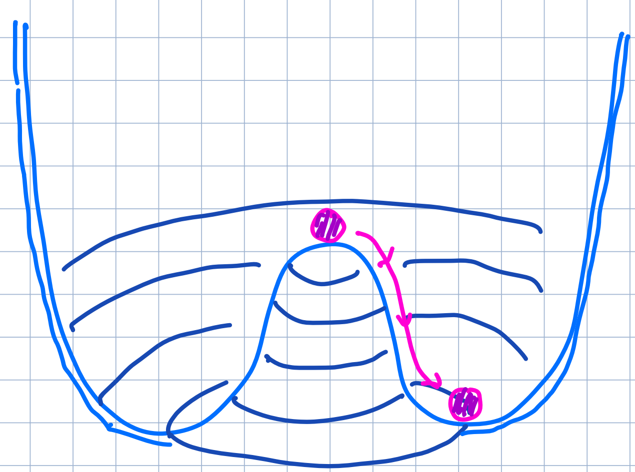
Lecture 3: Soft Bootstrap

We showed that L.I. and factorization imply:

- massless spin 2 $\Longrightarrow$ gravity
- massless spin 1 $\Longrightarrow$ gauge theory
- massless spin 0 $\Longrightarrow$ largely unconstrained !!!

How are scalar EFTs constrained ??

How do we describe symmetry without fields ???



nonlinearly realized symmetry $\quad (\phi \to \phi + c)$

$\Downarrow$

effective Lagrangian $\quad \left(\mathcal{L} = \sum_n c_n (\partial\phi^2)^n \right)$

soft bootstrap

$\Downarrow$

soft zeros $\quad \left(A(p, \dots) \overset{p \to 0}{\sim} p \right)$

Via the bootstrap we can discover new symmetries from amplitudes.

Classification of Theories by Derivative Counting

Consider a general parameterization of a derivatively coupled EFT of a scalar:

allow for various Lorentz + flavor structs.

$$\mathcal{L} = \partial^2\phi^2 \sum_{m,n=0}^{\infty} \lambda_{m,n} \underbrace{\partial^m \phi^n}$$

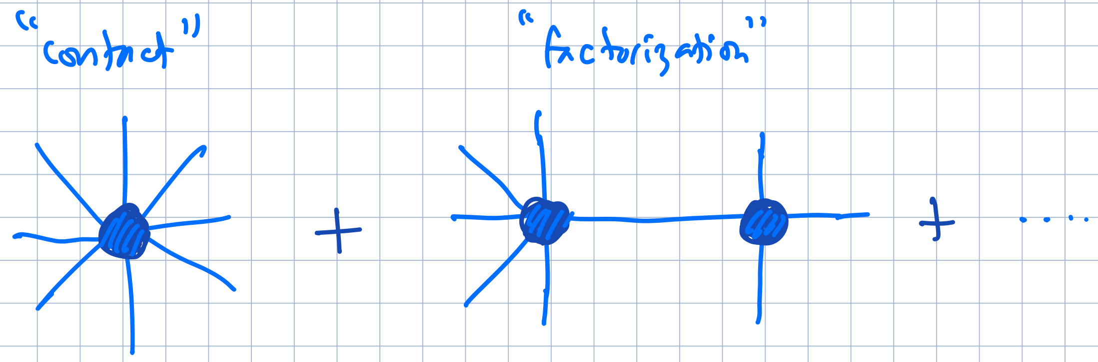
Consider any nonlinear symmetry whose generators T satisfy:

generator of N.L. symmetry

$$[T, D] = 0$$

dilation / rescaling generator

Such a N.L. symmetry must induce interference between terms that scale as the same overall power of momentum p.

$A_n =$ 

$$\lambda_{m,n}\, p^{m+2} + \lambda_{m',n'}\, p^{m'+2}\, \frac{1}{p^2}\, \lambda_{m'',n''}\, p^{m''+2}$$

$$\left.\begin{aligned} m &= m' + m'' \\ n &= n' + n'' \end{aligned}\right\}$$

sufficient condition for "contact" and "factorization" to have same overall power of $p^{\#}$.

