delta\rho}} \propto k^2\hat{\phi}\left(a + \frac{3\kappa\bar{\rho}c^2}{2k^2}\right) \quad ✗$$

unsuitable unless $a \gg \kappa\bar{\rho}c^2/k^2$:

sufficiently small distances where cosmological screening is irrelevant

# Unified approach: effective screening length

- Combine the screening mechanisms

- Replace the source $\propto \Xi$ by the source $\propto \nu \propto \Phi$

- peculiar motion contribution to the gravitational field is insignificant at small enough scales

- nonlinearity scale $(\sim 15\ \mathrm{Mpc}$ today$)$ is much less than the investigated screening ranges

in the regions where the linear perturbation theory fails, the source $\propto \Xi$ is ignored completely together with the term $\propto \Phi$ so as to yield the standard Poisson equation

$$\Delta\Phi = \frac{\kappa c^2}{2a}\delta\rho$$

# Effective screening length

$$\Delta\Phi - \frac{a^2}{\lambda_{\mathrm{eff}}^2}\Phi = \frac{\kappa c^2}{2a}\delta\rho$$

single velocity-independent source (scheme of linear perturbation theory)

Source $\propto \delta\rho$ : analytically determined by the positions of gravitating masses (scheme of discrete cosmology)

$\lambda_{\mathrm{eff}}(\eta)$: effective physical (non-comoving) screening length

$$\frac{1}{\lambda_{\mathrm{eff}}^2} \equiv \frac{1}{\lambda^2} + \frac{1}{a^2 l^2}$$

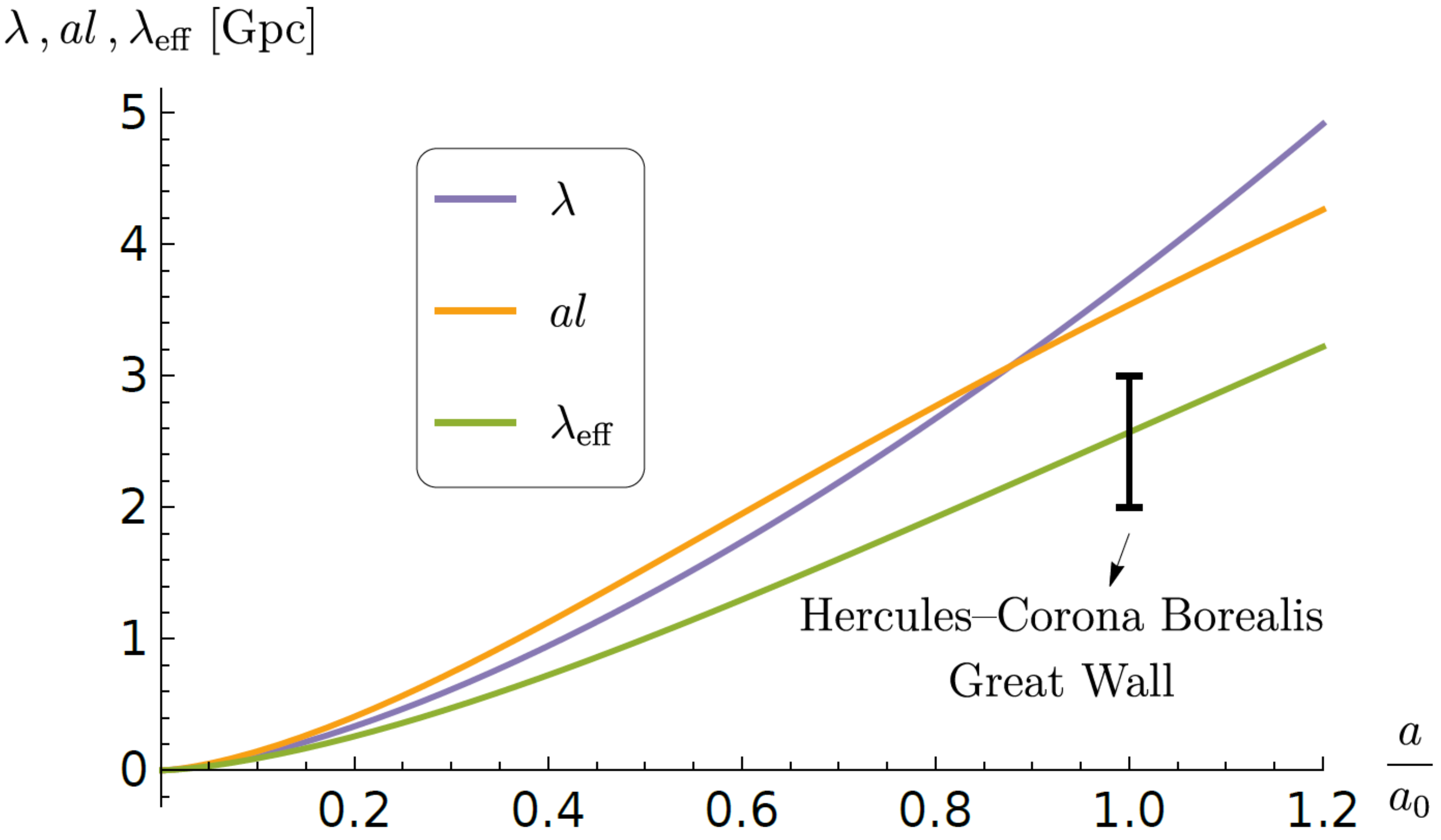
$$\frac{1}{\lambda_{\mathrm{eff}}^2} = \frac{3}{a\mathcal{H}}\left(\int \frac{da}{\mathcal{H}^3}\right)^{-1} = \frac{3}{c^2 a^2 H}\left(\int \frac{da}{a^3 H^3}\right)^{-1}$$

