The moving tetrahedron mesh hydrodynamics for numerical simulation of galaxies

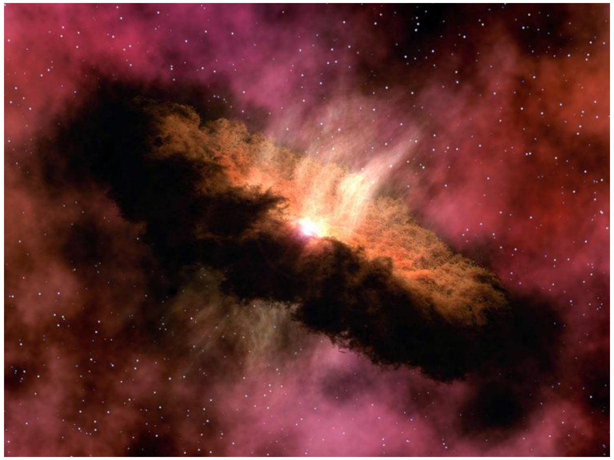
Dr. of Sc. Igor Kulikov

Institute of Computational Mathematics and Mathematical Geophysics SB RAS

Novosibirsk, Russia

kulikov@ssd.sscc.ru

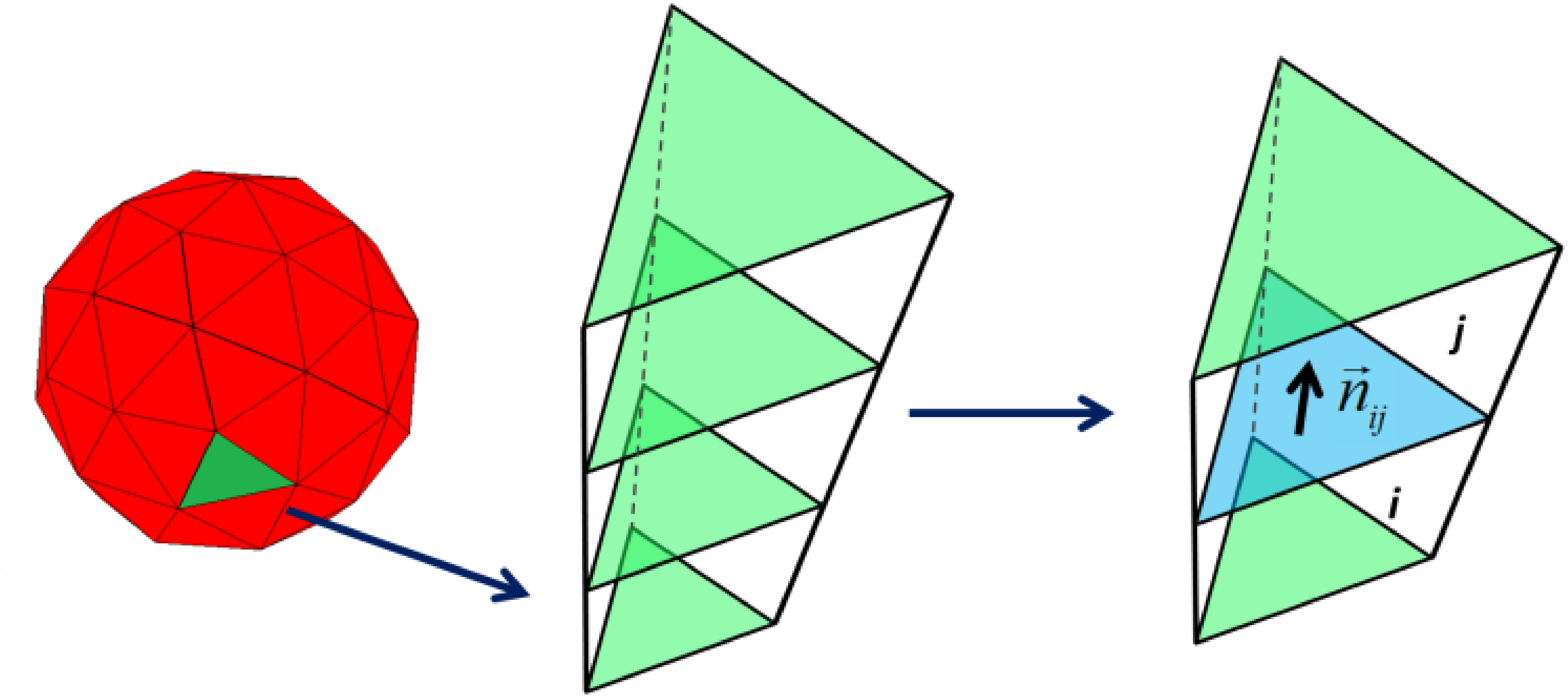
