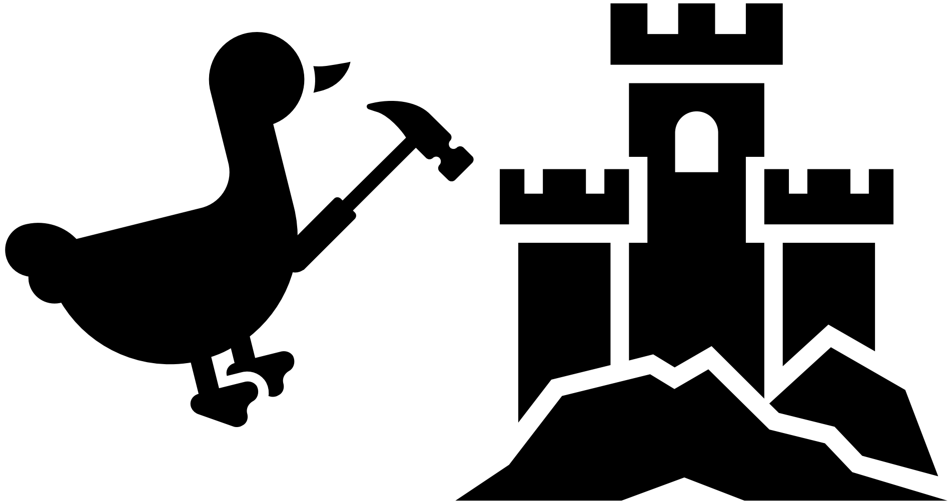
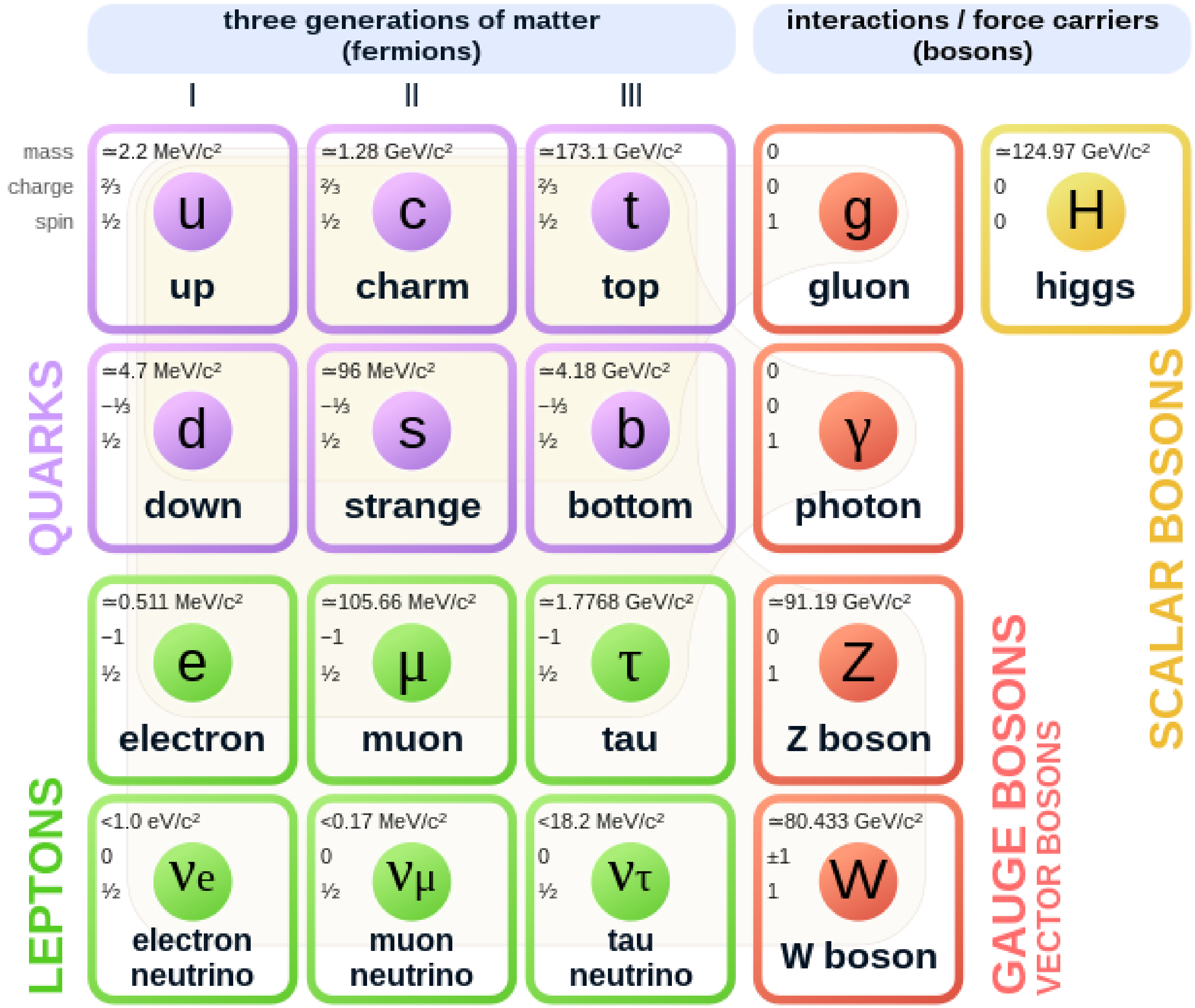
Standard Model
:Bottom to Top Building



