Investigating scale of cosmic homogeneity using multi-fractal analysis of SDSS quasars (Ongoing Study)

Survey Science Group Workshop 2023

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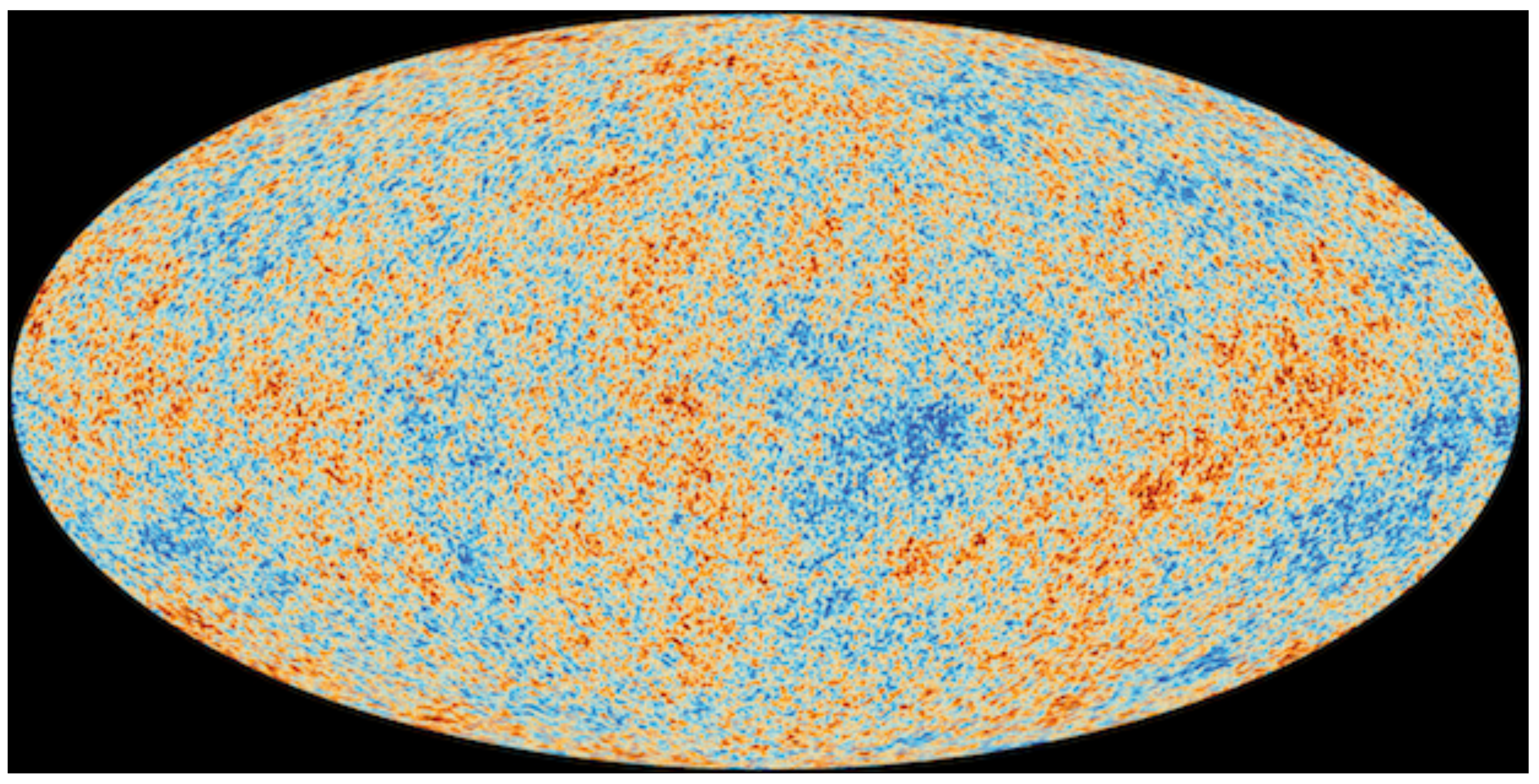
Cosmological Principle

- ΛCDM : concordance model of cosmology is consistent with observations of CMB & large scale structure of the Universe.
- Our Universe is statistically,
  - Homogeneous ~ translation invariance
  - Isotropic ~ rotationally invariance

on large scale.

