KIAS Workshop on Nonlinear Analysis

강문진 (KAIST *moonjinkang81@gmail.com*)

Title: Compressible Navier-Stokes flows perturbed from Riemann datum: long-time behavior and inviscid limits

This talk deals with results on long-time behavior and inviscid limits of compressible Navier-Stokes flows perturbed from Riemann initial datum. Those results play a key role for stability of physically relevant solutions to the associated compressible Euler equations.

강현석 (GIST *kang@gist.ac.kr*)

Title: alpha-mean curvature flow of non-compact complete convex hypersurfaces in Euclidean space

In this talk, we discuss the development of study for the mean curvature flow on convex graphs in Euclidean space. Going back to the result by Ecker and Huisken, the graphness of hypersurfaces moving by the mean curvature flow has been a popular theme. In particular, we consider the alpha-mean curvature flow with the initial projected graph being complete and strictly convex. Typically, a gradient estimate is obtained and subsequently uniform curvature bounds depending on initial data are derived. With appropriate cut-off functions multiplied on the curvature quantity, the curvature bounds are shown to be independent of the height of the graph. Moreover, the projected graphs follows the mean curvature flow of co-dimension two. This is a joint work with Kiahm Lee and Taehun Lee.

김대환 (대구대 *daehwan@daegu.ac.kr*)

Title: Some curvature estimates and Bernstein type theorem of complete f-minimal hypersurfaces in Minkowski space.

The prescribed mean curvature submanifolds have great physical importance both in the Riemannian geometry and in the pseudo-Riemannian geometry. For example, space-like hypersurfaces with a prescribed mean curvature were constructed as the stationary limits of a geometric evolution equation. We firstly show several curvature estimates and a particular inequality related to the mean curvature of a space-like hypersurface in the Minkowski space. Secondly, there is a well-known result of Cheng and Yau: the only complete maximal hypersurface is a hyperplane in the Minkowski space. A extended result for the weighted minimal space-like hypersurface with a restricted condition of the weighte function is provided.

김민현 (Bielefeld *minhyun.kim@uni-bielefeld.de*)

Title: Nonlocal problems with non-standard growth

In the calculus of variations, functionals with non-standard growth have been studied extensively since the late 1980s. We introduce several nonlocal analogues of these classical models, attempting to develop parallel or unified theory. We study local regularity properties such as local boundedness, weak Harnack inequality, and local Hölder regularity.

김수정 (숭실대 *soojungkim@ssu.ac.kr*)

Title: Smectic liquid crystal flows in a magnetic field

Liquid crystal is an intermediate phase between solid and liquid states. In the nematic phase of a liquid crystal, molecules point towards a preferred direction but have no positional order. When lowering the temperature, the smectic phase having layered molecular arrangement occurs. In this talk, we consider the de Gennes model for smectic liquid crystals in an applied magnetic field. Under a planar ansatz, we discuss dynamical instabilities of pure smectic states and pure nematic states induced by magnetic fields. This is joint work with Xing-Bin Pan.

김승혁 (한양대 *shkim0401@gmail.com*)

Title: Positive solutions of the Lane-Emden system near the critical hyperbola

We are concerned with positive solutions of the Lane-Emden system on a smooth bounded convex domain, one of the simplest Hamiltonian-type elliptic systems. Given an arbitrary family of solutions, we thoroughly analyze its asymptotic behavior as the exponents of the nonlinearities tend to the critical ones, establishing a detailed description. In particular, we derive a priori energy bound and prove that the multiple bubbling phenomena arise. The interaction between bubbles may be so strong for this problem, so the determination process of the blow-up locations and rates is very different from that of the classical Lane-Emden equation. As a by-product of our analysis, we obtain a general existence theorem that is also valid on non-convex domains. This is joint work with Sang-Hyuck Moon (National Center for Theoretical Sciences, Taiwan).

김진수 (Postech *jinsukim@postech.ac.kr*)

Title: Stability of stochastically modeled reaction networks

A reaction network is a graphical configuration that describes an interaction between species (molecules). If the abundances of the network system are small, then the randomness inherent in the molecular interactions is important to the system dynamics, and the abundances are modeled stochastically as a jump by jump fashion continuous-time Markov chain. One of the challenging issues facing researchers who study biological systems is the often extraordinarily complicated structure of their interaction networks. Thus, how to characterize network structures that induce characteristic behaviors of the system dynamics is one of the major open questions in this literature. In this talk, I will provide an analytic approach to find a class of reaction networks whose associated Markov process has a stationary distribution.