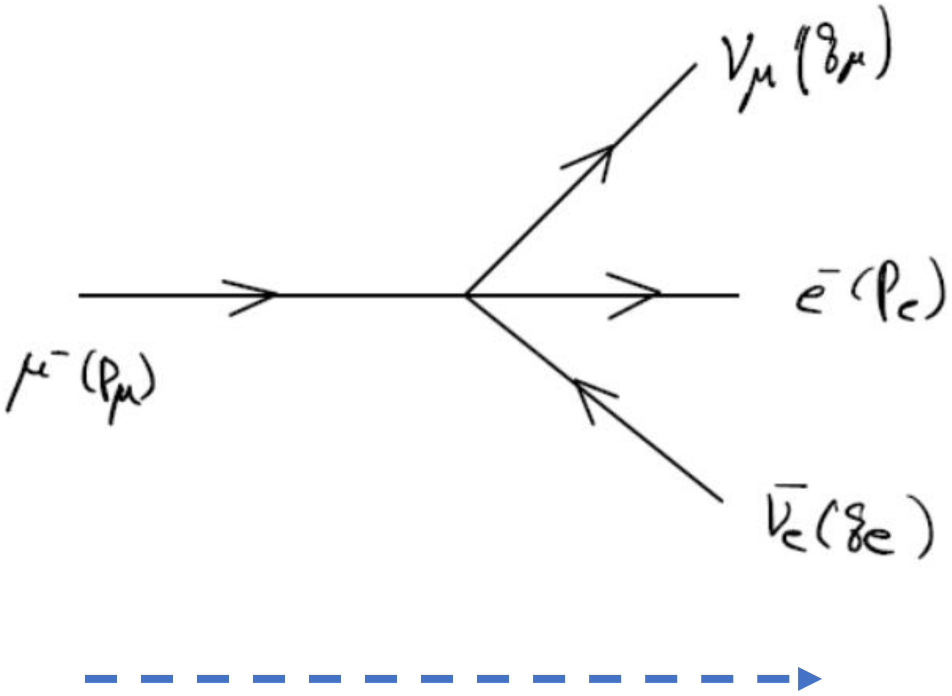
이지헌
KAIST
2022 KIAS Summer camp

Standard Model of Elementary Particles



Feynman Diagram

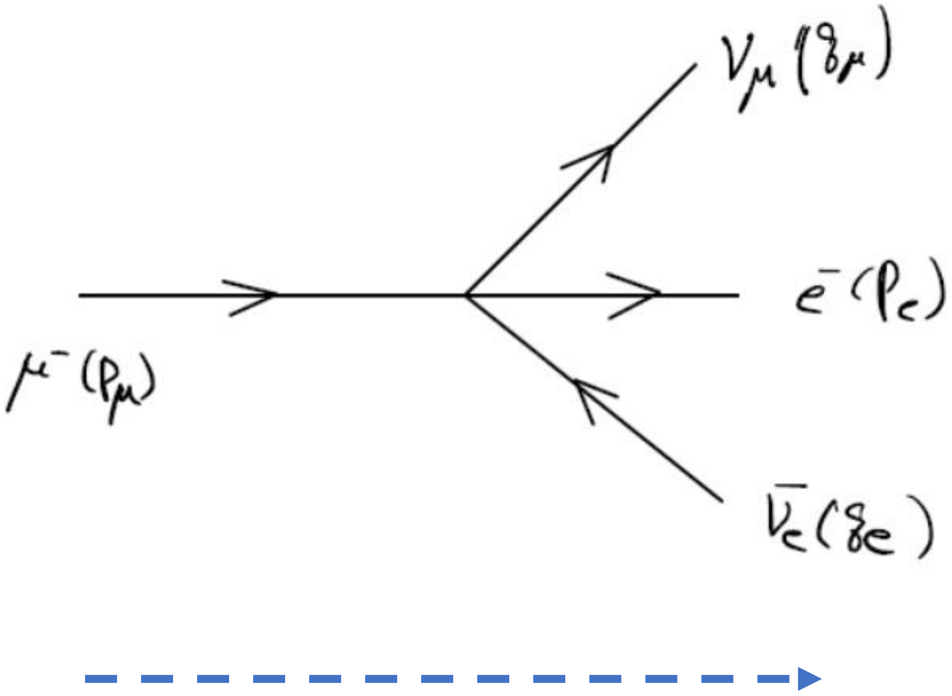
- What physical process does this diagram represent?



Time flow direction

Feynman Diagram

- What physical process does this diagram represent?



Time flow direction

Answer:

Muon (μ^-) decays to muon neutrino (ν_μ), electron (e^-) and electron neutrino (ν^-).

Top-Down vs Bottom to Top

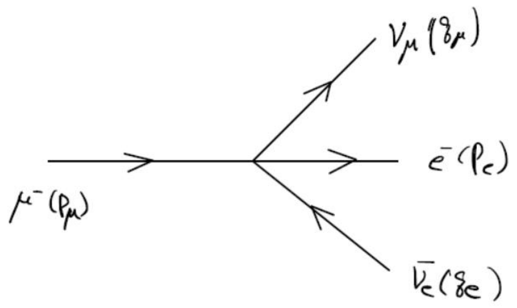
- Top down:
  1. Assign matter field to a particular representation
  2. Add scalar and yukawa coupling
  3. Impose the local symmetries and broke some of them
  4. Work out with the phenomenology

- Bottom to Top
  1. Starts from QED, Fermi model
  2. Use thought experiments and field theory
  3. Add new fields and interactions when it is required

*R.Kleiss, Physics upto 200 TeV(Derivation of the minimal standard model lagrangian),1991

Bottom to Top

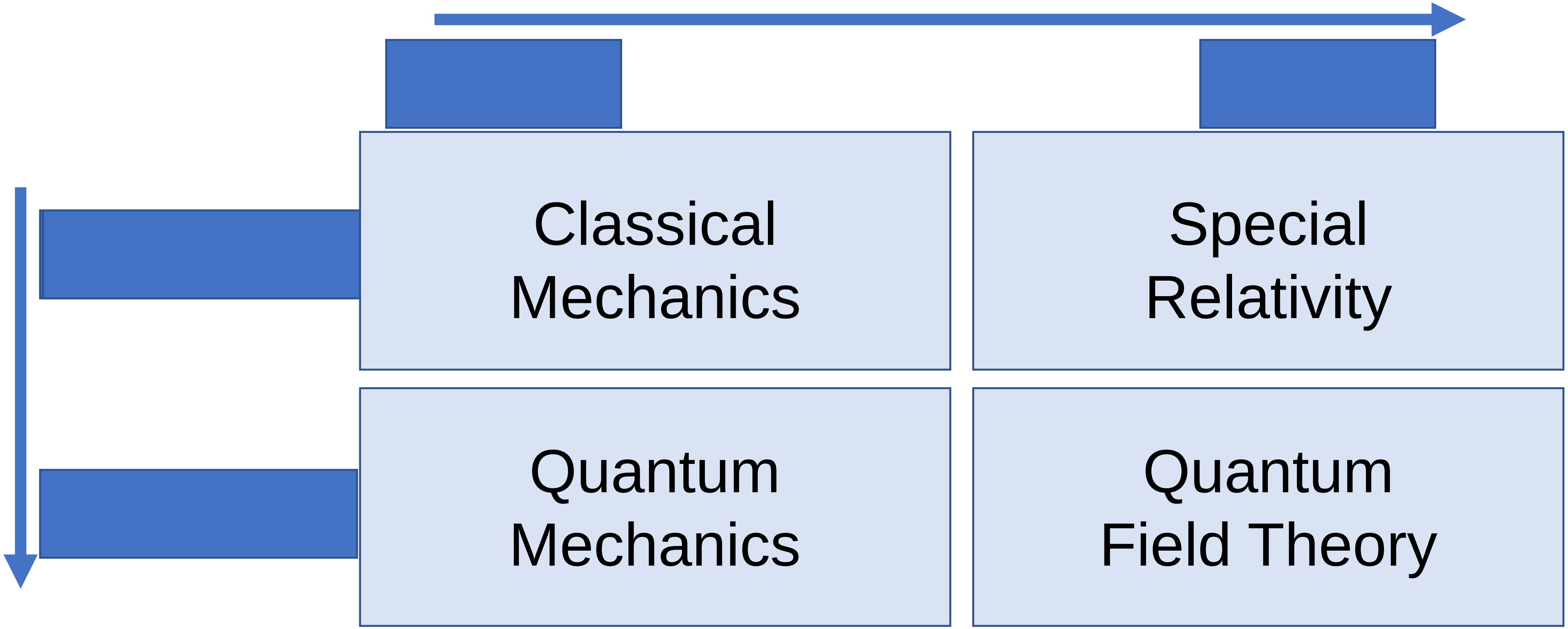
- Fermi Theory : Effective theory of weak interaction. (eg. Muon decay)



