Galactic Clustering with GUP modified Newtonian potential

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Outline

- Structure formation.
- Many models.
- Thermodynamic model.
- Emergent gravity.
- Emergent gravity and uncertainty.
- From uncertainty to GUP.
- Clustering under GUP.

Structure formation

In the standard hierarchical structure formation scenario objects are thought to form via gravitational collapse of peaks in the initial primordial density field characterized by the density contrast field:

$$\delta(x) = \frac{(\rho(x) - \bar{\rho}_m)}{\bar{\rho}_m},$$

where $\bar{\rho}_m$ is the mean mass density of the universe.

Many models

We have many models for cluster formation:

- Spherical collapse model (Gott and Gunn 1972, Lahav et al. 1991)
- Self-similar model (Kaiser 1986)
- Thermodynamic model(Saslaw and Hamilton, 1984)
- Statistical mechanical model (Saslaw 20002)

Thermodynamic model

Thermodynamics, originally a dynamical description on a macroscopic level, originally developed for equilibrium states, can be extended to non-equilibrium systems(de Groot and Mazur, 1962), when

- Globally averaged thermodynamic quantities change more slowly than the local particle configuration.
- The expansion of the universe cancels the long range nature of the gravitational field(rigorous mathematics). Hence the local thermodynamics depends on the local fluctuations of the gravitational field.

Standard procedure

- Calculate equations of state for the equilibrium case, and let these values change slowly compared to the time on which detailed local configuration changes.
- The main result of the theory is that the probability of finding N galaxies in a volume V of arbitrary shape is

$$f(N) = \exp^{-\bar{N}(1-b)-Nb} \frac{\bar{N}(1-b)}{N!} \left[\bar{N}(1-b) + Nb\right]^{N-1}$$

where

$$b = -\frac{W}{2K} = \frac{2\pi G m^2 \bar{n}}{3T} \int_0^R \zeta(\bar{n}, T, r) dr$$

is clustering parameter and represents deparures from noninteracting system.

Emergent gravity

- Erik Verlinde proposed entropic nature of gravity (Verlinde 2011).
- The space-time is an information device made of holographic surfaces on which the information about the physical dynamics can be stored and can be retrieved by the variation of this information.
- For a particle of mass m, the change in entropy as a function of seperationΔx from the screen is

$$\Delta S = 2\pi k_B \frac{\Delta x}{l_c} \tag{1}$$

where $l_c = \hbar/mc$ is the Compton length.

Emergent gravity and Uncertainty principle

- Consider a system composed of a holographic screen S and a quantum test particle of mass m. Due to quantum nature of the particle , Heisenberg's Uncertainty principle applies

$$\delta x \delta p = \frac{\hbar}{2}$$

- The uncertainty of position and momentum leads to uncertainty of entropy on screen.
- The uncertainty of entropy can be related to the uncertainty in the total energy via first law of thermodynamics

$$\delta W = \delta S$$

- The uncertainty of entropy is given by(M. A. Santos and I. V. Vancea,2011)

$$\delta S = 2\pi k_B \left(\frac{\delta x}{l_c} + \frac{p\delta p}{m^2 c^2}\right)$$

From Uncertainty principle to GUP

- Once quantum correction has been introduced it is quite natural to introduce generalized uncertainty principle.

$$\delta x \delta p = \frac{\hbar}{2}(1 + \beta(\delta p)^2)$$

- This leads to GUP corrected entropy which in turn leads to GUP modified Newtonian potential

$$F = F_0 \left(1 + \frac{\alpha^2 \beta}{R^4} + \mathcal{O}(\alpha^4)\right),$$

where $F_0 = \frac{MmQ^2c^3}{8\pi\hbar\eta R^2}, \beta = \gamma\left(\frac{2Mc^2Q}{4\pi k_B}\right), l_p^2(planklength) = \frac{G\hbar}{c^3}$, and $Q = \sqrt{8\pi k_B \eta l_p^2}$.

