양자장론과 표준모형

QUANTUM FIELD THEORY AND STANDARD MODEL

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The Standard Model

LECTURE 4

1. While QFT is a plausible framework describing relativistic QM, it does not tell us how the nature should behave, e.g., which gauge group? Which representations for matter?

2. Due to our limit of accessibility, we just have a phenomenological effective model

(energy below TeV, feeble interaction with dark sector, gravity…)

3. Nevertheless, quite often, the low energy behavior can be a smoking gun (떡밥) of the physics beyond our probe when combined with the fundamental principles of e.g., QM

4. Standard Model (Weinberg-Salam-Glashow model : unification of electrodynamics and weak interaction)

Recommended reading : R. Kleiss, 'Derivation of the minimal standard model Lagrangian' in 'physics up to 200TeV' (Erice school 1990)

See also Particle Data Group https://pdg.lbl.gov

Parity Violation : discovery of chiral nature of fermions

Parity : $\boxed{\begin{array}{c} t \to t \\ \mathbf{x} \to -\mathbf{x} \end{array}}$ **or** $\boxed{\begin{array}{c} \mathbf{x} \to -\mathbf{x} \\ \mathbf{p} \to -\mathbf{p} \end{array}}$

just like $\mathbf{L} = \mathbf{x} \times \mathbf{p}$ **,** $J_i \to J_i$ **but** $K_i \to -K_i$ **: under the parity,** $J_{+,i} \leftrightarrow J_{-,i}$

That is, $(j_+, j_-) \to (j_-, j_+)$ **, in particular,** $\left(\frac{1}{2}, 0\right) \leftrightarrow \left(0, \frac{1}{2}\right)$

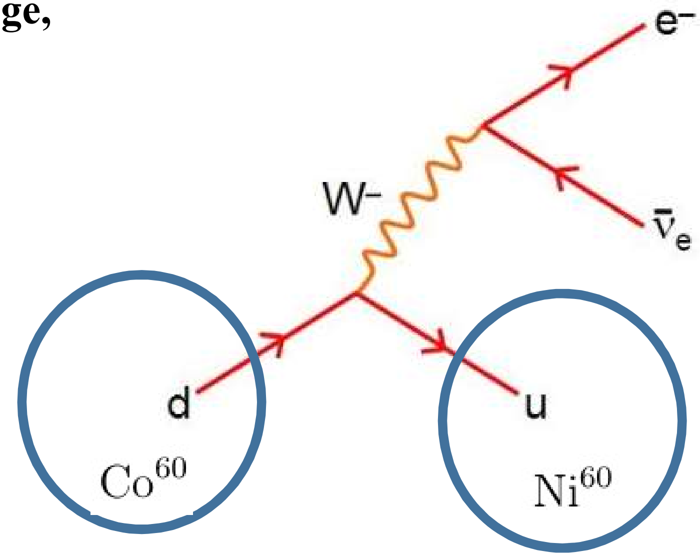
: the parity invariance of spin-1/2 particle means that the left-handed spinor (1/2, 0) and the right handed spinor (0, 1/2) must behave in the same way (always vector-like)

However, investigation of β decay shows that the weak interaction does not respect parity

(T. D. Lee and C. N. Yang, 1956 ; C. S. Wu, E. Amber, R. W. Hayward, D. D. Hoppes, R. P. Hudson, 1957)

Consider the β decay process $\mathrm{Co}^{60} \to \mathrm{Ni}^{60} + e^{-} + \overline{\nu}_e$