$\Longrightarrow$ define $\varsigma \equiv m/n$ = "power counting parameter"

$$\mathcal{L}^{(\rho)} = (\partial\phi)^2 F(\partial^\rho\phi) \Bigg\}$$ classification applies generally, even to include spin

$\rho=0$: $\mathcal{L}^{(0)} = (\partial\phi)^2 F(\phi)$ $\mathcal{L}_{NLSM} = \partial U^\dagger \partial U$

$\mathcal{L}_{GR} = \sqrt{-g}\,R$

$\rho=1$: $\mathcal{L}^{(1)} = (\partial\phi)^2 F(\partial\phi)$ $\mathcal{L}_{U(1)\,NGB} = f(\partial_\mu\phi)$

$\mathcal{L}_{Euler\text{-}Heis.} = f(F_{\mu\nu})$

$\rho=2$: $\mathcal{L}^{(2)} = (\partial\phi)^2 F(\partial\partial\phi)$ $\mathcal{L}_{Galileon} = (\partial\phi)^2 + \partial\phi\,\partial\phi\,\partial\partial\phi + \cdots$

$\rho=-1$: $\mathcal{L}^{(-1)} = (\partial\phi)^2 F(\partial^{-1}\phi)$ $\mathcal{L}_{YM} = \partial A\,\partial A + \partial A A A + A^4$

$\mathcal{L}_{\phi^4} = (\partial\phi)^2 + \phi^4$

$\rho=-2$: $\mathcal{L}^{(-2)} = (\partial\phi)^2 F(\partial^{-2}\phi)$ $\mathcal{L}_{\phi^3} = (\partial\phi)^2 + \phi^3$

Classification of Theories by Soft Behavior

Next, we define a soft parameter σ, such that

$$A(p, \dots) \overset{p \to 0}{\sim} p^{\sigma}$$

where σ: non-neg integer : "soft parameter"

$$\sigma \le \frac{\#\text{ of derivatives}}{\text{per field}} = \frac{m+2}{n+2} = \frac{\rho n + 2}{n+2} \overset{n\to\infty}{=} \rho$$

$\sigma > \rho$ is not generically expected! Thus, enforcing this inequality will nontrivially constrain the theory.

$\sigma > \rho$ (enhanced soft behavior)

$\sigma \le \rho$ (non-enhanced soft behavior)

Soft Bootstrap

Let us bootstrap scalar theories with enhanced $\sigma > \rho$.

3pt

No amplitudes in derivatively coupled scalar EFT.

4pt

$$A_4 = \sum_{\substack{a,b,c \\ \underbrace{a+b+c=\rho+1}}} k_{abc}\, \overset{(p_1p_2)}{s^a}\, \overset{(p_1p_4)}{t^b}\, \underset{(p_1p_3)}{u^c} = \mathcal{O}(p_1^{\rho+1})$$

crucial to yield theory with fixed ρ

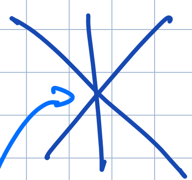
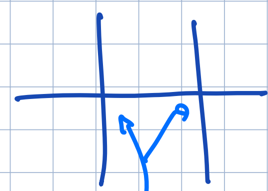
$$\Rightarrow \quad A_4 \overset{p_1\to 0}{\sim} p_1^{\rho+1} \quad \Rightarrow \quad \boxed{\sigma = \rho + 1 > \rho}$$

Four-particle scattering automatically has enhanced soft behavior.

6pt

First, assume some (ρ,σ). Second, build an ansatz.

$$A_6 = A_6^{(\text{cont})} + A_6^{(\text{fact})}$$

$=$  $+$ ( $+ \cdots$)

ansatz polynomial in $p_i p_j$

insert A_4 for given (ρ,σ)

Then impose $A_6(p_1,\cdots) \overset{p_1\to 0}{\sim} p_1^{\sigma}$ to constrain ansatz.

Exceptionally Soft Theories

$\boxed{(\rho,\sigma) = (0,1)}$

$$\mathcal{L}^{(\rho=0)} = \frac{1}{2}\partial_\mu\phi^i\partial^\mu\phi^j\left(\delta_{ij} + \lambda_{ijkl}\phi^k\phi^l + \cdots\right)$$

$$\Bigg\Downarrow \quad (\text{enforce } A \sim p^{\sigma=1})$$

$\mathcal{L}^{(NLSM)}$ for symmetric coset G/H (Susskind + Frye)

$$\boxed{(\rho,\sigma) = (1,2)}$$

$$\mathcal{L}^{(\rho=1)} = \frac{1}{2}(\partial\phi)^2 + \frac{\lambda_4}{4!}(\partial\phi)^4 + \frac{\lambda_6}{6!}(\partial\phi)^6 + \cdots$$

$$\Bigg\Downarrow \quad (\text{enforce } A \sim p^{\sigma=2})$$

$$\mathcal{L}^{(DBI)} = \frac{1}{2}(\partial\phi)^2 + \frac{\lambda_4}{24}(\partial\phi)^4 + \frac{\lambda_4^2}{144}(\partial\phi)^6 + \frac{5\lambda_4^3}{3456}(\partial\phi)^8 + \cdots$$

$$= -\frac{3}{\lambda_4}\sqrt{1 - \frac{\lambda_4}{3}(\partial\phi)^2} + \text{const}$$