Clustering with the modified gravity law

Paper under review in *Astronomy and Astrophysics* journal

- We employ the statistical approach to understand the structure formation.
- We study the distribution function through the construction of partition function for the cluster.
- The Grand canonical partition function for our system of N gravitationally interacting galaxies is

$$Z_N(T,V) = \frac{1}{N!}\left(\frac{2\pi MT}{\Lambda^2}\right)^{3N/2} V^N \left(1 + (\alpha_1 + \alpha_2)x\right).$$

- The general distribution function with the corrected parameter α_2 is

$$F(N) = \frac{\bar{N}}{N!}(1-b_g)[\bar{N}(1-b_g) + Nb_g]^{N-1} e^{-Nb_g - \bar{N}(1-b_g)}$$

where b_g is the modified correction parameter.

Equations of state

- Helmholtz free energy

$$F = NT\left(\ln\frac{N}{V}T^{-3/2}\right) - NT - NT\ln\left[1 + (\alpha_1 + \alpha_2)\,x\right] - \frac{3}{2}NT\ln\left(\frac{2\pi M}{\lambda^2}\right).$$

- Entropy

$$S = N\ln\left(\frac{V}{N}T^{3/2}\right) + N\ln[1 + (\alpha_1 + \alpha_2)x] - 3N\frac{(\alpha_1 + \alpha_2)\,x}{(1 + (\alpha_1 + \alpha_2)\,x)} + \frac{5}{2}N + \frac{3}{2}N\ln\frac{2\pi M}{\lambda^2}.$$

- Internal energy

$$U = \frac{3}{2}NT\left[1 - 2\frac{(\alpha_1 + \alpha_2)\,x}{1 + (\alpha_1 + \alpha_2)x}\right].$$

Plot of general distribution function with and without correction

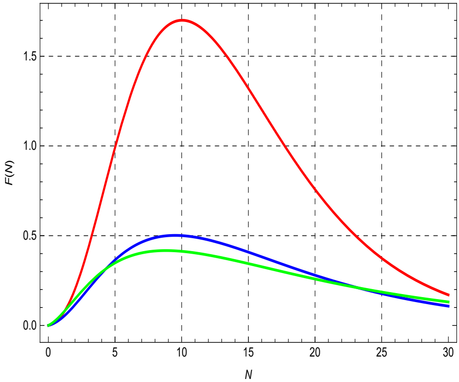

Figure: Behavior of the distribution function $F(N)$ as a function of particle number, N. Red, blue , and green curves represent three different values for the correction parameter α_2. Red curve is for α_2=0 i,e. no correction, blue for α_2=0.5, and green for α_2=1. From the plot we confer that the correction parameter shifts the peak downwards without

- Pressure

$$P = \frac{NT}{V}\left[1 - \frac{(\alpha_1 + \alpha_2)\,x}{1 + (\alpha_1 + \alpha_2)x}\right].$$

- Chemical potential

$$\mu = T\left(\ln \frac{N}{V} T^{-3/2}\right) + T \ln\left[1 - \frac{(\alpha_1 + \alpha_2)\,x}{1 + (\alpha_1 + \alpha_2)x}\right] - T\frac{(\alpha_1 + \alpha_2)\,x}{1 + (\alpha_1 + \alpha_2)x} - \frac{3}{2} T \ln\left(\frac{2\pi M}{\lambda^2}\right).$$

Conclusion

Upon comparison of the equations of state, derived utilizing the GUP corrected potential, to their standard form (Saslaw et al.) , the effect of the correction on the clustering parameter b_g is seen, while the basic structure of the equations is preserved. The modified clustering parameter is given by

$$b_g = \frac{(\alpha_1 + \alpha_2)\, x}{F' + (\alpha_1 + \alpha_2)x}. \tag{2}$$

In the limit $\alpha_2 \to 0$, the modified parameter, b_g, reduces to the original parameter, b, defined as

$$b = \frac{\alpha_1 x/F'}{1 + \alpha_1 x/F'}. \tag{3}$$

THANK YOU