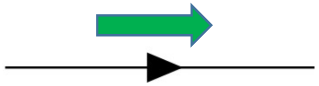
In modern language,



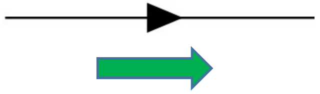
Note also that

(1/2,0) : e^{-} **helicity= −1/2**

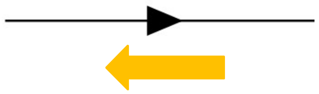
$\overline{\nu}_e$ **helicity= +1/2**

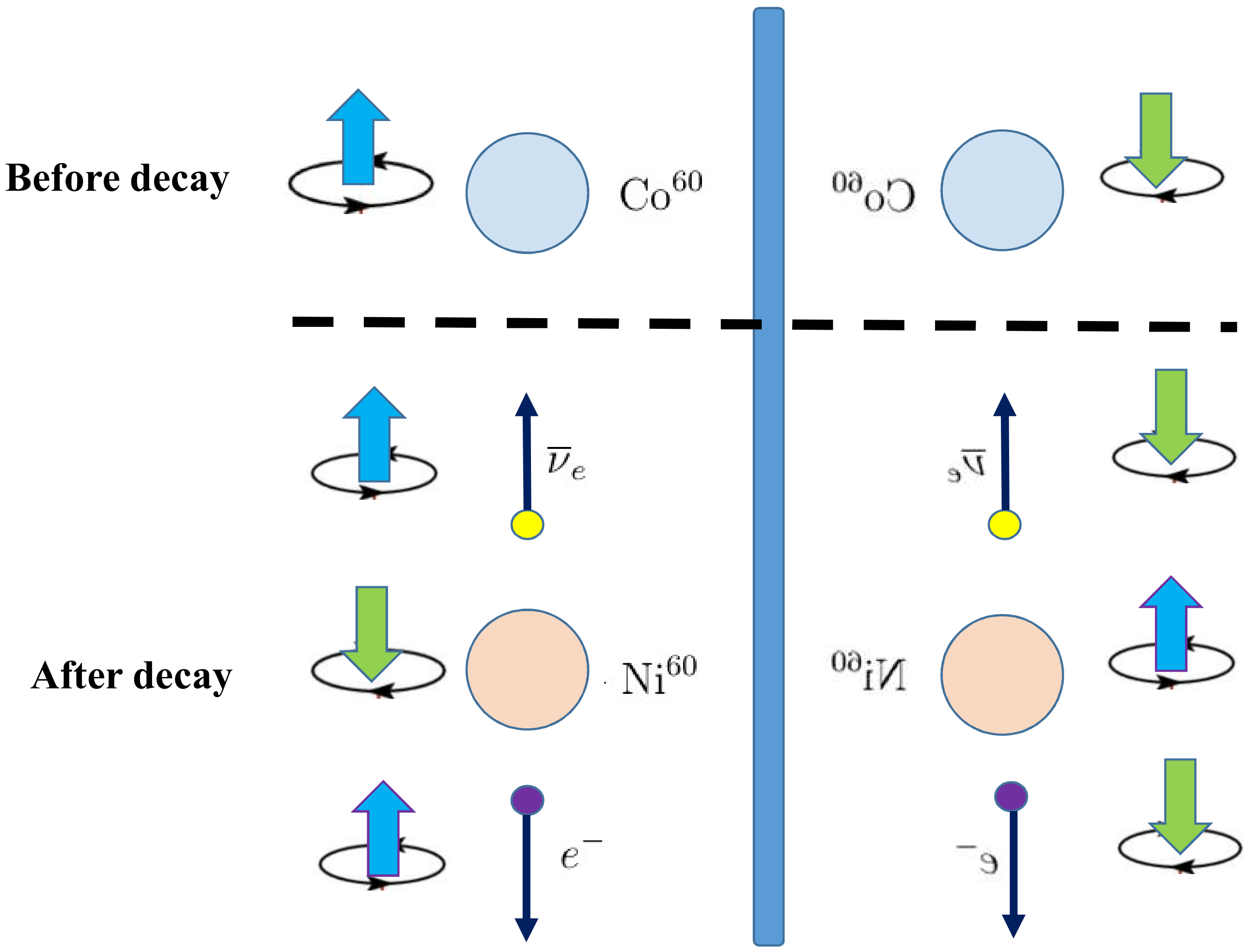


(0,1/2) : e^{-} **helicity= +1/2**



$\overline{\nu}_e$ **helicity = −1/2**





(1) What actually happens

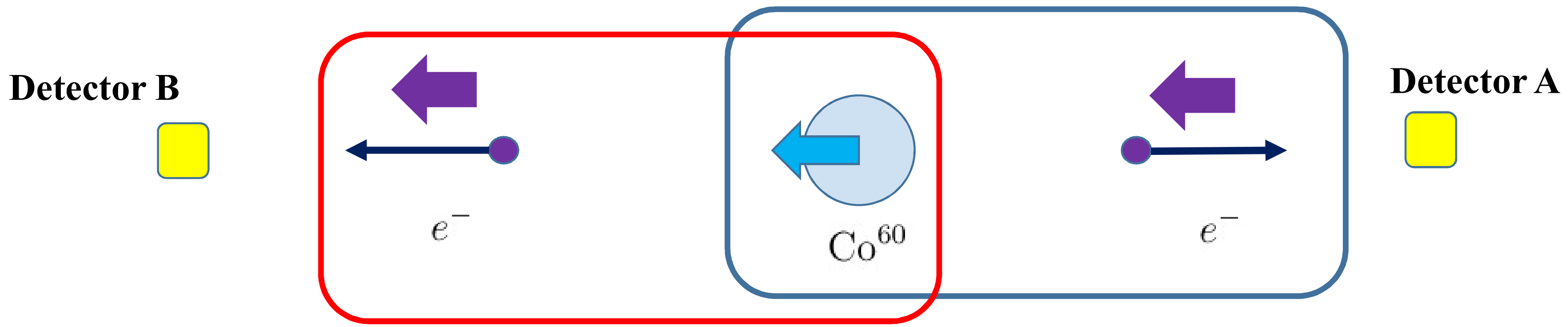
mirror image

(2) Expected one in the presence of parity invariance

In the left figure, $\boldsymbol{J} \rightarrow -\boldsymbol{J}$
Since the mirror image is a transformation
$$x \rightarrow -x, y \rightarrow y, z \rightarrow z$$
However, this is equivalent to the parity
$$x \rightarrow -x, y \rightarrow -y, z \rightarrow -z$$
in the presence of rotational invariance since

$$\underset{\text{parity}}{\begin{pmatrix} -1 & & \\ & -1 & \\ & & -1 \end{pmatrix}} = \underset{\text{mirror}}{\begin{pmatrix} -1 & & \\ & 1 & \\ & & 1 \end{pmatrix}} \times \underset{\substack{\text{rotation}\\(\pi\hat{x})}}{\begin{pmatrix} 1 & & \\ & -1 & \\ & & -1 \end{pmatrix}}$$

Depicting Co^{60} and e^- only,



(2) Expected one in the presence of parity invariance

(1) What actually happens

If parity is a symmetry, detectors A and B should detect the same amount of electrons.

However, C. S. Wu et al's experiment shows that detector A finds much more electrons

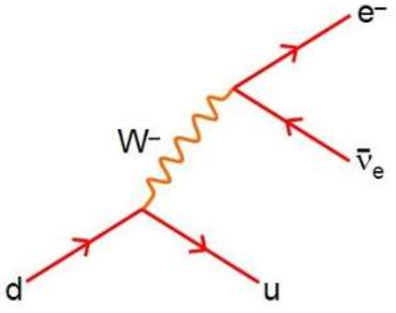
Electron at detector B can be thought of as a result of the transition of the right handed to the left handed through the <u>left-right mixing</u> in the mass term (suppressed by electron mass)

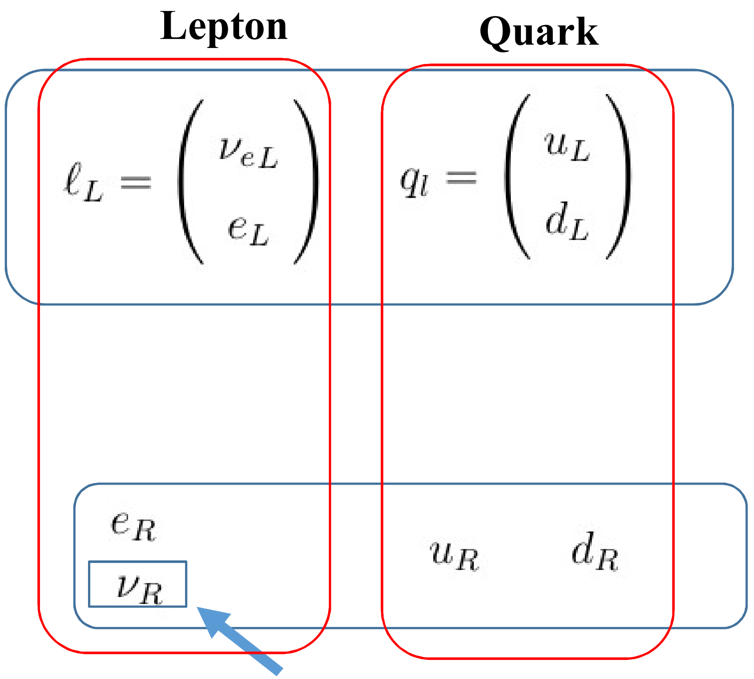
$$\mathcal{L} = i\bar{\chi}\bar{\sigma}^\mu\partial_\mu\chi + i\bar{\eta}\bar{\sigma}^\mu\partial_\mu\eta \boxed{- m(\chi\eta + \overline{\chi\eta})}$$

This shows that only the left handed electron / neutrino are charged under the weak interaction

: fermions are chiral

Let's ignore the masses for a moment (assume that all particles are massless)





Lepton **Quark**

$$\ell_L = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \qquad q_l = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

(1/2, 0) : SU(2) doublet

e_R $\boxed{\nu_R}$ u_R d_R

(0, 1/2) : SU(2) singlet

Not discovered (not charged in any known interaction)
Can be ignored if neutrino is massless

Pattern from electric charge : U(1) hypercharge *Y*

let $T^a = \frac{1}{2}\sigma^a$: SU(2) generators acting on doublet (fundamental representation)

	T^3	Y	Q
$\ell_L = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$	$1/2$ $-1/2$	$-1/2$ $-1/2$	0 -1
e_R	0	0	-1
$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$1/2$ $-1/2$	$1/6$ $1/6$	$2/3$ $-1/3$
u_R	0	$2/3$	$2/3$
d_R	0	$-1/3$	$-1/3$

$$\boxed{Q = T^3 + Y}$$

Y acts like U(1)

Since Q and T^3 are gauged, it is reasonable to gauge the hypercharge Y.