박향동 (연세대 *hyangdong.park@yonsei.ac.kr*)

Title: Shock and Contact discontinuity

In this talk, I will present recent results on the existence of shocks and contact discontinuities for the steady Euler system

$$\begin{cases} \operatorname{div}(\rho \mathbf{u}) = 0, \\ \operatorname{div}(\rho \mathbf{u} \otimes \mathbf{u}) + \nabla p = \mathbf{0}, \\ \operatorname{div}(\rho \mathbf{u} B) = 0. \end{cases}$$

This talk is based on joint works with Myoungjean Bae.

석진명 (경기대 *jmseok@kgu.ac.kr*)

Title: Kinetic description of stable white dwarfs

In this talk, I will present some results on fermion ground states of the relativistic Vlasov-Poisson system arising in the semiclassical limit from relativistic quantum theory of white dwarfs. I'd like to discuss the existence and orbital stability of fermion ground states of the three dimensional relativistic Vlasov-Poisson system for subcritical mass. We will also see that the mass density of such fermion ground states satisfies the Chandrasekhar equation for white dwarfs. This is a joint work with Prof. Juhi Jang at USC.

안재욱 (동국대 *jaewookahn@dgu.ac.kr*)

Title: Asymptotics of logarithmic chemotaxis models

In this talk, we consider two logarithmic chemotaxis models. As for the first model, we construct a global weak solution that becomes smooth after some waiting time. This solution stabilizes to a constant steady-state under further assumptions on the domain and the system parameters. As for the second model, we construct a global classical solution using a convenient energy method and find the condition on system parameters that make a constant steady-state globally stable or linearly unstable. Related works on the non-existence of non-constant steady states would be also discussed.

양민석 (연세대 *m.yang@yonsei.ac.kr*)

Title: Regularity criteria for the Navier-Stokes equations in terms of pressure

We shall show that a suitable weak solution to the Navier-Stokes equations is regular if the pressure satisfies some mild integrability condition.

옥지훈 (서강대 *jihoonok@sogang.ac.kr*)

Title: Regularity theory for non-autonomous elliptic equations in divergence form

Optimal regularity theory for the non-autonomous problems

$$\mathrm{div} A(x, Du) = 0 \quad \text{and} \quad \min_u \int_\Omega F(x, Du)\, dx$$

is a long-standing issue; the classical approach by Giaquinta and Giusti involves assuming that the nonlinearity A satisfies a structure condition. This means that the growth and ellipticity conditions depend on a given special function, such as t^p, $\phi(t)$, $t^{p(x)}$, $t^p + a(x)t^q$, and not only A but also the given function is assumed to satisfy suitable continuity conditions. Hence these regularity conditions depend on given special functions.

In this talk, we discuss on local $C^{1,\alpha}$-regularity for some $\alpha \in (0, 1)$ and C^α-regularity for any $\alpha \in (0, 1)$ of weak solutions and local minimizers of the above problems introducing new ellipticity and continuity assumptions on A or F with a general (p, q)-growth without recourse to special function structure and without assuming Uhlenbeck structure.

We establish a new ellipticity condition using A or F only, which entails that the function is quasi-isotropic, i.e. it may depend on the direction, but only up to a multiplicative constant. Moreover, we formulate the continuity condition on A or F without specific structure and without direct restriction on $\frac{q}{p}$ (the parameters from the (p, q)-growth condition). Previously known, essentially optimal, regularity results are included as special cases.

유상현 (고려대 *sanghyeon_yu@korea.ac.kr*)

Title: Mathematical analysis of meta-materials

Meta-materials, which are materials made of artificially designed atoms, is an emerging field between physics and materials science. Meta-materials exhibit many exotic phenomena such as negative refractive index, super-focusing and invisibility cloaks. The rational design of meta-materials requires to solve interesting problems involving PDEs, spectral geometry, operator theory and topology. In this talk, we discuss our recent works on surface plasmons, acoustic bubbles and their applications to meta-materials.

정인지 (서울대 *injee_j@snu.ac.kr*)

Title: Instabilities in vortex ring dynamics

The evolution of incompressible inviscid fluids is governed by the Euler equations. We consider the dynamics of vortex rings, which are axisymmetric solutions to the three dimensional Euler equations with concentrated axial vorticity. We prove the following infinite norm growth results: (i) filamentation (formation of a long tail) behavior from a single vortex ring, and (ii) vortex stretching from the "collision" of two vortex rings with opposite signs. Joint work with Kyudong Choi (UNIST).