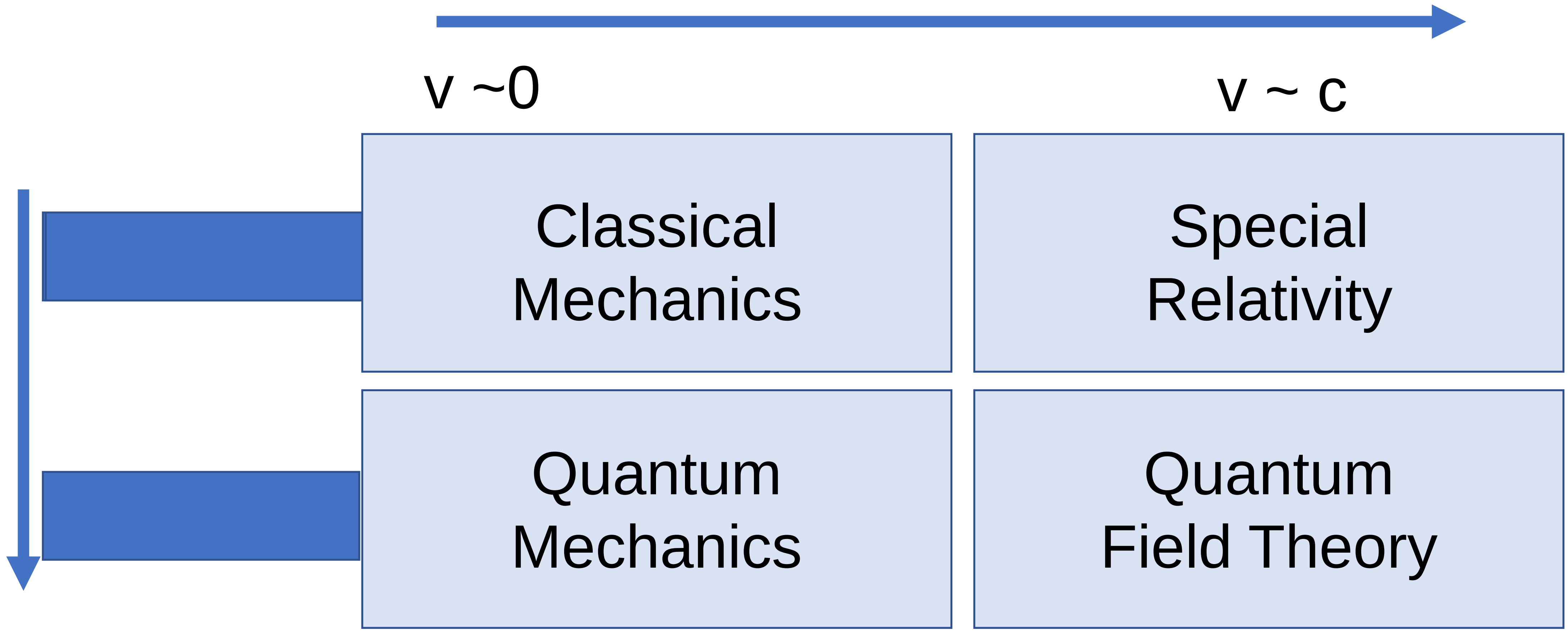
Muon Decay

What is an Effective theory?