: U(1)$_{EM}$ is a combination of SU(2)$_W$ and U(1)$_Y$:

SU(2) doublet : $D_\mu = \partial_\mu - igW_\mu^a T^a - ig'YB_\mu$

SU(2) singlet : $D_\mu = \partial_\mu - ig'YB_\mu$

$$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g\epsilon_{abc}W_\mu^b W_\nu^c$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

Problems

1. In terms of SU(2)$_W$×U(1)$_Y$, e_L and e_R are completely different

But in U(1)$_{EM}$, why they behave as the spin components of the single particle?

2. Mass issue

Because of 1, the mass term $\bar{e}_L e_R$ is NOT a singlet under SU(2)$_W$×U(1)$_Y$

Moreover, why particles in SU(2)$_W$ doublet do not have the same mass?

Hint : Indeed, the ‘gauge boson’ $W^{\pm}$ is charged and massive

1. Charged: photon and $W^{\pm}$ were originally parts of the non-Abelian gauge boson but separated for some reason…

2. Massive (~80 GeV) : for it to be consistent with the gauge invariance, the Higgs (Stuckelberg) mechanism is required.

When Higgs is in the nontrivial representation of $\mathrm{SU(2)}_{W}$ and has a VEV, it can be used to provide fermion masses by forming from the gauge singlet consists of Higgs and fermion bi-spinor : Higgs should be a $\mathrm{SU(2)}_{W}$ doublet

: 1 and 2 may be a connected issue, and this can resolve problems discussed in the previous page.

$$\mathrm{SU}(2)_{\mathrm{W}} \times \mathrm{U}(1)_{\mathrm{Y}} \underset{\substack{\Uparrow \\ \boxed{\text{Higgs mechanism}}}}{\longrightarrow} \mathrm{U}(1)_{\mathrm{EM}}$$

Higgs scalar : $$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$ **SU(2)$_W$ doublet, Y=1/2**

$$\phi^+ \ : \ Q = +1$$
$$\phi^0 \ : \ Q = 0$$

Y=1/2 is chosen such that the Higgs has a Q=0 component
VEV should be given to the Q=0 component not to break electromagnetic gauge invariance.
(one may choose Y=−1/2) : As we will see, in terms of fermion quantum numbers Y=−1/2 indeed is good for the gauge singlet for fermion mass

In terms of renormalizable potential,

$$V(\phi) = \frac{\lambda}{2}\left(\phi^\dagger\phi - \frac{v^2}{2}\right)^2 \qquad \langle\phi\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

While SU(2)$_W$×U(1)$_Y$ has 4 gauge bosons corresponding to 4 generators (3 for SU(2)$_W$ and 1 for U(1)$_Y$), 3 of them must be massive ($v = 246$ GeV)

: 3 massive gauge bosons absorb 3 of 4 real components of the Higgs, fluctuations in the direction of broken gauge transformation. One remains as a real scalar (Higgs boson)

‘Gauge singlet consists of the Higgs and bi-spinor’ : fermion mass term (Yukawa coupling)

1. Charged lepton mass (for a moment we assume neutrino to be massless)

$y_e\overline{\ell_L}\cdot\phi e_R + \text{h.c.}$ **: obviously SU(2)$_W$ singlet**

Y: $\frac{1}{2}$ $\frac{1}{2}$ -1 **: U(1)$_Y$ singlet**

$$y_e\overline{\ell_L}\cdot\phi e_R + \text{h.c.} = y_e\left[\overline{v_L}\phi^+ e_R + \overline{e_L}\phi_0 e_R\right] + \text{h.c.} \longrightarrow \left(\frac{y_e}{\sqrt{2}}v\right)\overline{e_L}e_R + \text{h.c.}$$

Higgs VEV

$$\boxed{m_e = \frac{y_e}{\sqrt{2}}v}$$

2. d-quark mass

$$y_d\overline{q_L}\cdot\phi d_R + \text{h.c.} = y_d\left[\overline{u_L}\phi^+ d_R + \overline{d_L}\phi_0 d_R\right] + \text{h.c.} \longrightarrow \left(\frac{y_d}{\sqrt{2}}v\right)\overline{d_L}d_R + \text{h.c.}$$

Y: $-\frac{1}{6}$ $\frac{1}{2}$ $-\frac{1}{3}$

3. u-quark mass

$\overline{q_L}\cdot\phi u_R \sim (v/\sqrt{2})\overline{d_L}u_R$ **does not provide the u-quark mass and not a U(1)$_Y$ singlet**

Y: $-\frac{1}{6}$ $\frac{1}{2}$ $\frac{2}{3}$

(but becomes a singlet if the Higgs hypercharge= $-1/2$, just like ϕ^* **)**

Indeed we have another way to construct SU(2)$_W$ singlet : $\overline{q_L}\epsilon\phi^*$ $\epsilon = i\sigma^2$

Check :

$$\overline{q_L}\epsilon\phi^* \to \overline{q_L}U^\dagger\epsilon U^*\phi^* = \overline{q_L}U^\dagger(\sigma_2U^*\sigma_2)\epsilon\phi^* = \overline{q_L}U^\dagger U\epsilon\phi^* = \overline{q_L}\epsilon\phi^*$$

$$\boxed{\sigma_2\sigma_i^*\sigma_2 = -\sigma_i}$$

Thus,

$$y_u\epsilon^{ab}\overline{q_L}_a\phi_b^*u_R + \text{h.c.} = y_u\left[\overline{u_L}\phi_0^*u_R - \overline{d_L}\phi^-u_R\right] + \text{h.c.} \to \left(\frac{y_u}{\sqrt{2}}v\right)\overline{u_L}u_R + \text{h.c.}$$

Higgs VEV

How Higgs VEV splits the gauge bosons as we observe them?