There are no preferred positions or directions in the Universe

- One of the greatest challenges of standard model today is to test these assumptions.



$$\frac{\Delta T}{T} \sim 10^{-5}$$

CMB Temperature fluctuations map by ESA Planck Collaboration

Testing the cosmological principle in the radio sky

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Article PDF

References ▾

+ Article information

Abstract

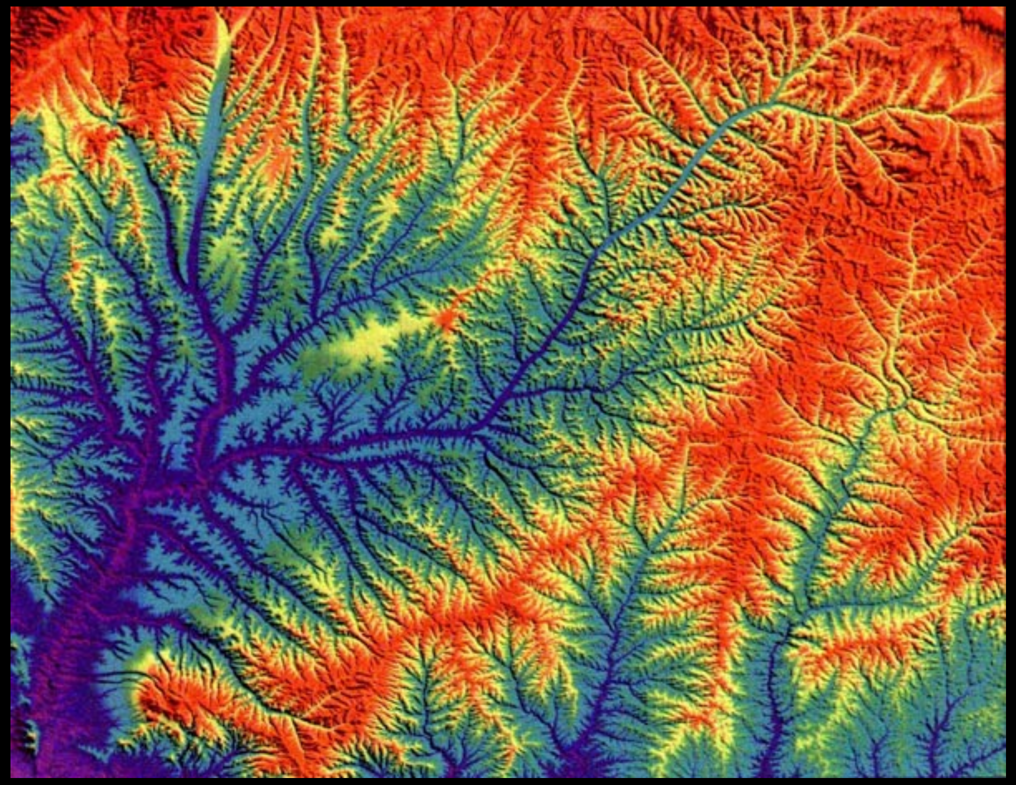
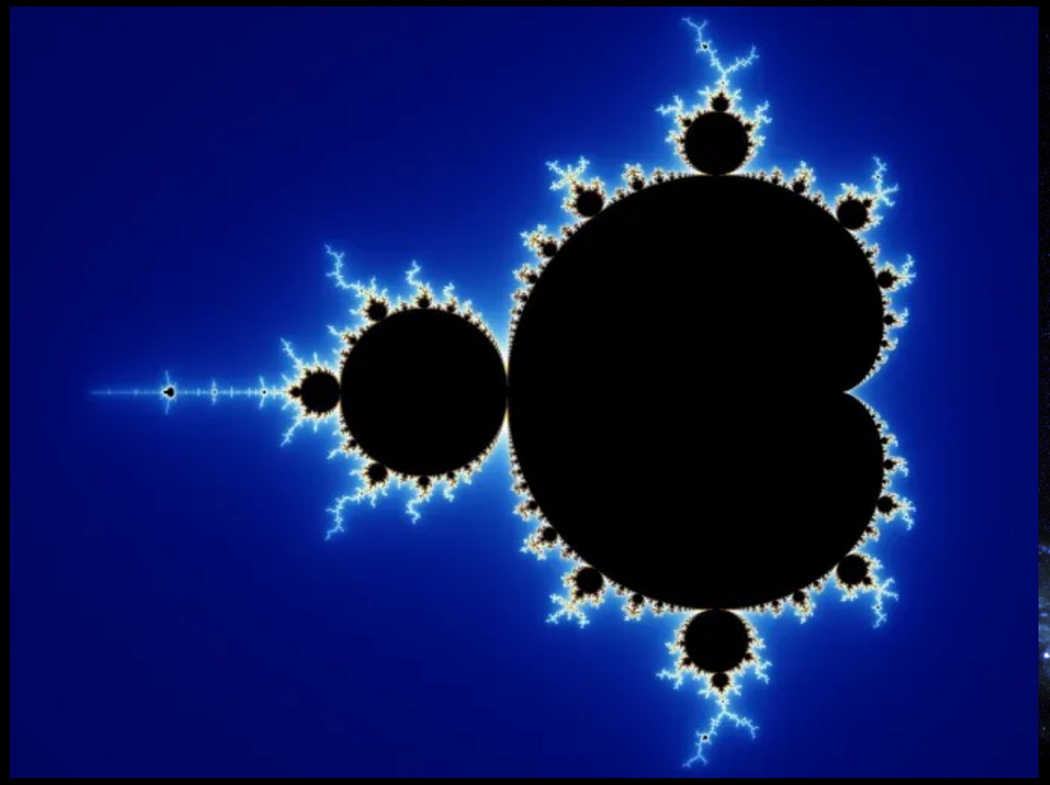
The Cosmological Principle states that the Universe is statistically isotropic and homogeneous on large scales. In particular, this implies statistical isotropy in the galaxy distribution, after removal of a dipole anisotropy due to the observer's motion. We test this hypothesis with number count maps from the NVSS radio catalogue. We use a local variance estimator based on patches of different angular radii across the sky and compare the source count variance between and within these patches. In order to assess the statistical significance of our results, we simulate radio maps with the NVSS specifications and mask. We conclude that the NVSS data is consistent with statistical isotropy.