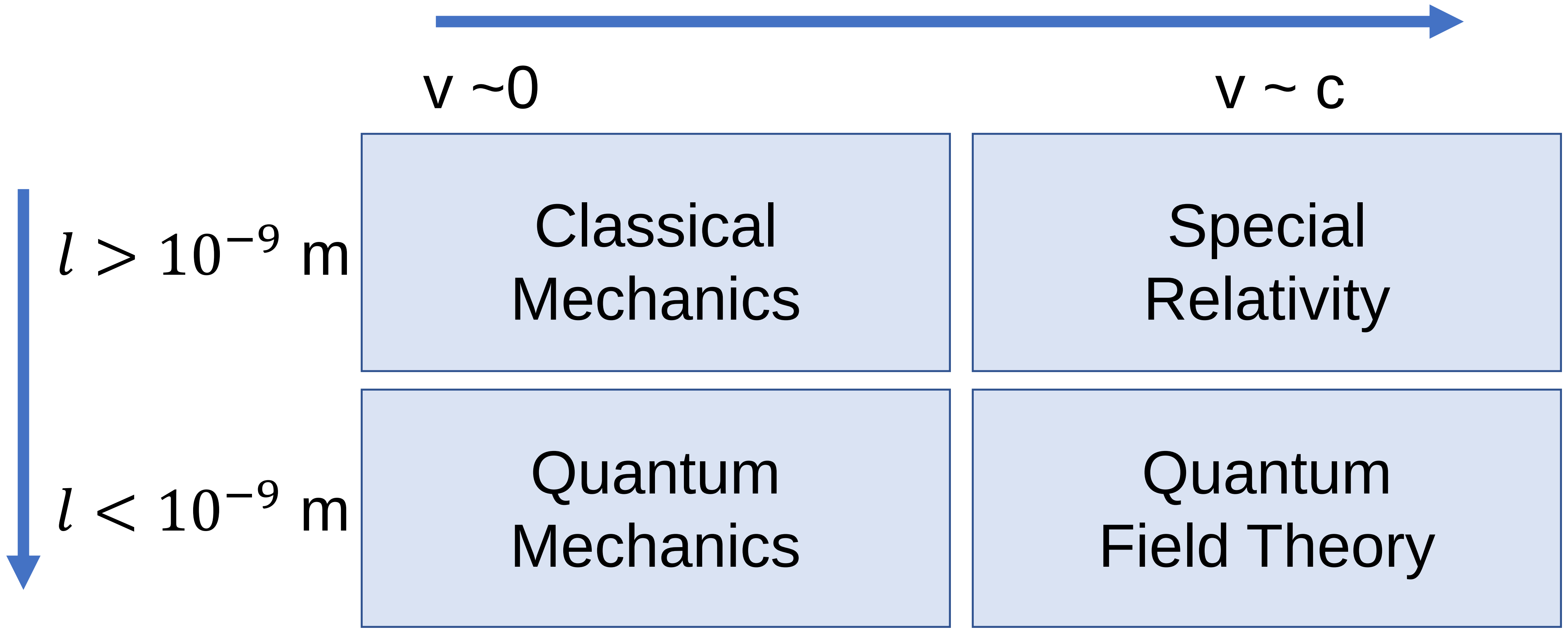
Branches of Physics



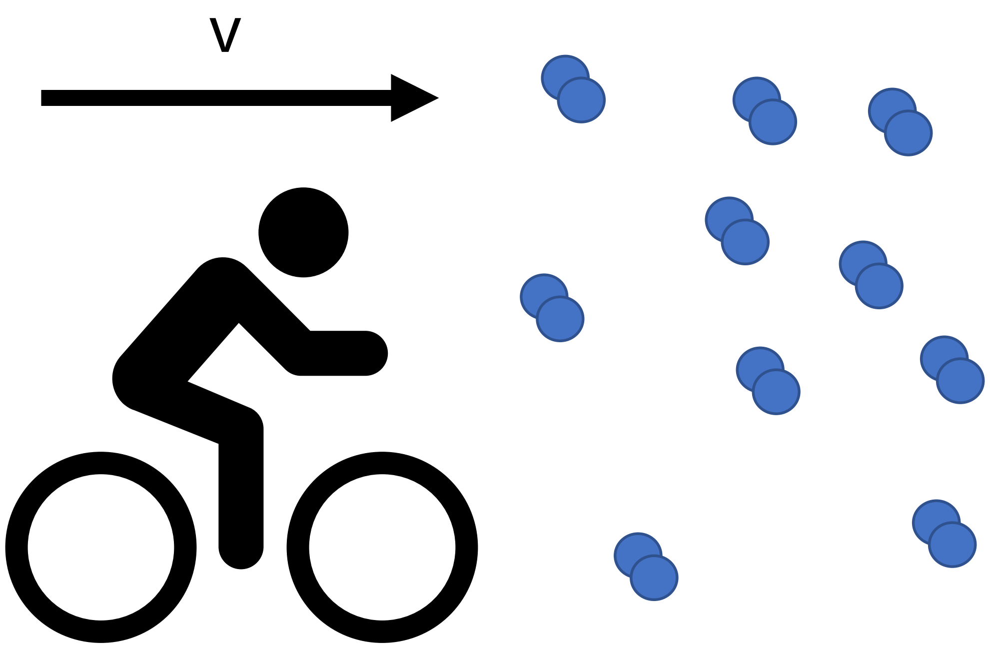
Branches of Physics



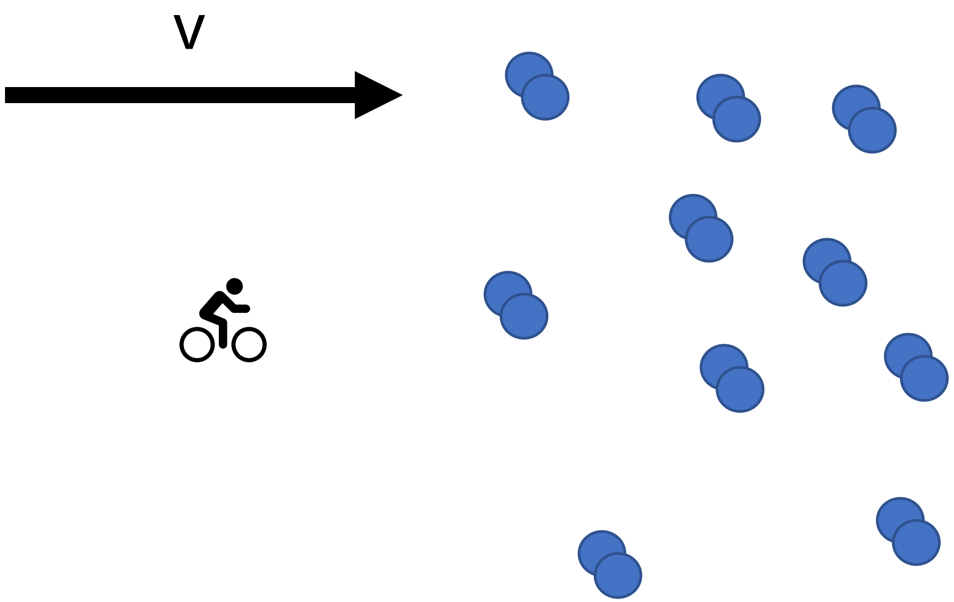
Branches of Physics



Effective Parametrization of Theory



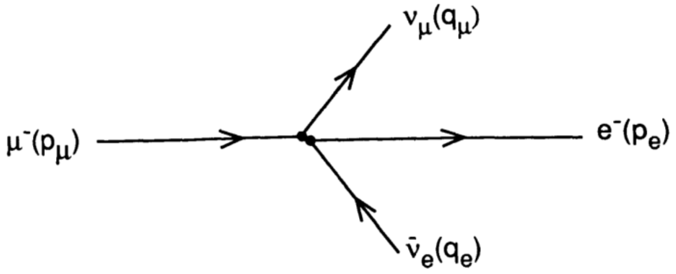
$F = C\,v$



$F = C\,v$ No longer valid!

Bottom to Top

- Fermi Theory : Effective theory of weak interaction. (eg. Muon decay)



Muon Decay

Field Operators

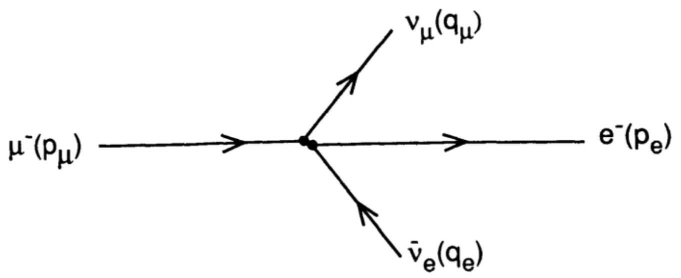
$$\langle \nu_\mu(q_\mu) e^-(p_e) \bar{\nu}_e(q_e) |\, \nu_e\, \bar{e}\, \bar{\nu}_\mu\, \mu \,| \mu^-(p_\mu) \rangle$$

Initial state

Final state

Bottom to Top

- Fermi Theory : Effective theory of weak interaction. (eg. Muon decay)



Muon Decay

$$\langle \nu_\mu(q_\mu) e^-(p_e) \bar{\nu}_e(q_e) | \, \nu_e \, \bar{e} \, \bar{\nu}_\mu \, \mu \, | \mu^-(p_\mu) \rangle$$

Martix Element

$$\mathcal{M} = -i \frac{G}{\sqrt{2}} \bar{u}(q_\mu)(1+\gamma^5)\gamma^\alpha u(p_\mu) \bar{u}(p_e)(1+\gamma^5)\gamma_\alpha v(q_e)$$

After simple calculations…

The decay rate is

$$\Gamma = \frac{G^2 m_\mu^5}{192\pi^3}$$

$$m_\mu = 0.10566 \text{ GeV}$$

$$\Gamma = 2.996 \times 10^{-19} \text{ GeV}$$

$$G = 1.16 \times 10^{-5} \text{ GeV}^{-2}$$

Fermi model at high energy

$$\sigma(e^- \bar{\nu}_e \to \mu^- \bar{\nu}_\mu) = \frac{G^2 s}{3\pi}$$

Diverging limit?

- Invariant mass

$$s = (p(e^-)^\mu + p(\bar{\nu}_e)^\mu)(p(e^-)_\mu + p(\bar{\nu}_e)_\mu)$$

In the COM frame : $s = (E(e^-) + E(\bar{\nu}_e))^2$

Unitarity Limit

- A scattering in the Quantum Field Theory is :

$$_{t_f}\langle f_1 f_2 ... | i_1 i_2 ... \rangle_{t_i} =_t \langle f_1 f_2 ... | S | i_1 i_2 ... \rangle_t$$

- S is unitary

$$SS^\dagger = 1$$

Why S should be unitary?

Unitarity Limit

- A scattering in the Quantum Field Theory is :

$$_{t_f}\langle f_1 f_2 ... | i_1 i_2 ... \rangle_{t_i} =_t \langle f_1 f_2 ... | S | i_1 i_2 ... \rangle_t$$

- S is unitary

$$SS^\dagger = 1$$

- Answer : Probability conservation

$$(\langle i | S^\dagger)(S | i \rangle) = \langle i | i \rangle = 1$$

Unitarity Limit

- Define a T matrix : $S = 1 + iT$

$$\begin{aligned} 1 &= SS^\dagger \\ &= (1+iT)(1-iT^\dagger) \\ &= 1 + i(T - T^\dagger) + TT^\dagger \end{aligned}$$

$\rightarrow$ $$i(T - T^\dagger) = -TT^\dagger$$

- If the initial and final state is same :

$$2\mathrm{Im}(T_{kk}) = (TT^\dagger)_{kk} = \sum_n T_{kn} T^\dagger_{nk} = \sum_n |T_{kn}|^2$$

$$T_{ab} \equiv \langle a | T | b \rangle$$

Unitarity Limit

$$2\mathrm{Im}(T_{kk}) = (TT^\dagger)_{kk} = \sum_n T_{kn}T^\dagger_{nk} = \sum_n |T_{kn}|^2$$

- Optical Theorem

$$\sum_n \sigma_{\text{tot}}(k \to n) = \frac{1}{s}\mathrm{Im}\mathcal{M}_{k\to k}$$

- eg)

$$\sum_n \sigma_{\text{tot}}(e^+e^- \to f\bar{f}) = \frac{1}{s}\mathrm{Im}\mathcal{M}_{e^+e^-\to e^+e^-}$$