Star Formation



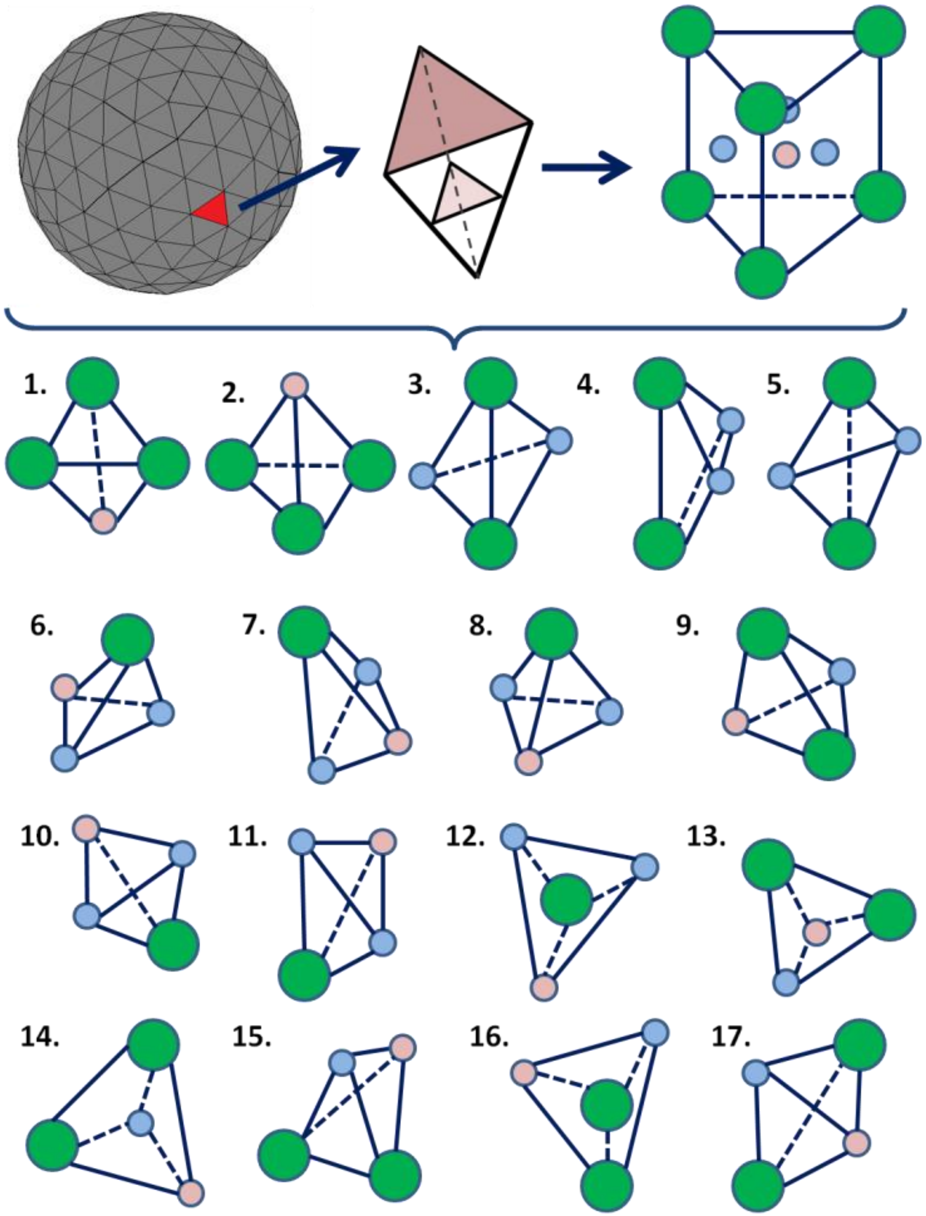
A new numerical method based on the mathematical apparatus of moving geodesic tetrahedron meshes is proposed. This approach allows to simulate spherical objects without features that occur when the spherical or cylindrical coordinates are used.