1. $W^{\pm}$

$$D_\mu = \partial_\mu - igW_\mu^a T^a - ig'YB_\mu = \partial_\mu - i \begin{pmatrix} \frac{g}{2}W_\mu^3 + g'YB_\mu & \frac{g}{2}(W_\mu^1 - iW_\mu^2) \\ \frac{g}{2}(W_\mu^1 + iW_\mu^2) & -\frac{g}{2}W_\mu^3 + g'YB_\mu \end{pmatrix}$$

We identify off diagonal terms by $W_\mu^{\pm} = \frac{1}{\sqrt{2}}(W_\mu^1 - iW_\mu^2)$ **: connecting $(\nu_{e\,L}, e_L), (u_L, d_L)$**
(charged current)

Justification of electric charge :

W_μ^a belong to the adjoint representation : $(t_G^b)_{ac} = if_{abc}$

for $\langle t_G^a | t_G^b \rangle = \delta_{ab}, \quad t_G^b | t_g^c \rangle = [[t^b, t^c]\rangle = if_{abc}|t_G^a\rangle$

Charge = +1

W_μ^+ is accompanied with $\frac{1}{2}(T^1 + iT^2)$ **:** $\left[Q, \frac{1}{2}(T^1 + iT^2)\right] = \left[T^3 + Y, \frac{1}{2}(T^1 + iT^2)\right] = \left[T^3, \frac{1}{2}(T^1 + iT^2)\right] = 1 \times \frac{1}{2}(T^1 + iT^2)$

In the same way, one finds that photon (charge=0) must be one <u>massless</u> combination of W_μ^3 and B_μ

another combination is neutral but <u>massive</u> (Z-boson)

Determined by the coupling to Higgs

2. Neutral gauge bosons

Since Higgs has Y=1/2,

$$|D_\mu\langle\phi\rangle|^2 = \left| \begin{pmatrix} \frac{g}{2}W_\mu^3 + \frac{g'}{2}B_\mu & \frac{g}{\sqrt{2}}W_\mu^+ \\ \frac{g}{\sqrt{2}}W_\mu^- & -\frac{g}{2}W_\mu^3 + \frac{g'}{2}B_\mu \end{pmatrix} \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \right|^2$$

$$= \left(\frac{g}{2}v\right)^2 W_\mu^- W^{+\mu} + \frac{g^2+g'^2}{8}v^2\left(-\frac{g}{2}W_\mu^3 + \frac{g'}{2}B_\mu\right)^2$$

$$= \left(\frac{g}{2}v\right)^2 W_\mu^- W^{+\mu} + \frac{1}{2}\left(\frac{\sqrt{g^2+g'^2}}{2}v\right)^2 Z_\mu Z^\mu$$

The massive, neutral combination

$$Z_\mu = -\frac{g}{\sqrt{g^2+g'^2}}W_\mu^3 + \frac{g'}{\sqrt{g^2+g'^2}}B_\mu \equiv -\cos\theta_W W_\mu^3 + \sin\theta_W B_\mu$$

$$\boxed{\begin{aligned} m_Z &= \frac{1}{2}\sqrt{g^2+g'^2}v \\ m_W &= \frac{1}{2}gv \end{aligned}}$$

Then remaining combination is neutral, massless : photon

$$A_\mu = \cos\theta_W B_\mu + \sin\theta_W W_\mu^3$$

$$\boxed{\begin{aligned} W_\mu^3 &= \cos\theta_W Z_\mu + \sin\theta_W A_\mu \\ B_\mu &= \cos\theta_W A_\mu - \sin\theta_W Z_\mu \end{aligned}}$$

$$A_\mu = \cos\theta_W B_\mu + \sin\theta_W W^3_\mu$$

$$Z_\mu = -\cos\theta_W W^3_\mu + \sin\theta_W B_\mu$$

$\boldsymbol{\theta_W}$ (weak mixing angle / Weinberg angle) measures how much U(1)$_Y$ is contained in the massive Z-boson through the mixing with SU(2)$_W$.

If $\boldsymbol{\theta_W = 0}$ Z-boson is purely $\boldsymbol{W^3_\mu}$: SU(2)$_W$ and U(1)$_Y$ are completely separated. In this case, only SU(2)$_W$ is spontaneously broken and photon is purely $\boldsymbol{B_\mu}$. (The Higgs Y=0 : hypercharge does not couple to Higgs so not spontaneously broken) Then W and Z bosons have the same mass, $\boldsymbol{\left(\frac{1}{2}\right) g\, v}$.

(and fermions cannot have masses at the renormalizable level as the fermion mass terms cannot be gauge singlets for Y=0)

Note: $\boldsymbol{g'}$in $\boldsymbol{\theta_W}$ is in fact $\boldsymbol{2Y_{Higgs}g'}$

Then

$$T^3 = \frac{\sigma^3}{2}$$

$$\begin{aligned} D_\mu &= \partial_\mu - ig\frac{\sigma^a}{2}W^a_\mu - ig'YB_\mu \\ &= \partial_\mu - ig\frac{1}{2}(\sigma^1 W^1_\mu + \sigma^2 W^2_\mu) - igT^3W^3_\mu - ig'B_\mu \\ &= \partial_\mu - i\frac{g}{\sqrt{2}}\left[(\sigma^1 + i\sigma^2)W^+_\mu + (\sigma^1 - i\sigma^2)W^-_\mu\right] \end{aligned}$$

$$\boxed{\begin{aligned} &-i\underline{\sqrt{g^2+g'^2}}\,\underline{(\cos^2\theta_W T^3 - \sin^2\theta_W Y)}Z_\mu \\ &-i\underline{\sqrt{g^2+g'^2}\cos\theta_W \sin\theta_W}\,\underline{(T^3+Y)}A_\mu \end{aligned}}$$

$$\boxed{-ig_ZQ_ZZ_\mu - ieQA_\mu} \Leftarrow$$

$$\boxed{\begin{aligned} W^3_\mu &= \cos\theta_W Z_\mu + \sin\theta_W A_\mu \\ B_\mu &= \cos\theta_W A_\mu - \sin\theta_W Z_\mu \end{aligned}}$$

$$\boxed{\begin{aligned} g &= \sqrt{g^2+g'^2}\cos\theta_W \\ g' &= \sqrt{g^2+g'^2}\sin\theta_W \end{aligned}}$$

$$\boxed{g_Z = \sqrt{g^2+g'^2},}$$

$$\boxed{Q_Z = \cos^2\theta_W T^3 - \sin^2\theta_W Y = T^3 - \sin^2\theta_W Q}$$

$$\boxed{e = \sqrt{g^2+g'^2}\sin\theta_W\cos\theta_W = \frac{gg'}{\sqrt{g^2+g'^2}}}$$

$$\boxed{Q = T^3 + Y}$$

Summary

$$W_\mu^3 = \cos\theta_W Z_\mu + \sin\theta_W A_\mu$$

$$B_\mu = \cos\theta_W A_\mu - \sin\theta_W Z_\mu$$

or

$$A_\mu = \cos\theta_W B_\mu + \sin\theta_W W_\mu^3$$

$$Z_\mu = -\cos\theta_W W_\mu^3 + \sin\theta_W B_\mu$$

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp W_\mu^2)$$

$$\cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}}$$

$$\sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}$$

⇒

$$e = \frac{gg'}{\sqrt{g^2 + g'^2}}$$

$$g_Z = \sqrt{g^2 + g'^2}$$

$$Q = T^3 + Y$$

$$Q_Z = T^3 - \sin^2\theta_W Q$$

$$m_A = 0$$

$$m_Z = \frac{1}{2}\sqrt{g^2 + g'^2}v$$

$$m_W = \frac{1}{2}gv$$

$$v = 246\text{GeV}$$

$$m_Z = 91\text{GeV}$$

$$m_W = 80\text{GeV}$$

Summary

$$\begin{aligned}\mathcal{L} = & -\frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} + (D_\mu\phi)^\dagger(D^\mu\phi) - \frac{\lambda}{2}\left(\phi^\dagger\phi - \frac{v^2}{2}\right)^2 \\ & + \overline{q_L}(i\not\partial - ig\not W^a T^a - ig'Y\not B - ig_s\not G^a t^a)q_L \\ & + \overline{u_R}(i\not\partial - ig'Y\not B - ig_s\not G^a t^a)u_R + \overline{d_R}(i\not\partial - ig'Y\not B - ig_s\not G^a t^a)d_R \\ & - y_u\epsilon^{ab}\overline{q_L}_a\phi^*_b u_R - y_d\overline{q_L}\cdot\phi d_R + \text{h.c.} \\ & + \overline{\ell_L}(i\not\partial - ig\not W^a T^a - ig'Y\not B)\ell_L + \overline{e_R}(i\not\partial - ig'Y\not B)e_R \\ & - y_e\overline{\ell_L}\cdot\phi e_R + \text{h.c.}\end{aligned}$$