$$H = H_0\sqrt{\Omega_{\mathrm{M}}\left(\frac{a_0}{a}\right)^3 + \Omega_\Lambda}$$

$$\Omega_\Lambda \equiv \frac{\Lambda c^2}{3H_0^2}$$

$$\Omega_{\mathrm{M}} \equiv \frac{\kappa\bar{\rho}c^4}{3H_0^2 a_0^3}$$

# Effective screening length




Current values

$H_0 = 67.4 \text{ km s}^{-1}\text{Mpc}^{-1}$
$\Omega_M = 0.315$
$\Omega_\Lambda = 0.685$

$\lambda_0 = 3.74$ Gpc , $(al)_0 = 3.54$ Gpc , $(\lambda_{eff})_0 = 2.57$ Gpc

# Effective screening length

During **the matter-dominated stage**: $H^2 = H_0^2 \Omega_{\mathrm{M}} (a_0/a)^3 \to 1/H \propto a^{3/2}$

$$\lambda_{\mathrm{eff}} = \sqrt{\frac{2}{15}\frac{c}{H}}\,, \qquad \lambda = \frac{\sqrt{2}}{3}\frac{c}{H}, \qquad al = \frac{1}{\sqrt{3}}\frac{c}{H}$$

$$\lambda_{\mathrm{eff}} < \lambda < al$$

Below the characteristic comoving scale $k^{-1} = \sqrt{2a/(5\kappa\bar{\rho}c^2)} = \lambda_{\mathrm{eff}}/a$ matter overdensity $\widehat{\delta\rho}$ grows substantially.

$\Longrightarrow$

$\lambda_{\mathrm{eff}}$ : upper bound for cosmic structure size

$(\lambda_{\mathrm{eff}})_0 \approx 2.6\ \mathrm{Gpc}$ Her-CrB GW $\approx 2 \div 3\ \mathrm{Gpc}$


I. Horváth, J. Hakkila, Z. Bagoly, A&A 561, L12 (2014); arXiv:1401.0533

I. Horváth, Z. Bagoly, J. Hakkila, L.V. Toth, A&A 584, A48 (2015); arXiv:1510.01933

# Conclusion

$$\Delta\Phi - \frac{a^2}{\lambda_{\mathrm{eff}}^2}\Phi = \frac{\kappa c^2}{2a}\delta\rho \ ,$$

$$\Phi = \frac{1}{3}\left(\frac{\lambda_{\mathrm{eff}}}{\lambda}\right)^2 - \frac{\kappa c^2}{8\pi a}\sum_n \frac{m_n}{|\boldsymbol{r}-\boldsymbol{r}_n|}\exp\left(-\frac{a|\boldsymbol{r}-\boldsymbol{r}_n|}{\lambda_{\mathrm{eff}}}\right)$$

- valid for arbitrary distances
- ensures Newtonian gravitational interaction at sub-horizon scales & in the Minkowski background limit
- compact & convenient in comparison to

$$\Phi = \frac{1}{3} - \frac{\kappa c^2}{8\pi a}\sum_n \frac{m_n}{|\boldsymbol{r}-\boldsymbol{r}_n|}\exp(-q_n) + \frac{3\kappa c^2\mathcal{H}}{8\pi a}\sum_n \frac{m_n[\tilde{\boldsymbol{v}}_n(\boldsymbol{r}-\boldsymbol{r}_n)]}{|\boldsymbol{r}-\boldsymbol{r}_n|}\frac{1-(1+q_n)\exp(-q_n)}{q_n^2}$$

- its average value equals zero: $\bar{\Phi} = 0$
- divergent only at the positions of particles