Unitarity Limit

- Optical Theorem $$\sum_n \sigma_{\text{tot}}(k \to n) = \frac{1}{s}\text{Im}\mathcal{M}_{k\to k}$$
- This limits the partial cross section

$$\sigma_{\text{tot}}(k \to n) \leq \frac{1}{s}\text{Im}\mathcal{M}_{k\to k}$$

- These results are non perturbative

What we are going to do

1. Calculate a tree level cross section of a specific process.

2. Use the unitarity limit to find a energy limit that theory breaks

3. Introduce new particles or interactions

Question : Isn't the unitarity recovered if one includes all higher order terms?

$$H = H_1 + H_2 + \ldots$$

$$S = e^{iHt} = \sum_{n_1, n_2, \ldots} C_{n_1 n_2 \ldots} H_1^{n_1} H_2^{n_2} \ldots$$

Electron, Anti Electron Neutrino Scattering

- $e^-\bar{\nu}_e \to \mu^-\bar{\nu}_\mu$
- The unitarity limit : $\sigma_{\rm tot}(e^-\bar{\nu}_e \to \mu^-\bar{\nu}_\mu) \leq \dfrac{24\pi}{s}$
- The cross section is

$$\sigma(e^-\bar{\nu}_e \to \mu^-\bar{\nu}_\mu) = \frac{G^2 s}{3\pi}$$

Electron, Anti Electron Neutrino Scattering

The effective theory breaks down if

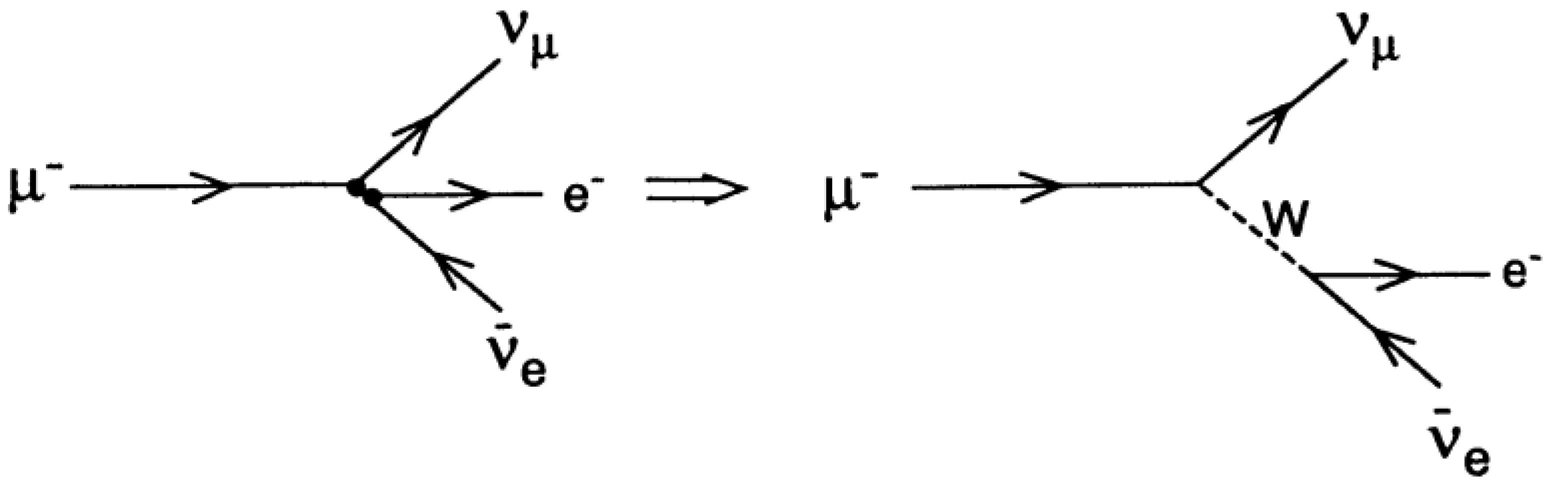
$$s \geq \left(\frac{6\pi\sqrt{2}}{G} \right)^{1/2} = 1516\,\mathrm{GeV}$$

- This shows the limit of the Fermi theory as an effective theory.

Introducing the W Boson

Now the fermion weak interaction is mediated by the W boson

- eg) Muon Decay



$$\mathcal{M} = -i\frac{G}{\sqrt{2}}\bar{u}(q_\mu)(1+\gamma^5)\gamma^\alpha u(p_\mu)\bar{u}(p_e)(1+\gamma^5)\gamma_\alpha v(q_e)$$

$$\mathcal{M} = i\frac{g_W^2}{(p_e+q_e)^2 - m_W^2}\bar{u}(q_\mu)(1+\gamma^5)\gamma^\alpha u(p_\mu)\bar{u}(p_e)(1+\gamma^5)\gamma_\alpha v(q_e)$$

Revisit Electron, Anti Electron Neutrino Scattering

$$\sigma(e^-\bar{\nu}_e \to \mu^-\bar{\nu}_\mu) = \frac{2g_W^4}{3\pi}\frac{s}{(s-m_W^2)^2}$$

In the Fermi model :

$$\sigma(e^-\bar{\nu}_e \to \mu^-\bar{\nu}_\mu) = \frac{G^2 s}{3\pi}$$

- $s \to 0$ limit :

$$\sigma(e^-\bar{\nu}_e \to \mu^-\bar{\nu}_\mu) = \frac{2g_W^4}{3\pi}\frac{s}{m_W^4}$$

In the Fermi model :

$$\sigma(e^-\bar{\nu}_e \to \mu^-\bar{\nu}_\mu) = \frac{G^2 s}{3\pi}$$

- $s \to \infty$ limit :

$$\sigma(e^-\bar{\nu}_e \to \mu^-\bar{\nu}_\mu) = \frac{2g_W^4}{3\pi}\frac{1}{s}$$

Unitarity limit :

$$\sigma_{\text{tot}}(e^-\bar{\nu}_e \to \mu^-\bar{\nu}_\mu) \leq \frac{24\pi}{s}$$

Propagator for unstable particle

- Something is uncomfortable…

$$\sigma(e^-\bar{\nu}_e \to \mu^-\bar{\nu}_\mu) = \frac{2g_W^4}{3\pi}\frac{s}{(s-m_W^2)^2}$$

Can you guess?

Propagator for unstable particle

- Recall the solution of Schrödinger equation

$$i\frac{\partial\psi(t)}{\partial t} = H\psi(t) = m\psi(t) \quad \Rightarrow \quad \psi(t) = e^{-imt}\psi(0)$$

- This imply the probability conservation

$$|\psi(t)|^2 = |\psi(0)|^2$$

- If the decay process is Poisson process

Decay Rate

$$|\psi(t)|^2 = |\psi(0)|^2 e^{-\Gamma t}$$

Propagator for unstable particle

- This implies that the wavefunction should take a form as

$$|\psi(t)|^2 = |\psi(0)|^2 e^{-\Gamma t} \Rightarrow \psi(t) = e^{-imt} e^{-\Gamma t/2} \psi(0)$$

- Propagator can be derived from the fourier transformation

$$\tilde{\psi}(E) \sim \int_0^\infty dt e^{i(E-m+i\Gamma/2)t} \psi(0) \propto \frac{1}{E-m+i\Gamma/2} \sim\propto \frac{1}{E^2-m^2+im\Gamma}$$

Revisit Electron, Anti Electron Neutrino Scattering

- Substitute new propagator to the matrix element

$$\frac{1}{p^2 - m^2 + i\epsilon} \to \frac{1}{p^2 - m^2 + im\Gamma}$$

New result

$$\sigma(e^-\bar{\nu}_e \to \mu^-\bar{\nu}_\mu) = \frac{2g_W^4}{3\pi}\frac{s}{(s-m_W^2)^2 + m_W^2\Gamma^2}$$

Old result

$$\sigma(e^-\bar{\nu}_e \to \mu^-\bar{\nu}_\mu) = \frac{2g_W^4}{3\pi}\frac{s}{(s-m_W^2)^2}$$

- Pole at $s = m_W$ vanished! Q : Why decay effect vanishes as $s \to \infty$?