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g f_{abc}G^b_\mu G^c_\nu$$

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\epsilon_{abc}W^b_\mu W^c_\nu$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$\ell_L = \begin{pmatrix}\nu_{eL} \\ e_L\end{pmatrix}$ $Y = -\frac{1}{2}$

e_R $Y = -1$

Lepton

$q_L = \begin{pmatrix}u_L \\ d_L\end{pmatrix}$ $Y = \frac{1}{6}$

u_R $Y = \frac{2}{3}$

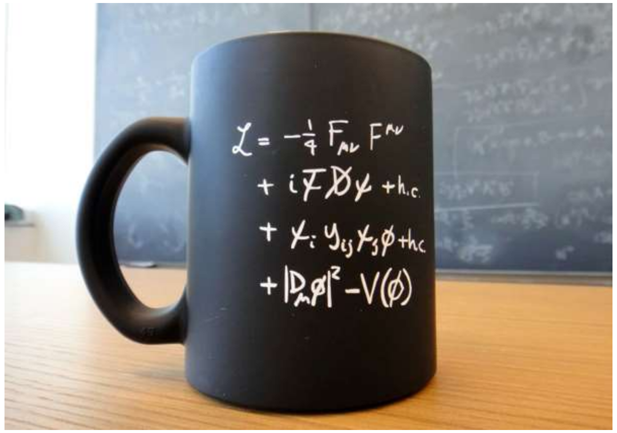
d_R $Y = -\frac{1}{3}$

Quark

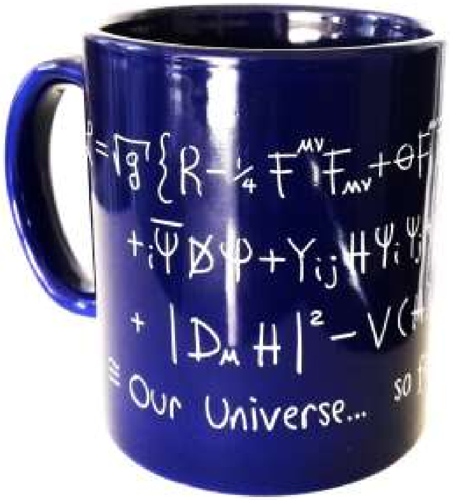
$\phi = \begin{pmatrix}\phi^+ \\ \phi^0\end{pmatrix}$ $Y = \frac{1}{2}$

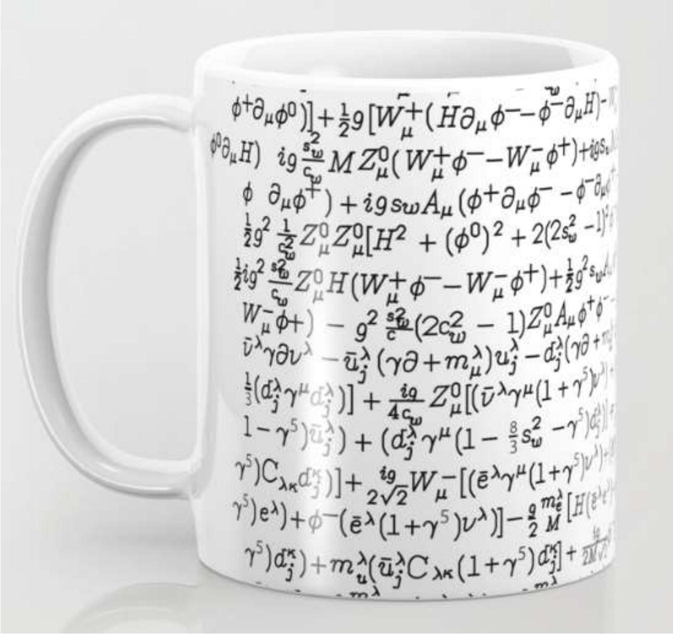
Higgs

-_-


$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$
$+ i\bar{\psi} \not{D} \psi + h.c.$
$+ \psi_i y_{ij} \psi_j \phi + h.c.$
$+ |D_\mu \phi|^2 - V(\phi)$

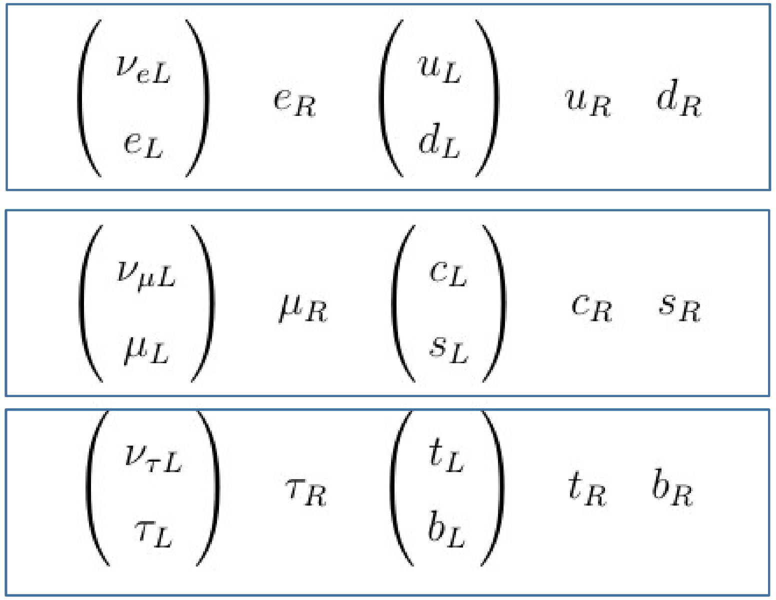

$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$
$+ i\bar{\psi} \not{D} \psi + h.c.$
$+ \psi_i y_{ij} \psi_j \phi + h.c.$
$+ |D_\mu \phi|^2 - V(\phi)$
$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$
$= \frac{8\pi G}{c^4} T_{\mu\nu}$


$= \sqrt{g} \{ R - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \theta F$
$+ i\bar{\Psi} \not{D} \Psi + Y_{ij} H \Psi_i \Psi_j$
$+ |D_\mu H|^2 - V($
≡ Our Universe... so f


$\phi^+ \partial_\mu \phi^0)] + \frac{1}{2} g [W_\mu^+ (H \partial_\mu \phi^- - \phi^- \partial_\mu H)$
$i g \frac{s_w^2}{c_w} M Z_\mu^0 (W_\mu^+ \phi^- - W_\mu^- \phi^+)$
$+ i g s_w A_\mu (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu$
$\frac{1}{3}(\bar{d}_j^\lambda \gamma^\mu d_j^\lambda)] + \frac{ig}{4c_w} Z_\mu^0 [(\bar{\nu}^\lambda \gamma^\mu (1 + \gamma^5)$
$+ \frac{ig}{2\sqrt{2}} W_\mu^- [(\bar{e}^\lambda \gamma^\mu (1 + \gamma^5)$
$+ m_u^\lambda (\bar{u}_j^\lambda C_{\lambda\kappa} (1 + \gamma^5) d_j^\kappa]$

1. three generations (families)

Three copies of $(\ell_L, e_R, q_L, u_R, d_R)$ have been found.