The method for solving hyperbolic equations is described in detail. The numerical method and the approach to constructing grids developed in the article made it possible to obtain a numerical solution that is invariant with respect to the rotation in Cartesian coordinates, which in turn allows to use this approach quite effectively for modeling arbitrary spherical astrophysical objects.

The geodesic mesh



Tetrahedron mesh



Hydrodynamics solver

$$\frac{\partial u}{\partial t}+\frac{\partial f(u)}{\partial x}+\frac{\partial g(u)}{\partial y}+\frac{\partial h(u)}{\partial z}=0$$

$$\int_{V_i}\frac{\partial u}{\partial t}dV+\int_{V_i}\left(\frac{\partial f(u)}{\partial x}+\frac{\partial g(u)}{\partial y}+\frac{\partial h(u)}{\partial z}\right)dV=0$$

$$V_i\frac{du}{dt}+\oint_{\Gamma_i}\left(fn^x+gn^y+hn^z\right)dS=0$$

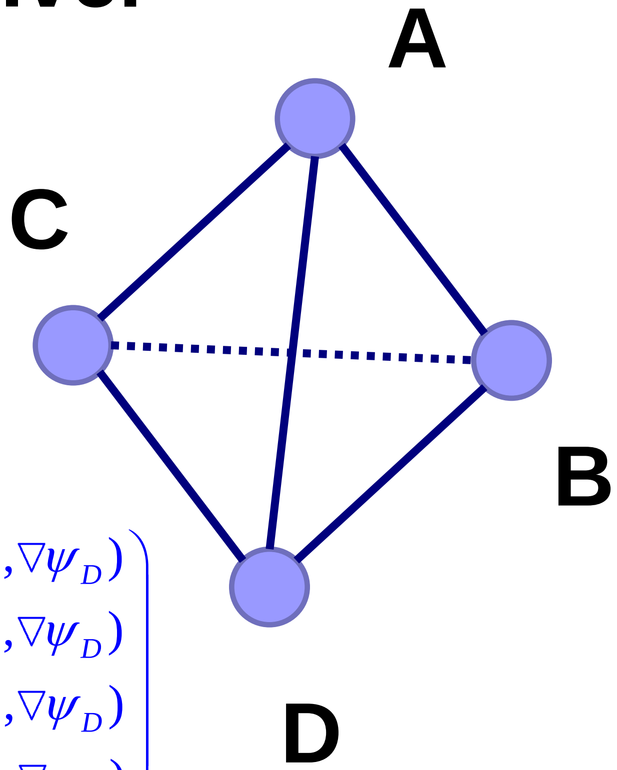
$$\frac{u_i^{n+1}-u_i^n}{\tau}+\sum_j\frac{S_{ij}}{V_i}\left(F_{ij}n_{ij}^x+G_{ij}n_{ij}^y+H_{ij}n_{ij}^z\right)=0$$

$$\mathcal{R}\left(w,u^L,u^R\right)=\frac{w\left(u^L\right)+w\left(u^R\right)}{2}+\frac{\left|\frac{\partial w}{\partial u}\right|}{2}\left(u^L-u^R\right).$$

