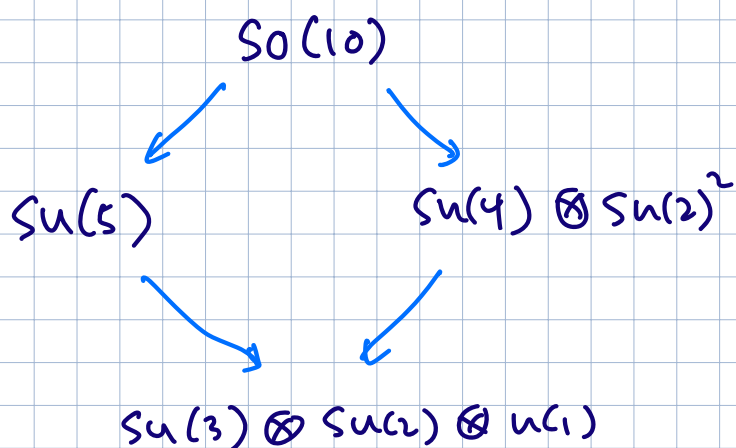


Lecture 4: Unity in Scattering

The theories that keep appearing in the S-matrix program are secretly unified!!

Unification in hep-ph

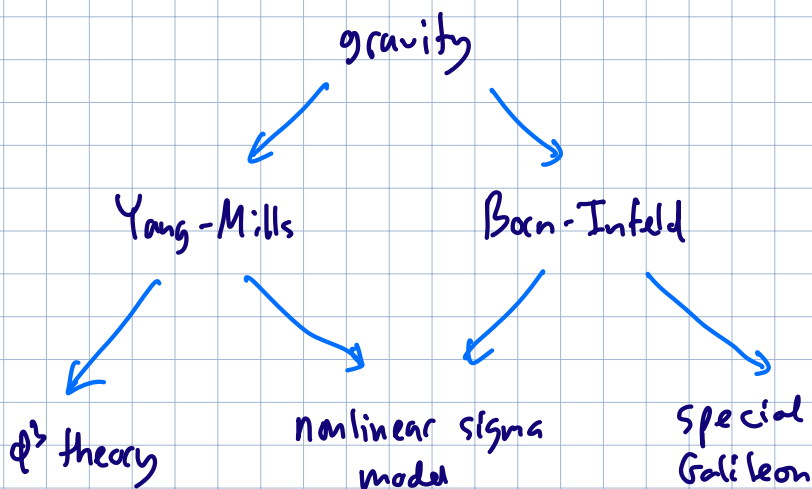


high energy



low energy

Unification in Amplitudes



high spin



low spin

"Gravity is the mother of all theories" : given the tree-level S-matrix for graviton + dilaton + two-form, one obtains all the other S-matrices.

Mandelstam Differential Operators

$$A = e_1^{\Gamma_1} e_2^{\Gamma_2} \dots e_n^{\Gamma_n} A_{p_1, p_2, \dots, p_n}$$

= function of $p_i p_j$, $p_i \epsilon_j$, $\epsilon_i \epsilon_j$ where $p_i^2 = p_i \epsilon_i = \epsilon_i^2 = 0$

Next, construct differential operators imposing physical constraints.

- Ward identity: $A|_{\epsilon_i \rightarrow p_i} \equiv W_i A = 0$

$$\Rightarrow W_i \equiv \sum_{v=p_i, \epsilon_j} (p_i v) \frac{\partial}{\partial (v \epsilon_i)}$$

- total momentum conservation: $P_\nu A = 0$

$$\Rightarrow P_\nu \equiv \sum_i p_i \nu$$

$$[P_\nu, P_{\nu'}] = 0$$

Commutation relations: $[W_i, W_j] = 0$

$$[W_i, P_\nu] = \delta_{\nu i} P_{\nu'} \sim 0$$

annihilates A

Amplitudes Transmutation

Let us construct a "transmutation operator", $\overline{\pi}$, that maps momentum-conserving, gauge invariant functions to new ones, so

$$\text{IF } [W_i, \overline{\pi}] = [P_\nu, \overline{\pi}] = 0$$

(modulo terms that annihilate A)

$$\text{THEN } W_i(\overline{\pi} A) = \overline{\pi}(W_i A) = 0$$

$$P_\nu(\overline{\pi} A) = \overline{\pi}(P_\nu A) = 0$$

transmuted object

$$\overline{T} \equiv \sum_{i,j} A_{ij} \partial p_i \epsilon_j + B_{ij} \partial p_i \epsilon_j + C_{ij} \partial \epsilon_i \epsilon_j$$