$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$	e_R	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	u_R	d_R
$\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}$	μ_R	$\begin{pmatrix} c_L \\ s_L \end{pmatrix}$	c_R	s_R
$\begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$	τ_R	$\begin{pmatrix} t_L \\ b_L \end{pmatrix}$	t_R	b_R

$m_e = 0.5\text{MeV}$ $\quad m_u = 1.5 - 3\text{MeV}$

$m_d = 3 - 7\text{MeV}$

$m_\mu = 105\text{MeV}$ $\quad m_c = 1.25\text{GeV}$

$m_s = 95\text{MeV}$

$m_\tau = 1.8\text{GeV}$ $\quad m_t = 174\text{GeV}$

$m_b = 4.2\text{GeV}$

Gauge charges (representations) are the same, but Yukawa couplings are different : masses are different

In fact, Yukawa coupling is not diagonal to generations. It is given by 3 × 3 matrix running over the generation index.

$$y^e_{ij}\overline{\ell_{Li}}\cdot\phi e_{Rj} + y^u_{ij}\epsilon^{ab}\overline{q_{La,i}}\phi^*_b u_{Rj} + y^d_{ij}\overline{q_{Li}}\cdot\phi d_{Rj}$$

masses are determined by eigenvalues of Yukawa coupling matrix.

1) Quarks :

diagonal

$$L_u y^u R_u^\dagger = \boxed{\widetilde{y}^u}, \qquad L_d y^d R_d^\dagger = \boxed{\widetilde{y}^d}$$

Then charged current (coupling to W-boson) becomes

$$\overline{u}_L\gamma^\mu d_L = \overline{\widetilde{u}}_L\gamma^\mu L_u L_d^\dagger \widetilde{d}_L \equiv \overline{\widetilde{u}}_L\gamma^\mu \underline{V_{\mathrm{CKM}}}\,\widetilde{d}_L$$

quarks in mass eigenbasis

Cabibbo-Kobayashi-Maskawa (CKM) matrix

While CKM matrix is very close to identity, for more than 3 generations, unremovable phases remain : for 3 generations, one phase remains.

: origin of T (=CP) violation in the weak interaction

2) If neutrinos are massless, such a mixing does not exist because any basis change in neutrinos gives the same, degenerate zero mass.

However, it turned out that neutrinos are massive and mixing exists : Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix

The PMNS matrix is not close to identity : 2nd and 3rd generations mix with $\pi/4$

There may be a special reason for it, e.g. discrete symmetry

The Standard model Lagrangian we have seen consists of renormalizable (relevant/marginal) operators only.

But the Standard Model is a low energy effective field theory, not a UV complete theory, so non-renormalzable (irrelevant) operators may be included.

Example : neutrino Majorana mass term

$$\frac{c}{M}(\epsilon^{ab}\overline{\ell_{La}}\phi_b^*)^2$$

singlet under the Standard Model gauge group and gives a neutrino Majorana mass term

$$\frac{cv^2}{2M}\overline{\nu}_L\overline{\nu}_L$$

If M is heavy enough, very tiny neutrino masses can be explained even with $O(1)$ value of the coefficient c.

UV completion: introducing right-handed neutrino with heavy Majorana mass :

$$M\nu_R\nu_R + y_\nu \epsilon^{ab}\overline{\ell_L}\phi_b^* \nu_R + \text{h.c.}$$

In the $(\boldsymbol{\nu_L}, \boldsymbol{\nu_R})$ basis, the mass matrix is given by

$$\begin{pmatrix} 0 & y_\nu \frac{v}{\sqrt{2}} \\ y_\nu \frac{v}{\sqrt{2}} & M \end{pmatrix}$$

After diagonalization, the leading term of the smaller mass eigenvalue is

$$\frac{y_\nu^2 v^2}{2M}$$

For $\boldsymbol{M \gg v}$ or $\boldsymbol{y_\nu \ll 1}$ mass eigenbasis is close to $\boldsymbol{\nu_L}$

(see-saw mechanism : cf. SO(10))

One more reason that the operator $\frac{c}{M}(\epsilon^{ab}\overline{\ell_L}\phi_b^*)^2$ is interesting: it breaks a global symmetry of the Standard Model (renormalizable) Lagrangian.

Two global symmetries of the Standard Model :

1. Baryon number $q_L \to e^{i\theta/3} q_L, \quad u_R \to e^{i\theta/3} u_R, \quad d_R \to e^{i\theta/3} d_R$

leptons are neutral

(point : all the quarks have the same charge; 1/3 is assigned to make baryons (qqq bound state) have the baryon number +1)

2. Lepton number $\ell_L \to e^{i\theta} \ell_L, \quad e_R \to e^{i\theta} e_R$

quarks are neutral

The operator $\frac{c}{M}(\epsilon^{ab}\overline{\ell_L}\phi_b^*)^2$ is not a singlet under the lepton number! Indeed, Majorana mass term breaks the lepton number. (indeed anomalous)

In general, global symmetry is feasible :

If we put global charges into the black hole, we do not have any way to measure it. (for U(1) gauge symmetry, charge inside the black hole changes the geometry and give the nontrivial electric field, the flux of which coincides with the amount of charge.)

If the global charge is preserved even inside the black hole, the entropy bound is violated

Global symmetry as an accidental symmetry

Global symmetry 'emerges' by some accidental structure of the model:

e.g., considering renormalizable operators only.

separation of quarks and leptons by the strong interaction

negligible Majorana mass term ….