Poisson FEM solver

$$\psi_i(x,y,z) = a_i x + b_i y + c_i z + d_i$$

$$\begin{pmatrix} A_x & A_y & A_z & 1 \\ B_x & B_y & B_z & 1 \\ C_x & C_y & C_z & 1 \\ D_x & D_y & D_z & 1 \end{pmatrix} \begin{pmatrix} a_A & a_B & a_C & a_D \\ b_A & b_B & b_C & b_D \\ c_A & c_B & c_C & c_D \\ d_A & d_B & d_C & d_D \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



$$S = -V_{ABCD} \begin{pmatrix} (\nabla\psi_A,\nabla\psi_A) & (\nabla\psi_A,\nabla\psi_B) & (\nabla\psi_A,\nabla\psi_C) & (\nabla\psi_A,\nabla\psi_D) \\ (\nabla\psi_B,\nabla\psi_A) & (\nabla\psi_B,\nabla\psi_B) & (\nabla\psi_B,\nabla\psi_C) & (\nabla\psi_B,\nabla\psi_D) \\ (\nabla\psi_C,\nabla\psi_A) & (\nabla\psi_C,\nabla\psi_B) & (\nabla\psi_C,\nabla\psi_C) & (\nabla\psi_C,\nabla\psi_D) \\ (\nabla\psi_D,\nabla\psi_A) & (\nabla\psi_D,\nabla\psi_B) & (\nabla\psi_D,\nabla\psi_C) & (\nabla\psi_D,\nabla\psi_D) \end{pmatrix}$$

$$M = \frac{V_{ABCD}}{20} \begin{pmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{pmatrix}$$

$$\nabla\Phi = \begin{pmatrix} \Phi_A a_A + \Phi_B a_B + \Phi_C a_c + \Phi_D a_D \\ \Phi_A b_A + \Phi_B b_B + \Phi_C b_c + \Phi_D b_D \\ \Phi_A c_A + \Phi_B c_B + \Phi_C c_c + \Phi_D c_D \end{pmatrix}$$

Moving mesh tetrahedron mesh

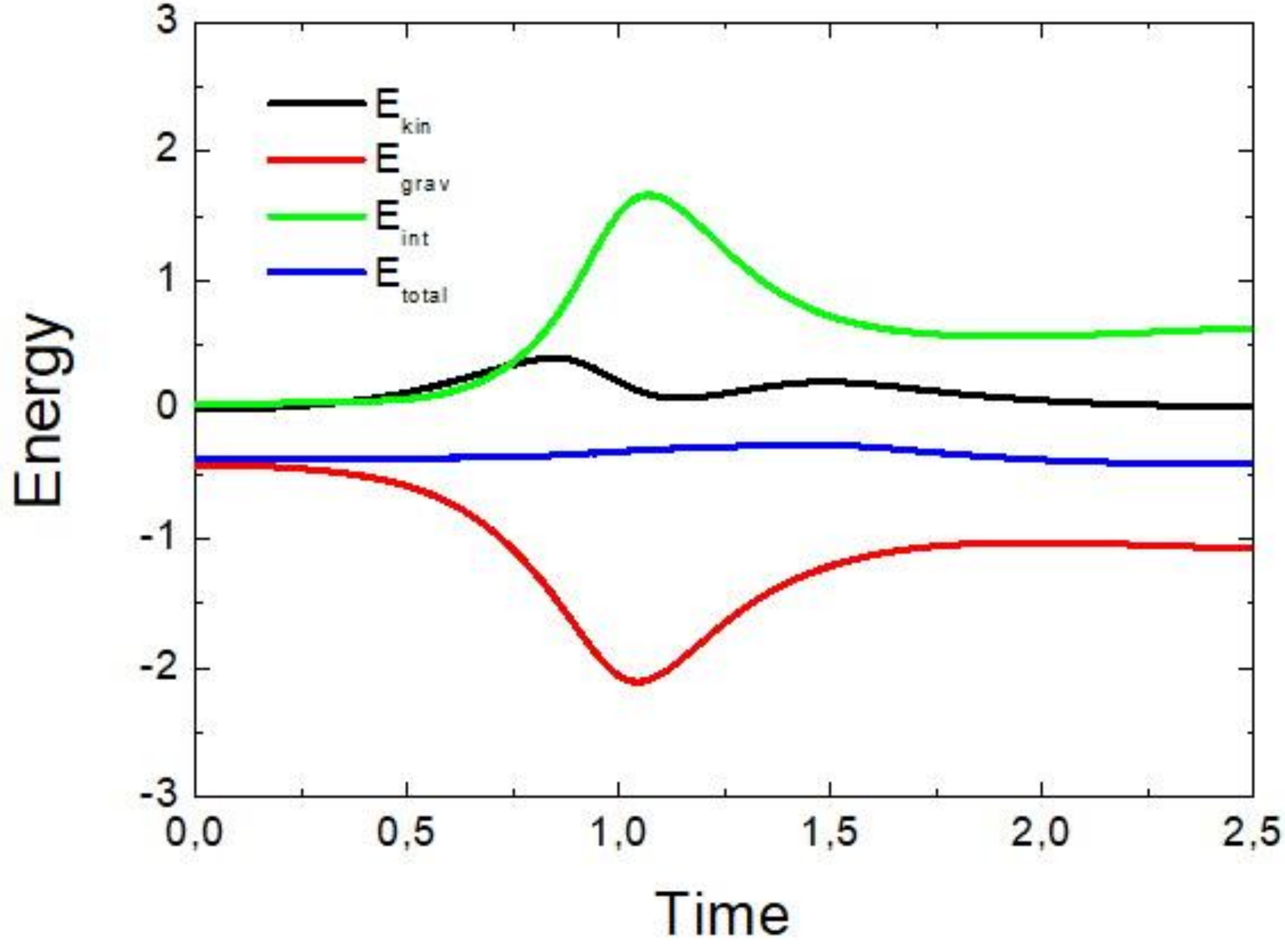
$$\frac{\partial}{\partial t}\begin{pmatrix}\rho\\ \rho\vec{u}\\ \rho E\end{pmatrix}+\nabla\cdot\begin{pmatrix}\rho(\vec{u}+\vec{w}-\vec{w})\\ \rho\vec{u}(\vec{u}+\vec{w}-\vec{w})+P\\ \left[\rho E+p\right](\vec{u}+\vec{w}-\vec{w})\end{pmatrix}=0$$