Necessity of Z Boson

- Pair production of W^+W^- :

$$\bar{u}(p_1)u(p_2) \to W^+(q_+, \epsilon_+)W^-(q_-, \epsilon_-)$$

- Notation: fermion of SU(2) doublets

$$\begin{pmatrix} u_f \\ d_f \end{pmatrix} \equiv \begin{pmatrix} \nu_e \\ e \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}, \begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}$$

- Why t (top), b (bottom) pair is not included ?

Necessity of Z Boson

- Pair production of W^+W^- :

$$\bar{u}(p_1)u(p_2) \to W^+(q_+, \epsilon_+)W^-(q_-, \epsilon_-)$$

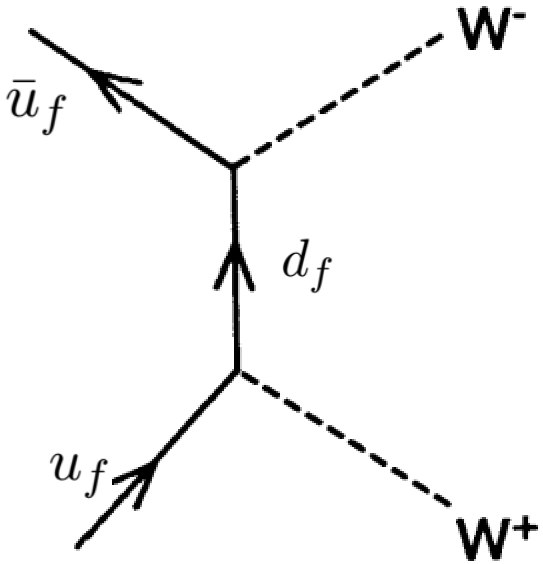
$$\epsilon_+ \sim E/m_W$$

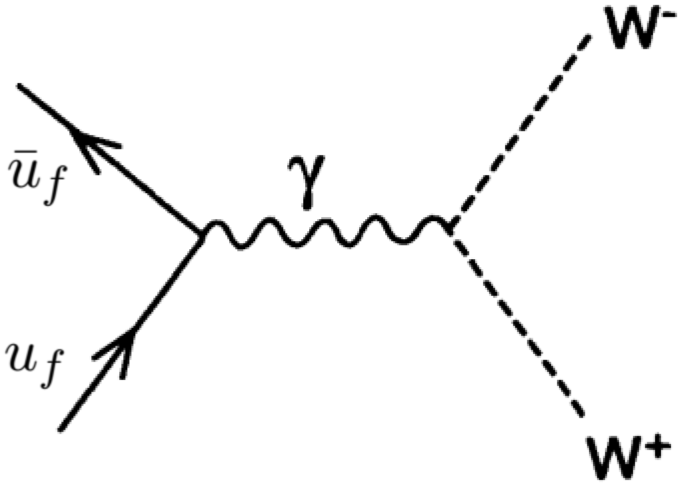
$$\epsilon_- \sim 1$$

- 2 diagrams for the given process :

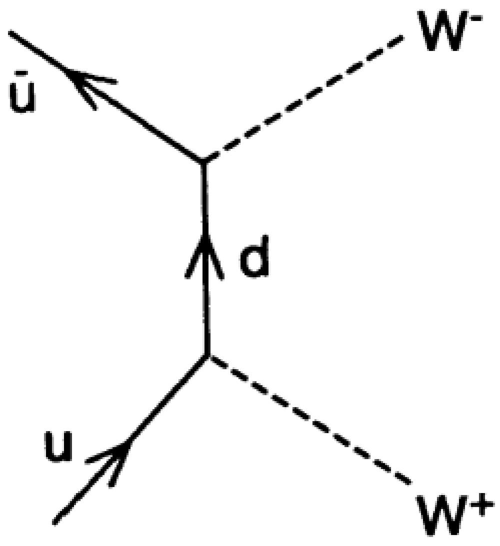
(Charge conservation)

$$Q_{u_f} - Q_{d_f} = Q_{W^+}$$





Necessity of Z Boson



$$= 2ig_W^2 \bar{v}(p1)(1+\gamma^5)\not{\epsilon}_- \frac{\not{p}_2 - \not{q}_+ + m_{d_f}}{(p_2 - q_+)^2 - m_{d_f}^2 + i\epsilon} \not{\epsilon}_+ u(p_2)$$

$$\sim E$$

For power counting :

$$\sum_{\text{spins}} u(p)\bar{u}(p) = \not{p} + m \qquad \epsilon_+ \sim E/m_W \qquad \epsilon_- \sim 1$$

Longitudinal Mode of Spin 1 Particle

- Let the momentum on the z-direction

$$k^\mu = (k^0, 0, 0, |\vec{k}|)$$

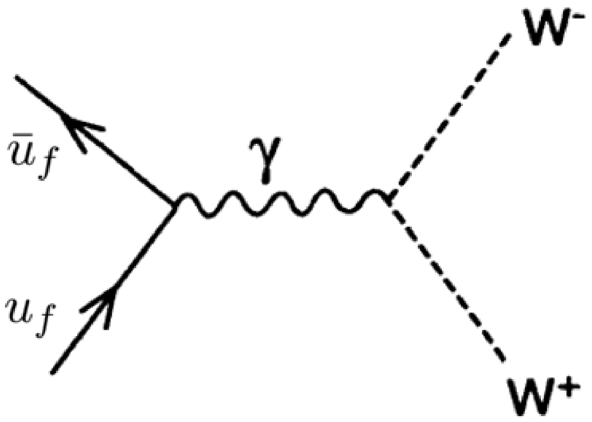
- Then there are two transverse mode, one longitudinal mode

$$\epsilon^1 = (0,1,0,0) \qquad \epsilon^2 = (0,0,1,0) \qquad \epsilon^3 = (\frac{|\vec{k}|}{m_W}, 0, 0, \frac{k^0}{m_W})$$

- In the high energy limit, (Goldston boson equivalence theorem)

$$\epsilon^3 \sim \frac{k^\mu}{m_W}$$

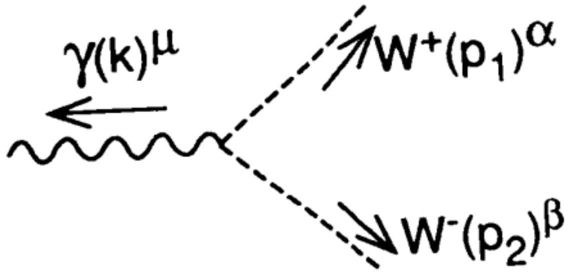
Necessity of Z Boson



~ E?