(ansatz)

functions of $p_i \epsilon_j, p_i \epsilon_j, \epsilon_i \epsilon_j$
 where $A_{ii} = B_{ii} = C_{ii} = 0$

There are many such operators, but a handful send (color-ordered) amplitudes to other amplitudes!!

transmutation operators

$$\overline{T}_{ij} \equiv \frac{\partial}{\partial \epsilon_i \epsilon_j}$$

(2 gluon \rightarrow 2 scalar)

$$\overline{T}_{ijk} \equiv \frac{\partial}{\partial p_i \epsilon_j} - \frac{\partial}{\partial p_k \epsilon_j}$$

(1 gluon \rightarrow 1 scalar)

$$\underline{L}_i \equiv \sum_j p_i \epsilon_j \frac{\partial}{\partial p_j \epsilon_i}$$

(1 gluon \rightarrow 1 pion)

$$\mathbb{T}[123 \dots n] = \mathbb{T}_{1,n} \prod_{i=2}^{n-1} \mathbb{T}_{i-1,i,n} \quad (n \text{ gluons} \rightarrow n \text{ scalar})$$

$$\mathbb{L}[123 \dots n] = \mathbb{T}_{1,n} \prod_{i=2}^{n-1} \mathbb{L}_i \quad (n \text{ gluons} \rightarrow n \text{ pion})$$

Unifying Relations

BAS = "biadjoint scalar theory" $\sim \phi^3$
 $\mathcal{L}_{BAS} = \frac{1}{2} \partial^{\mu\alpha} \phi^a \partial^{\mu\beta} \phi^a + f^{abc} f^{cde} \phi^a \phi^b \phi^c$

$$A_{BAS} = \mathbb{T}[123 \dots n] A_{YM}$$

$$A_{NLSM} = \mathbb{L}[123 \dots n] A_{YM}$$

graviton + dilaton + 2-form
 $E_{\mu\nu} = E_{\mu} \bar{E}_{\nu}$

$$A_{YM} = \mathbb{T}[123 \dots n] A_{grav}$$

$$A_{BI} = \mathbb{L}[123 \dots n] A_{grav}$$

$$A_{NLSM} = \mathbb{T}[123 \dots n] A_{BI}$$

$$A_{SG} = \mathbb{L}[123 \dots n] A_{BI}$$

Special Galileon

Born-Infeld

e.g. #1) YM $\rightarrow \mathcal{P}^3$

take any representation in
terms of $p_i p_j, l_i l_j, \epsilon_i \epsilon_j$

$$\begin{aligned} A_{\text{BAS}}(\varphi_1, \varphi_2, \varphi_3, \varphi_4) &= \overline{\Pi} [1234] A_{\text{YM}}(g_1, g_2, g_3, g_4) \\ &= \frac{1}{s} + \frac{1}{t} \end{aligned}$$

e.g. #2) YM $\rightarrow \text{NLSM}$

$$\begin{aligned} A_{\text{NLSM}}(\pi_1, \pi_2, \pi_3, \pi_4) &= \mathbb{L} [1234] A_{\text{YM}}(g_1, g_2, g_3, g_4) \\ &= s+t \end{aligned}$$

e.g. #3) YM $\rightarrow \text{SQCD}$

$$\begin{aligned} A_{\text{SQCD}}(\varphi_1, \varphi_2, \varphi_3, \varphi_4) &= \overline{\Pi} [12] \overline{\Pi} [34] A_{\text{YM}}(g_1, g_2, g_3, g_4) \\ &= \frac{u}{s} \end{aligned}$$

Web of Theories

"Gravity as the mother of all theories"

