Exercises

(An Introduction to Low-dimensional Topology)

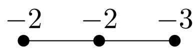
Lecture 1 - Dehn surgery along a framed link

Problem 1. Let Y be a closed oriented 3-manifold. Show that $H_2(Y;\mathbb{Z})$ is determined by $H_1(Y;\mathbb{Z})$, and that the Euler characteristic of Y equals to 0.

Problem 2. Let K be a knot in S^3. Show that $H_*(S^3 \backslash nb(K);\mathbb{Z}) \cong H_*(S^1;\mathbb{Z})$. If Y denotes the 3-manifold obtained by p/q-surgery along K, show that $H_1(Y;\mathbb{Z}) \cong \mathbb{Z}/p\mathbb{Z}$.

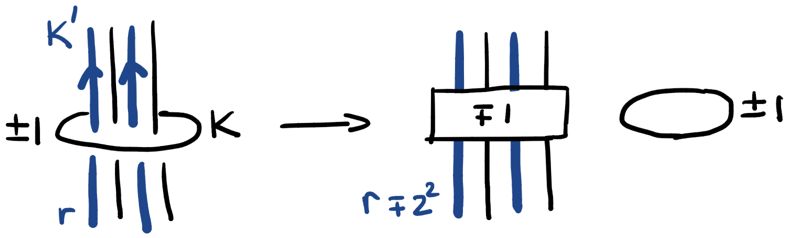
Problem 3. Use van Kampen's theorem to calculate $\pi_1(L(p,q))$.

Problem 4. Show that the following plumbing graph describes $L(7,3)$ and $L(7,5)$ simultaneously, and hence $L(7,3) \cong L(7,5)$. In fact, one can generalize this to show that $L(p,q) \cong L(p,q')$ if $qq' \equiv 1 \bmod p$.



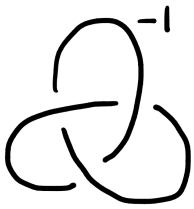
Lecture 2 - Kirby Moves

Problem 5. Let K be a ± 1 framed unknot in a link surgery diagram and K' be another component of the link. Apply Kirby moves to show that, after blowing down K, ∓ 1 full twist is applied to all curves running through K and the framing of K' is added by $\mp(lk(K,K'))^2$.



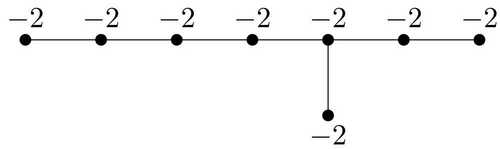
Problem 6. Find a plumbing graph description of Seifert homology sphere $\Sigma((2,3,7))$ and more generally $\Sigma((2,3,6m+1))$.

Problem 7. Use Kirby moves to show that the 3-manifold obtained by (-1)-surgery on the right-handed trefoil knot yields the Seifert homology sphere $\Sigma(2,3,7)$.



Lecture 3 - Intersection forms of 4-manifolds and Rohlin invariants

Problem 8. Show that the E_8 form, which is represented by the incidence matrix of the following graph, is negative definite and unimodular.



Problem 9. Show that the Seifert homology sphere $\Sigma(2,3,7)$ can be represented by the following pluming graph, and that $\mu(\Sigma(2,3,7)) = 1$.