진상돈 (중앙대 *sdjin@cau.ac.kr*)

Title: Uniqueness and orbital stability of standing waves for the nonlinear Schr ödinger equation with a partial confinement

We consider the 3d cubic nonlinear Schrödinger equation with a strong 2d harmonic potential. The model is physically relevant to observe the lower-dimensional dynamics of the Bose-Einstein condensate, but its ground state cannot be constructed by the standard method due to its supercritical nature. In Bellazzini-Boussaïd-Jeanjean-Visciglia, the authors constructed a proper ground state introducing a constrained energy minimization problem. In this talk, we further investigate the properties of the ground state. First, we show that as the partial confinement is increased, the 1d ground state is derived from the 3d energy minimizer with a precise rate of convergence. Then, by employing this dimension reduction limit, we prove the uniqueness of the 3d minimizer provided that the confinement is sufficiently strong. Consequently, we obtain the orbital stability of the minimizer, which improves that of the set of minimizers in the previous work of Bellazzini-Boussaïd-Jeanjean- Visciglia.

최미란 (서강대 *miranchoi831030@gmail.com*)

Title: Averaging for dispersion managed nonlinear Schrödinger equations

In this talk, we consider the nonlinear Schrödinger equation with periodically varying coefficients and its averaged equation, the so-called Gabitov–Turitsyn equation, which models strong dispersion management in nonlinear optics. We justify the averaging procedure, via the convergence theorem. For sufficiently strong variation, we prove the global well-posedness in H^1 when the exponent of the nonlinearity is beyond the mass–critical power. This is joint work with Young-Ran Lee.

최범준 (Postech *cbj5212@gmail.com*)

Title: Liouville theorem for surfaces translating by sub-affine-critical powers of Gauss curvature

We classify the translators to the flows by sub-affine-critical powers of Gauss curvature in $\mathbb{R}^3$. If α denotes the power, this is a Liouville theorem for degenerate Monge-Ampere equations $\det D^2u = (1+|Du|^2)^{2-\frac{1}{2\alpha}}$ for $0 < \alpha < 1/4$. For the affine-critical-case $\det D^2u = 1$, the classical result by Jorgens, Calabi and Pogorelov shows the level curves of given solution are homothetic ellipses. In our case, the level curves converge asymptotically to a round circle or a curve with k-fold symmetry for some $k > 2$. More precisely, these curves are closed shrinking curves to the $\frac{\alpha}{1-\alpha}$-curve shortening flow that were previously classified by B. Andrews in 2003. This is a joint work with K. Choi and S. Kim.

최영필 (연세대 *ypchoi@yonsei.ac.kr*)

Title: Finite-time singularity formation for the Vlasov-Fokker-Planck equation with singular interaction forces

We discuss the finite time loss of smoothness of solutions to the Vlasov-Fokker-Planck equations with singular interaction forces. It is well known from that the any smooth solution to the Vlasov-Poisson system in the gravitational case can exist only on a finite interval of time when $d \geq 4$. We extend that classical result to the case with attractive Riesz potentials or even with the linear diffusion and damping in velocity, i.e. linear Fokker-Planck operator. This talk is based on a joint work with In-Jee Jeong.

최우철 (성균관대 *choiwc@skku.edu*)

Title: Asymptotic profile of solutions to the heat equation on thin plate with boundary heating

Surface heating on thin plate is widely used in engineering processes, which is described by the heat equation on thin plate of thickness $h > 0$ with Robin boundary condition on upper and lower faces of the plate. It is important for engineering to understand the solutions of the problem for small $h > 0$. In this paper we obtain a sharp asymptotic profile of the solutions as the thickness $h > 0$ approaches to zero.

표준철 (부산대 *jcpyo@pusan.ac.kr*)

Title: Translating solitons for the mean curvature flow

Translating solitons are not only special solutions of the mean curvature flow but also blow-up limit at a singularity. They are minimal submanifolds in a Riemannian manifold which is conformal with Euclidean space and so they share properties with minimal surfaces in Euclidean space. In this talk, we introduce translating solitons and some of their geometric properties. Moreover, we determine which half-space contains a complete translating soliton of the mean curvature flow and it is related to the well-known half-space theorem for minimal surfaces.