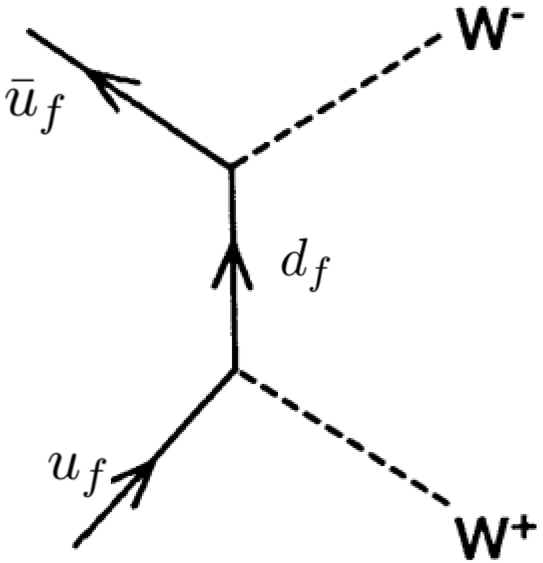
* The $\gamma W^{-} W^{+}$ vertex rule is :

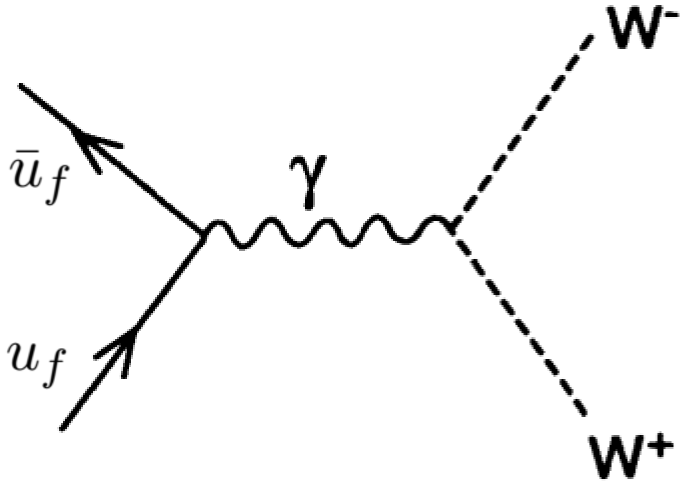
$$iQ_{\mathrm{w}} V(p_1, \alpha, p_2, \beta, k, \mu) \equiv iQ_{\mathrm{w}} \left[(p_1 - p_2)^{\mu} g^{\alpha\beta} + (p_2 - k)^{\alpha} g^{\beta\mu} + (k - p_1)^{\beta} g^{\mu\alpha} \right]$$



Necessity of Z Boson

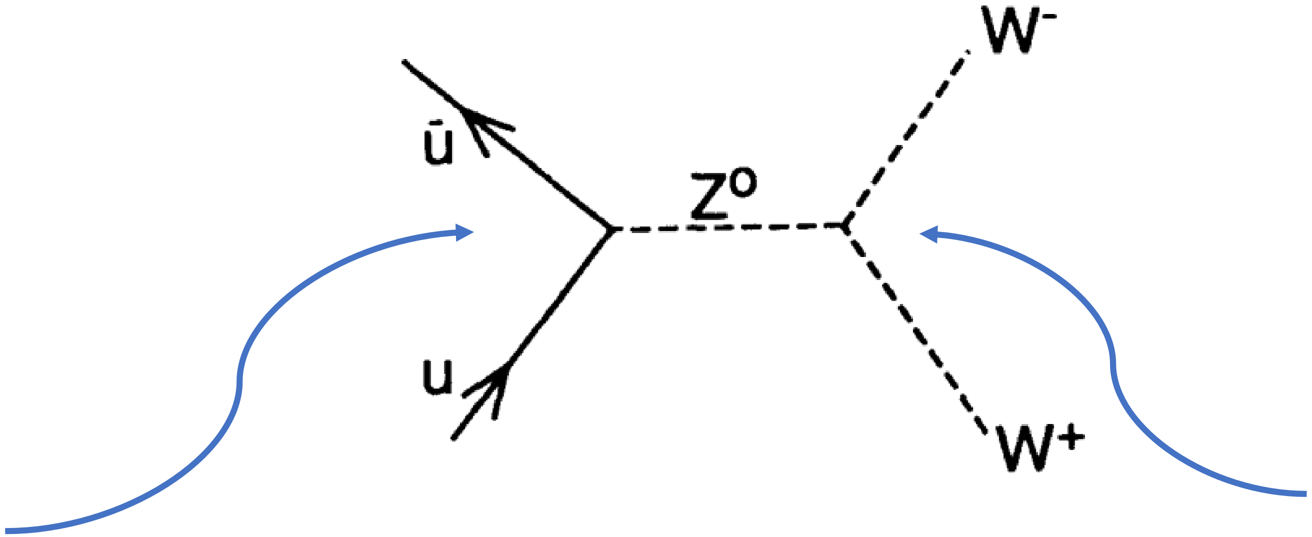
- But these diagram cannot cancel : Why?





Necessity of Z Boson

Introducing the Z boson give new diagram:



$i(v_f + a_f \gamma^5)\gamma^\mu$

$ig_{\rm WWZ} V(p_1, \alpha, p_2, \beta, k, \mu)$

$$\mathcal{M}_3 = ig_{\rm WWZ}\bar{v}(p_1)(v_u + a_u\gamma^5)\gamma_\mu u(p_2)\frac{1}{s - m_{\rm Z}^2}V(q_+, \epsilon_+, q_-, \epsilon_-, -q_+ - q_-, \mu)$$

Coupling Constant Determination

- Require the cancel of ~E behavior of various processes :

$$u\bar{u} \to W^+W^- \quad \Rightarrow \quad \begin{aligned} 2g_W^2 + Q_u Q_W + v_u g_{WWZ} &= 0 \\ 2g_W^2 + a_u g_{WWZ} &= 0 \end{aligned}$$

$$d\bar{d} \to W^+W^- \quad \Rightarrow \quad \begin{aligned} -2g_W^2 + Q_d Q_W + v_d g_{WWZ} &= 0 \\ -2g_W^2 + a_d g_{WWZ} &= 0 \end{aligned}$$

$$u\bar{d} \to W^+Z^0 \quad \Rightarrow \quad g_{WWZ} = v_d + a_d - v_u - a_u$$

Coupling Constant Determination

- This uniquely determine the couplings and coefficients:

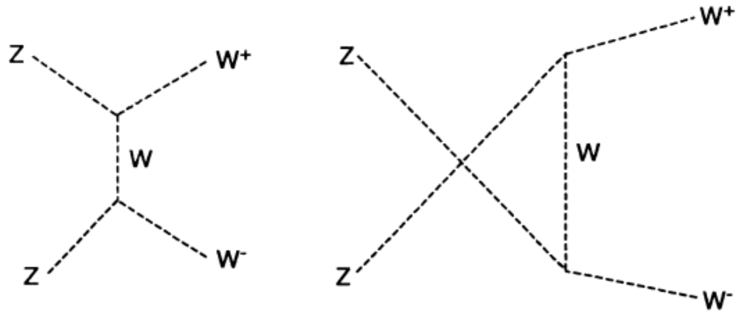
$$
\begin{aligned}
g_{\mathrm{W}} &= e\frac{1}{s_w\sqrt{8}} \ , \\
Q_{\mathrm{W}} &= -e \ , \\
g_{\mathrm{WWZ}} &= -e\frac{c_w}{s_w} \ , \\
a_u &= -a_d = e\frac{1}{4s_w c_w} \ , \\
v_u &= e\frac{1}{4c_w s_w}\left[1 - 4\frac{Q_{\mathrm{u}}}{e}s_w^2\right] \ , \\
v_d &= e\frac{-1}{4c_w s_w}\left[1 + 4\frac{Q_{\mathrm{d}}}{e}s_w^2\right] \ .
\end{aligned}
$$