Chiral Perturbation Theory

Theory of mesons :

quark confinement, spontaneous breaking of accidental global symmetry…

Every quark consists of three ‘colors’ : $SU(3)_c$ gauge invariance

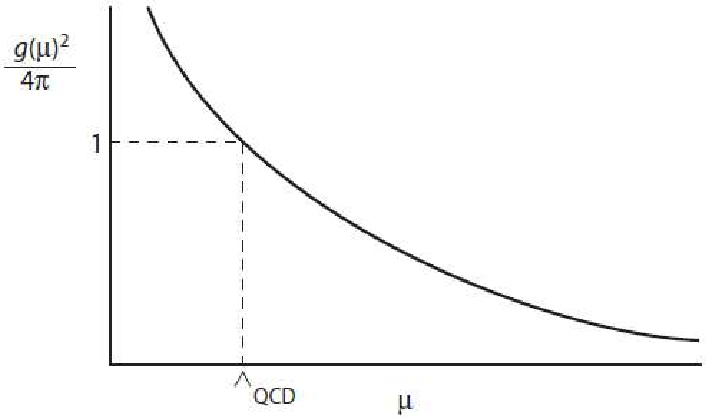
(quantum chromodynamics : QCD)

$$\begin{pmatrix} u_L^r \\ u_L^g \\ u_L^b \end{pmatrix}$$

Asymptotic freedom : the interaction gets stronger as the probe scale gets lower

it becomes non-perturbative at some scale Λ_{QCD}~ 100 MeV

$$\alpha(\mu) \equiv \frac{g^2(\mu)}{4\pi} > 1 \quad \text{for} \quad \mu < \Lambda_{\rm QCD}$$



As QCD strongly bound quarks at low energy, we can observe SU(3)$_c$ singlets (hadrons)

meson : $\bar{q}^a q^a$ **(spin-0, 1)**

baryon : $\epsilon_{abc} q^a q^b q^c$ **(spin-1/2, 3/2) p, n : light spin-1/2 baryons**

Even though heavy (needs more energy to overcome electric repulsion), uuu bound state can exist (Han-Nambu)

'heavy' quark : (quark mass) > Λ_{QCD} c, b, t

← **s ?**

'light' quark : (quark mass) < Λ_{QCD} u, d

$m_u = 1.5 - 3\text{MeV}$

$m_d = 3 - 7\text{MeV}$

$m_c = 1.25\text{GeV}$

$m_s = 95\text{MeV}$

$m_t = 174\text{GeV}$

$m_b = 4.2\text{GeV}$

Light quark dynamics (u, d, and possibly s)

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{2}\text{tr}\big[F_{\mu\nu}F^{\mu\nu}\big] + \sum_{\alpha=u,d} \overline{q}_\alpha (i\not{D} - M) q_\alpha$$

If we consider u and d quarks only, since $M \ll \Lambda_{\text{QCD}}$, we can neglect the quark mass as a leading approximation.

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{2}\text{tr}\big[F_{\mu\nu}F^{\mu\nu}\big] + \sum_{\alpha=u,d} \overline{q}_\alpha i\not{D} q_\alpha$$

In the massless limit,

1) u and d quarks behave in the exactly same way

2) (1/2,0) and (0,1/2) are separated

(since only mass term mixes (1/2,0) and (0,1/2))

$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix}$$

Caution :

1. q_L is NOT a SU(2)$_W$ gauge doublet : we are probing the scale below electroweak breaking so quarks here behave like vector-like. (obvious if we include s-quark)

2. Every quark is a SU(3)$_c$ triplet

Then, the accidental chiral symmetry $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$ emerges (slightly broken by quark masses)

$$\sum_{\alpha=u,d} \overline{q}_\alpha i\not{D} q_\alpha = \sum_{\alpha=u,d} \left(\overline{q_L}_\alpha i\not{D} q_{L\alpha} + \overline{q_R}_\alpha i\not{D} q_{R\alpha}\right) \qquad q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix}$$

$$\boxed{\begin{array}{lcl} \mathrm{SU}(2)_L & : & q_{L\alpha} \to V_{L\alpha}^{\ \beta} q_{L\beta}, \quad q_{R\alpha} \to q_{R\alpha} \\ \mathrm{SU}(2)_R & : & q_{L\alpha} \to q_{L\alpha}, \quad q_{R\alpha} \to V_{R\alpha}^{\ \beta} q_{R\beta} \end{array}}$$

Through the quark confinement, the vacuum configuration

$$\langle 0 | q_{L\alpha} \overline{q_R}_\beta | 0 \rangle = \Lambda^3_{\mathrm{QCD}}\, \delta_{\alpha\beta}$$

respects the subgroup of $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$, $\mathrm{SU}(2)_V$ only ($\boldsymbol{V_L = V_R}$)

$$\langle 0 | q_{L\alpha} \overline{q_R}_\beta | 0 \rangle = \Lambda^3_{\mathrm{QCD}}\, \delta_{\alpha\beta} \to \Lambda^3_{\mathrm{QCD}} (V_L I V_R^\dagger)_{\alpha\beta}$$

To be concrete, note that

$$q_L = \frac{1}{2}(1-\gamma_5)q, \quad q_R = \frac{1}{2}(1+\gamma_5)q$$

Then the SU(2)L×SU(2)R transformation is given by

$$e^{\underbrace{i\theta_{L\alpha}T^\alpha \frac{1}{2}(1-\gamma_5)}_{\text{SU(2)}_{\text{L}}}+\underbrace{i\theta_{R\alpha}T^\alpha \frac{1}{2}(1+\gamma_5)}_{\text{SU(2)}_{\text{R}}}}$$

parity : (1/2,0) ↔ (0,1/2)
means that
q : parity even
$\gamma_5 q$: parity odd

by redefinition $\theta_V^\alpha = \frac{1}{2}(\theta_L + \theta_R) \qquad \theta_A^\alpha = -\frac{1}{2}(\theta_L - \theta_R)$

The above transformation can be rewritten as

$$e^{\underbrace{i\theta_V^\alpha T^\alpha}_{\text{SU(2)}_{\text{V}}}+\underbrace{i\theta_A^\alpha T^\alpha \gamma_5}_{\text{SU(2)}_{\text{A}}}}$$

‘vector’
:parity even

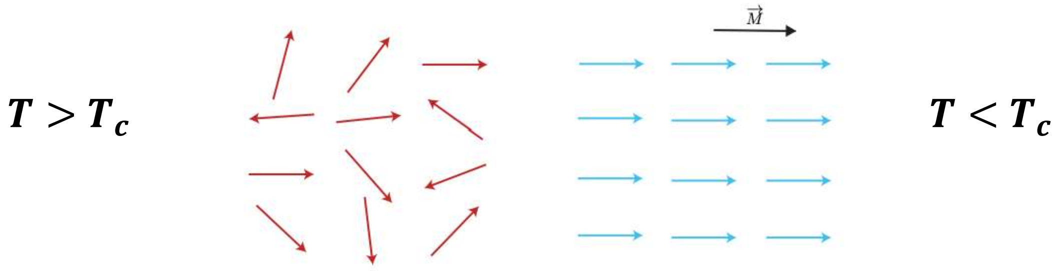
‘axial’
: parity odd

Vacuum configuration is invariant only when $\boldsymbol{\theta_A = 0}$ i.e., $\boldsymbol{\theta_L = \theta_R}$

