Group Research Project
KIAS-SNU Physics Winter Camp 2022

PART 1

Suppose a spin system

$$a|\uparrow\rangle_s + b|\downarrow\rangle_s$$

with an environment of N identical spin states

$$\otimes_{k=1}^{N}(\alpha|\uparrow\rangle + \beta|\downarrow\rangle)_k.$$

The system-environment interaction Hamiltonian is assumed to be

$$\hat{H}_{se} = (|\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|)_s \otimes \sum_{k=1}^{N} g_k(|\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|)_k \otimes_{(k'\neq k)} 1_{k'} \qquad (1)$$

where g_k is the coupling constant and 1_j is the identity operator of the j'th environmental spin system. The initial total state is simply

$$|\psi(0)\rangle = (a|\uparrow\rangle + b|\downarrow\rangle)_s \otimes_{k=1}^{N} (\alpha|\uparrow\rangle + \beta|\downarrow\rangle)_k.$$

(1) Obtain the total state $|\psi(t)\rangle$ at time t.

(2) Obtain the density operator $\rho_s(t)$ of system s at time t by taking a partial trace of the total state se.

(3) Obtain the absolute magnitude $C(t)$ of the off-diagonal terms of the density matrix $\rho_s(t)$.

(4) Assuming reasonable conditions (*e.g.* sufficiently large size of the environment), obtain the limiting value of $C(t)$ for $t \to \infty$.

(4) Assuming reasonable conditions, show graphs of $C(t)$ against time t.

(5) Discuss the results.

PART 2

The spin operator of an arbitrary direction in the spherical coordinate is

$$\hat{S}(\theta) = \frac{\hbar}{2}\begin{pmatrix} \cos\theta & \sin\theta e^{-i\phi} \\ \sin\theta e^{i\phi} & -\cos\theta \end{pmatrix}.$$

The Bell-CHSH inequality is

$$B(\theta,\phi,\theta',\phi') \equiv |E(\theta,\phi)+E(\theta,\phi')+E(\theta',\phi)+E(\theta',\phi')| \leq 2$$

where $E(\theta,\phi) = \langle\Psi|\hat{S}(\theta)\otimes\hat{S}(\phi)|\Psi\rangle$ for a bipartite state $|\Psi\rangle$.

Consider an entangled bipartite state $|\Psi\rangle_{12}$ of two spin-1/2 particles 1 and 2:

$$|\Psi\rangle_{12} = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1|\downarrow\rangle_2 + |\downarrow\rangle_1|\uparrow\rangle_2).$$

(1) Find the angles for which the Bell-CHSH inequality is maximally violated.

(2) Suppose that *both* particles 1 and 2 undergo decoherence subject to the Hamiltonian in Eq. (1). Plot the change of the maximum value of $B(\theta,\phi,\theta',\phi')$ against time t.

(3) Discuss the results.

References

[1] M. Schlosshauer, Rev. Mod. Phys. 76, 1267 (2005).

[2] J.F. Clauser, M.A. Horne, A. Shimony, and R.A. Holt, Phys. Rev. Lett. 23, 880 (1969); Phys. Rev. Lett. 24, 549 (1970).