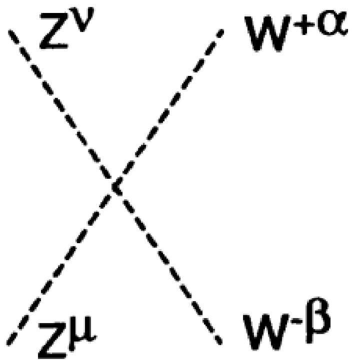
$$s_w \equiv \sin\theta_w$$

$$c_w \equiv \cos\theta_w$$

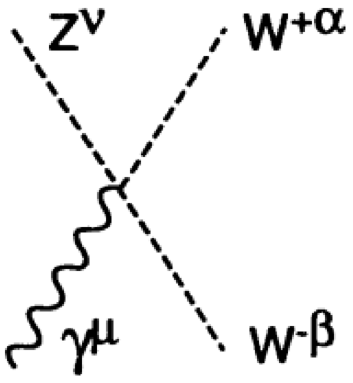
Four Boson Vertices

- Scattering of two bosons to two bosons

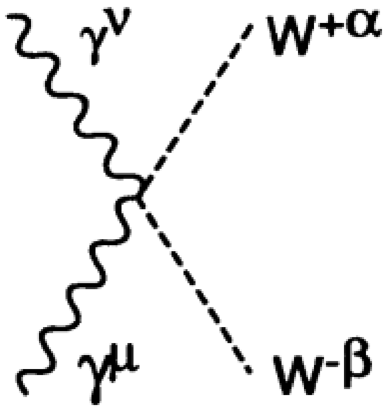




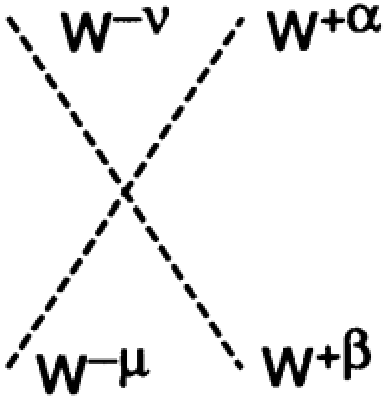
$Z^0\gamma \to W^+W^-$



$\gamma\gamma \to W^+W^-$

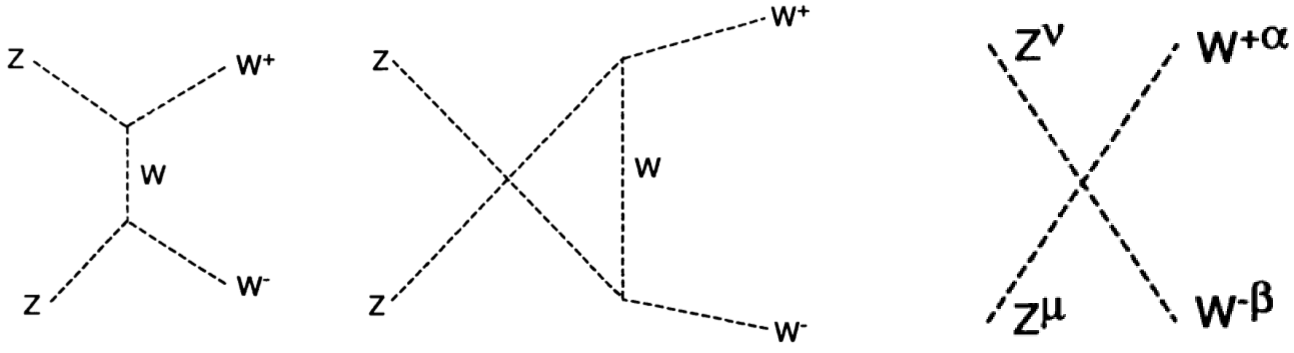


$W^+W^- \to W^+W^-$

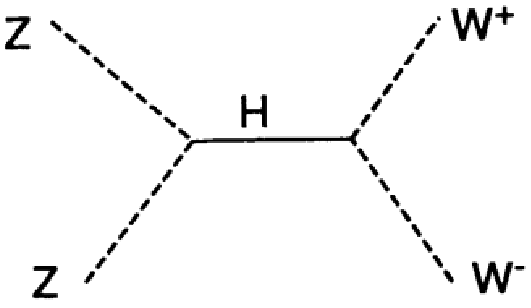


Introduction of Higgs

- Consider a $ZZ \to WW$ again with fully longitudinal modes



$$\sim i g_{\mathrm{WWZ}}^2 E^2 \frac{m_{\mathrm{Z}}^2}{m_{\mathrm{W}}^4}$$



$$\sim -i g_{\mathrm{WWH}} g_{\mathrm{ZZH}} \frac{s^2}{4 m_{\mathrm{W}}^2 m_{\mathrm{Z}}^2 (s - m_{\mathrm{H}}^2)}$$

Constraints on the Higgs to 2 Gauge Boson Coupling

$$Z^0 Z^0 \to W^+ W^- \quad \Rightarrow \quad g_{\mathrm{WWH}} g_{\mathrm{ZZH}} = g^2_{\mathrm{WWZ}} \frac{m^4_{\mathrm{Z}}}{m^2_{\mathrm{W}}}$$

$$W^+ W^- \to W^+ W^- \quad \Rightarrow \quad g^2_{\mathrm{WWH}} = g^2_{\mathrm{WWZ}} \left(4m^2_{\mathrm{W}} - 3m^2_{\mathrm{Z}}\right) + 4Q^2_{\mathrm{W}} m^2_{\mathrm{W}}$$

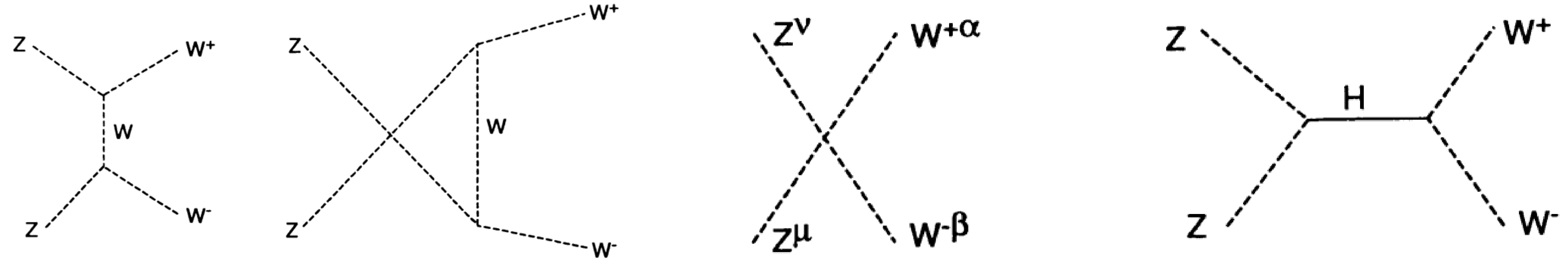
$$Z^0 H \to W^+ W^- \quad \Rightarrow \quad g_{\mathrm{ZZH}} = g_{\mathrm{WWH}} \frac{m^2_{\mathrm{Z}}}{m^2_{\mathrm{W}}}$$

$$\therefore \quad g_{\mathrm{WWH}} = -e \frac{m_{\mathrm{W}}}{s_w}$$

$$g_{\mathrm{ZZH}} = -e \frac{m_{\mathrm{Z}}}{c_w s_w}$$

Unitarity Bound on the Higgs Mass

$$Z^0 Z^0 \rightarrow W^+ W^-$$



$$\mathcal{M} \sim i \frac{e^2}{4 m_W^2 s_w^2} \left(\frac{s^2}{s - m_H^2} - s \right) = -i \frac{e^2 m_H^2 s}{4 m_W^2 s_w^2 (s - m_H^2)} = -i \sqrt{2} G m_H^2 \frac{s}{s - m_H^2}$$

$$\therefore \quad m_H \leq \left[\frac{16\pi}{G\sqrt{2}} \right]^{1/2} = 1750 \text{ GeV}$$