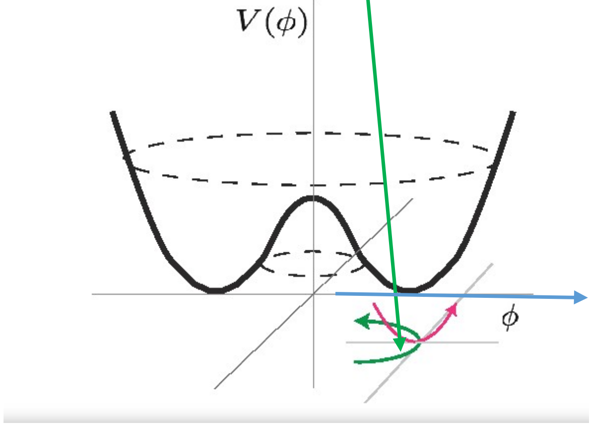
$$\langle 0|q_{L\alpha}\overline{q_R}_\beta|0\rangle = \Lambda^3_{\text{QCD}}\delta_{\alpha\beta}$$

That is, in the massless limit, while the interaction (Lagrangian) respects the global (chiral in our case) symmetry, vacuum configuration breaks it : spontaneous symmetry breaking ('hidden symmetry')

The similar situation can be found in the magnetization of ferromagnet



In this case, quantum fluctuation in the direction of (spontaneously broken) symmetry breaking is massless : (Nambu-)Goldstone boson



Vacuum configuration fixes the direction of spin

In magnetization, this Goldstone boson of the spin wave mediates the long-range force which leads to infinitely long correlation length
: explains how the spin direction here is the same as the spin direction there.

In the chiral symmetry case, SU(2)$_A$ is spontaneously broken so we expect 3 (= the number of SU(2)$_A$ generators) Goldstone bosons :

Let us parametrize the fluctuation in the direction of SU(2)$_A$ to be ξ^α ($\alpha = 1, 2, 3$): up to the vacuum configuration, we may express them as

SU(2)$_A$ generators

$$T^\alpha = \frac{\sigma^\alpha}{2}$$

$$q(x) = \begin{pmatrix} u(x) \\ d(x) \end{pmatrix} = e^{i\sum_\alpha \xi^\alpha T^\alpha \gamma_5}\tilde{q}$$

Would-be vacuum configuration

Since $q_L = e^{i\sum_\alpha \xi^\alpha T^\alpha \times(-1)}\tilde{q}_L$ $\quad q_R = e^{i\sum_\alpha \xi^\alpha T^\alpha \times(+1)}\tilde{q}_R$

the vacuum configuration and fluctuation around it are written as

$$\langle 0|q_R\overline{q_L}|0\rangle = e^{i\sum_\alpha \xi^\alpha T^\alpha}\langle 0|\tilde{q}_R\overline{\tilde{q}_L}|0\rangle e^{i\sum_\alpha \xi^\alpha T^\alpha} = e^{i\sum_\alpha \xi^\alpha T^\alpha}\Lambda^3_{\mathrm{QCD}} I e^{i\sum_\alpha \xi^\alpha T^\alpha}$$

$$= \Lambda^3_{\mathrm{QCD}} e^{2i\sum_\alpha \xi^\alpha T^\alpha} \equiv \Lambda^3_{\mathrm{QCD}} U(x)$$

$$U(x) = e^{2i\sum_\alpha \xi^\alpha T^\alpha} \qquad \sum_\alpha \xi^\alpha T^\alpha = \frac{\sqrt{2}}{F}\begin{pmatrix} \frac{\pi^0}{\sqrt{2}} & \pi^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} \end{pmatrix} \qquad F = 184\mathrm{MeV}$$

Pions (pi-mesons), the lightest mesons are interpreted as Goldstone bosons coming from spontaneously broken chiral symmetry $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$.

Of course, $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$ is explicitly broken by quark masses : the origin of the meson mass (pseudo-Goldstone bosons)

1. How mesons transform under $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$?

$$e^{i\sum_\alpha \theta_V^\alpha T^\alpha + \theta_A^\alpha T^\alpha \gamma_5} e^{i\sum_\alpha \xi^\alpha T^\alpha \gamma_5} \tilde{q} = e^{2i\sum_\alpha \xi'^\alpha T^\alpha} \underline{e^{i\sum_\alpha \theta^\alpha T^\alpha} \tilde{q}}$$

$\mathrm{SU}(2)_A$ transformation of the vacuum configuration:

$$\langle 0|\tilde{q}_R \overline{\tilde{q}_L}|0\rangle = \Lambda_{\mathrm{QCD}}^3 I$$

Invariant

For q_L $(\gamma_5 = -1)$: $e^{i\sum_\alpha \theta_L^\alpha T^\alpha} e^{-i\sum_\alpha \xi^\alpha T^\alpha} = e^{-i\sum_\alpha \xi'^\alpha T^\alpha} e^{i\sum_\alpha \theta^\alpha T^\alpha}$

$$\theta_L^\alpha = \theta_V^\alpha - \theta_A^\alpha$$

For q_R $(\gamma_5 = +1)$: $e^{i\sum_\alpha \theta_R^\alpha T^\alpha} e^{i\sum_\alpha \xi^\alpha T^\alpha} = e^{i\sum_\alpha \xi'^\alpha T^\alpha} e^{i\sum_\alpha \theta^\alpha T^\alpha}$

$$\theta_R^\alpha = \theta_V^\alpha + \theta_A^\alpha$$

$$\boxed{\begin{aligned} U'(x) = e^{2i\sum_\alpha \xi'^\alpha T^\alpha} &= e^{i\sum_\alpha \theta_R^\alpha T^\alpha} e^{2i\sum_\alpha \xi^\alpha T^\alpha} e^{-i\sum_\alpha \theta_L^\alpha T^\alpha} \\ &= e^{i\sum_\alpha \theta_R^\alpha T^\alpha} U(x) e^{-i\sum_\alpha \theta_L^\alpha T^\alpha} \end{aligned}}$$

Then SU(2)$_L$×SU(2)$_R$ invariant phenomenological Lagrangian is given by

$$\mathcal{L} = -\frac{F^2}{16}\mathrm{Tr}\big[\partial_\mu U(x)\partial^\mu U^\dagger(x)\big] + c_1\mathrm{Tr}\big[(\partial_\mu U(x)\partial^\mu U^\dagger(x))^2\big] + \cdots$$

Explicit breaking term by quark mass :

$$\mathcal{L}_{\mathrm{mass}} = -\overline{q}Mq + \mathrm{h.c.} \to \frac{1}{2}\Lambda^3\mathrm{Tr}\big[M(U+U^\dagger)\big]$$

meson mass ~ $M^{1/2}\frac{\Lambda_{\mathrm{QCD}}^{3/2}}{F}$

cf) for (u d s),

$$\sum_\alpha \xi^\alpha T^\alpha = \frac{\sqrt{2}}{F}\begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta^0 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta^0 & K^0 \\ K^- & \overline{K}^0 & -\sqrt{\frac{2}{3}}\eta^0 \end{pmatrix}$$

*** Why SU(3)$_A$, not U(1)$_A$?**

: U(1)$_A$ is anomalous (broken at quantum level) . The corresponding Goldston boson becomes heavy by non-perturbative effects (instanton).