$$\frac{\partial}{\partial t}\begin{pmatrix}\rho\\ \rho\vec{u}\\ \rho E\end{pmatrix}+\nabla\cdot\begin{pmatrix}\rho\vec{v}\\ \rho\vec{u}\vec{v}+P\\ \left[\rho E+p\right]\vec{v}\end{pmatrix}=0$$

$$\frac{\partial}{\partial t}\begin{pmatrix}\rho\\ \rho\vec{u}\\ \rho E\end{pmatrix}+\nabla\cdot\begin{pmatrix}\rho\vec{w}\\ \rho\vec{u}\vec{w}\\ \rho E\vec{w}\end{pmatrix}=0$$

$$\frac{d\vec{w}}{dt}\approx\nabla\Phi$$

Evrard collapse



Conclusion

- **A new moving mesh hydrodynamics 3D code**
- **A geodesic mesh using for spherical object**
- **A new parallel hydrodynamics & gravity solver**

Thank to

The RFBR- FWF project 19-51-14002

Thank You for Your attention!

CV

Igor Kulikov Doctor of Science, Professor
kulikov@ssd.sscc.ru
Specialist in solving hydrodynamics equation, computational astrophysics parallel computing, and HPC software development.

Kulikov I. A new code for the numerical simulation of relativistic flows on supercomputers by means of a low-dissipation scheme **// Computer Physics Communications. – 2020. – V. 257. – Article Number 107532.**

Kulikov I., Chernykh I., Tutukov A. A new hydrodynamic code with explicit vectorization instructions optimizations, dedicated to the numerical simulation of astrophysical gas flow. I. Numerical method, tests and model problems // **The Astrophysical Journal Supplement Series. – 2019. – V. 243. - Article Number 4.**

Kulikov I., Vorobyov E. Using the PPML approach for constructing a low-dissipation, operator-splitting scheme for numerical simulations of hydrodynamic flows **// Journal of Computational Physics. – 2016. – V. 317. – P. 318-346.**