DBI (Dirac-Born-Infield) has an nonlinearly realized symmetry corresponding to boosts in an extra dimension w/ coordinate ϕ.

$$\boxed{(\rho, \sigma) = (2, 3)}$$

many derivative structures

$$\mathcal{L}^{(\rho=2)} = \frac{1}{2}(\partial\phi)^2 + \underbrace{\lambda_1 \partial\phi\partial\phi\,\partial\partial\phi\,\partial\partial\phi + \cdots}$$

$\Downarrow$ (enforce $A \sim p^{\sigma=3}$)

$\mathcal{L}^{(\text{Special Galilean})}$ (first discovered w/ amplitudes!)

The Galilean (and its "special" version) is a scalar theory with a high order shift symmetry,

$$\phi \to \phi + a + b_\mu x^\mu + \underbrace{c_{\mu\nu} x^\mu x^\nu + \frac{c_{\mu\nu}}{\Lambda^{\nu}} \partial^\mu\phi\,\partial^\nu\phi}$$

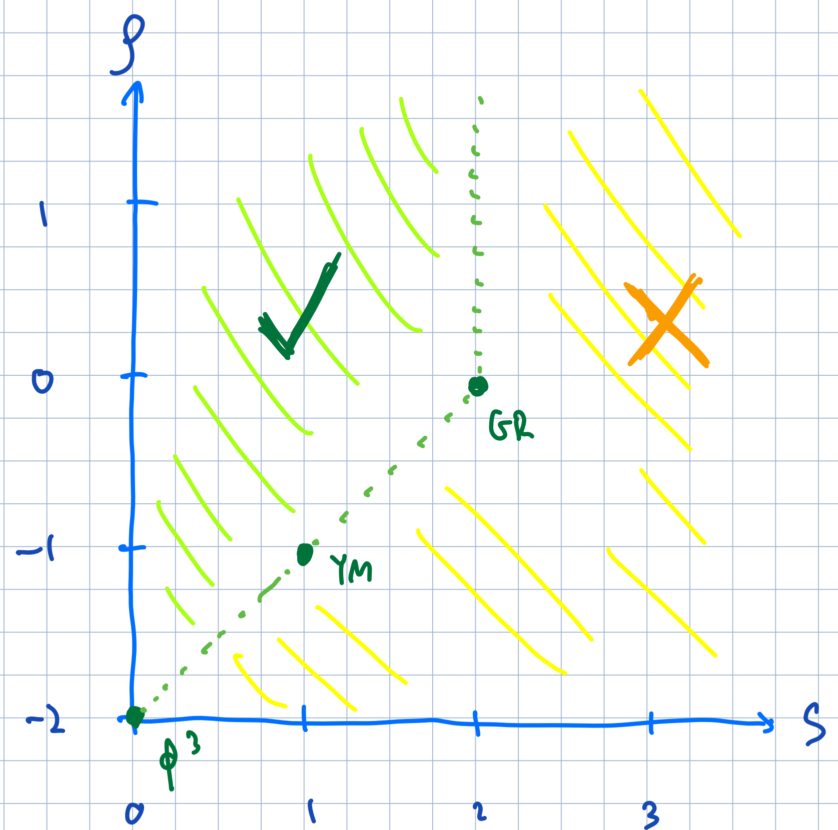
U(1) NGB symmetry

"Special" Galilean symmetry

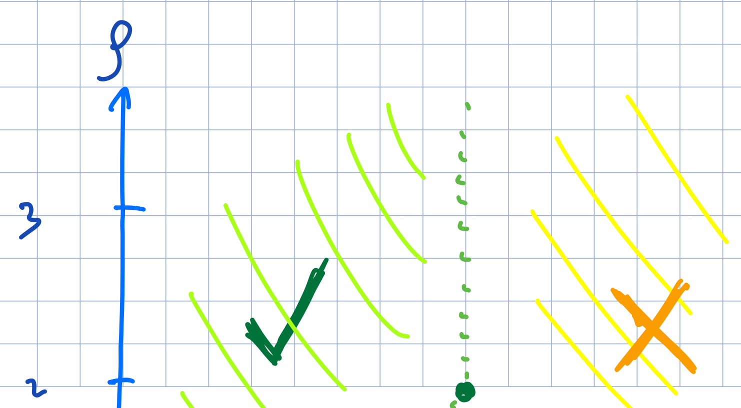
Galilean symmetry

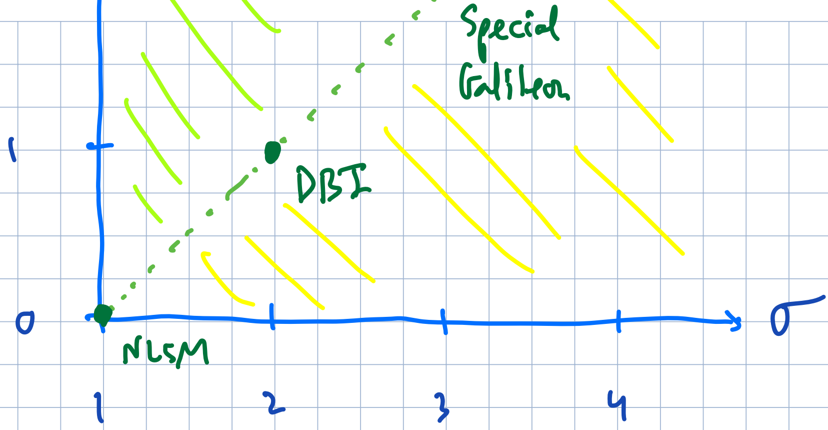
A Map of Theory Space

Lorentz + Factorization (with spin)



Lorentz + Factorization + Soft (without spin)

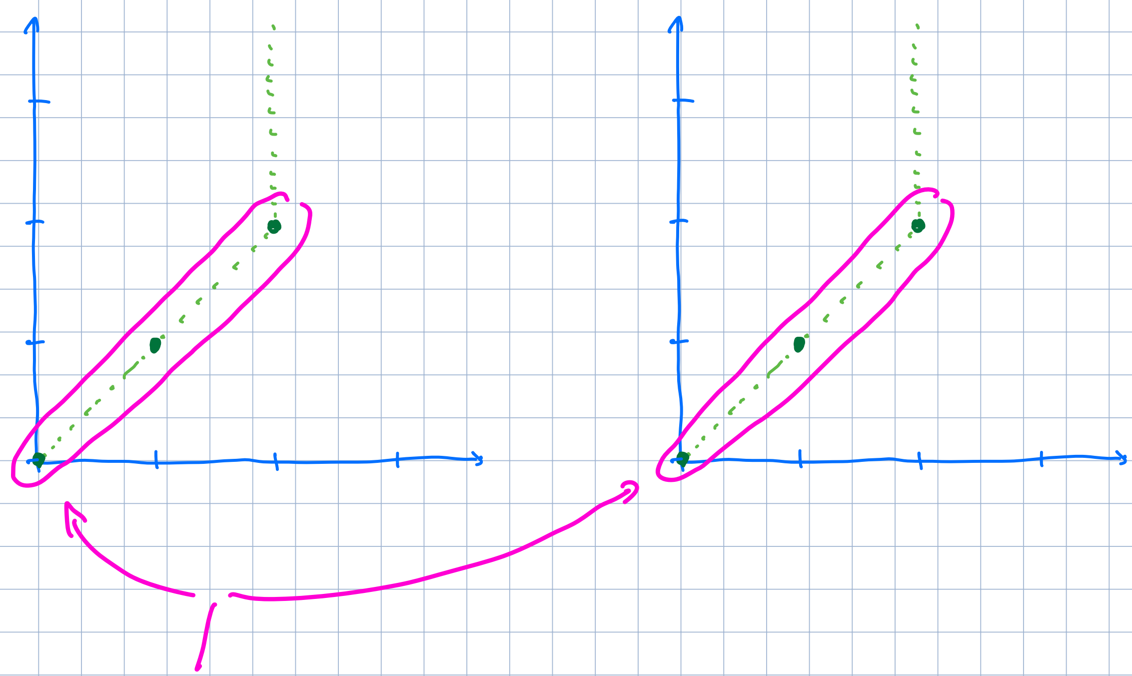




Remarkably, the theories at the edge of existence are connected!!!

Gauge Theory + Gravity

Scalar EFTs



inextricably linked!!!