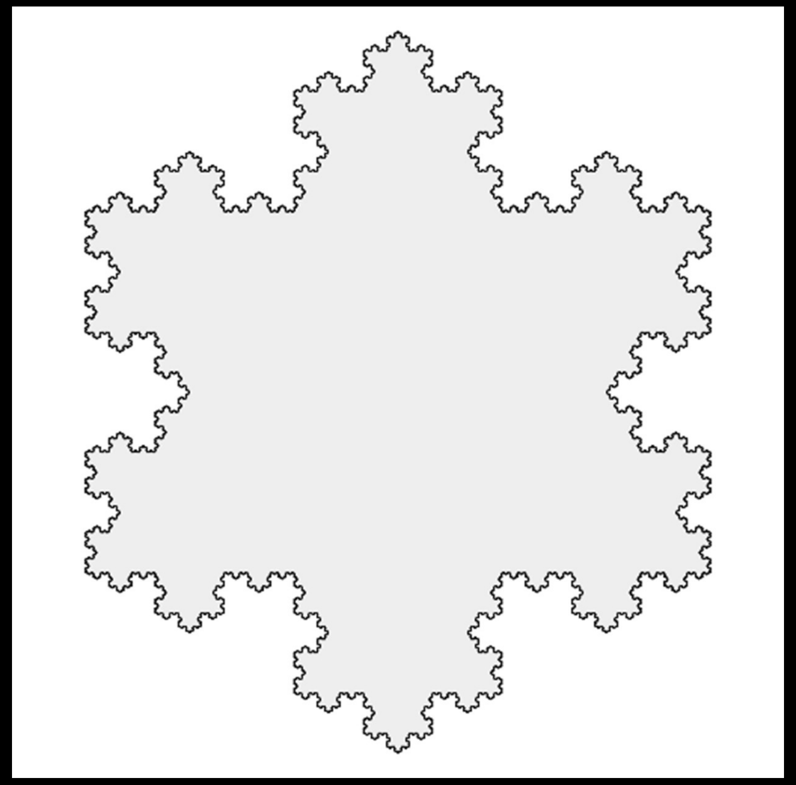
STATISTICAL HOMOGENEITY IS MUCH MORE CHALLENGING TO TEST

- Homogeneity is required by several important statistical probes of cosmology, such as the galaxy power spectrum and n-point correlation functions, in order for them to be meaningful.
- 3D mapping of universe through observations of galaxies, quasars makes it possible to test homogeneity in the nearby universe.
- Commonly used statistical methods are based on measuring counts in sphere, ie. number of objects (galaxies/quasars) in spheres of radius r centered on galaxies, then averaging this count over large number of such objects.
- This quantity scales in proportion to volume ($\propto r^3$) for a homogeneous distribution and homogeneity is said to be reached above which this hold true.
- This measurement can also be extended to a fractal analysis. Fractal dimensions can be used to quantify clustering; they quantify the scaling of different moments of (galaxy) counts-in-spheres, which in turn are related to the n-point correlation functions.

What is a Fractal?

- The term ‘fractal’ was coined by mathematician Benoit Mandelbrot in 1975, and he identified the domains of applicability of fractal geometry.
- Fractal: “a rough or fragmented geometric shape that can be split into parts each of which is reduced size copy of the whole”
- Fractal properties,
  - Self similarity
  - Fractal has a (suitably defined) dimension which need not be an integer.
  - Applications: Astronomy (Modeling our universe, Cloud & Coastline measurement, Human anatomy, Data compression (ML), fractal art, many more..)



Correlation dimension

- The most commonly used is the correlation dimension D_2(r), which quantifies the scaling of the two-point correlation function.
- Label the objects from 1 to N, draw a sphere of comoving radius r around ith object, and count the number of objects within this comoving sphere, $n_i(r)$.
- We then average over $n_i(r)$, by choosing M different objects as centers which gives us counts in sphere C_2 as,

$$C_2(r) = \frac{1}{MN} \sum_{i=1}^{M} n_i(r) \ ,$$

$$r = \sqrt{d(z_1)^2 + d(z_2)^2 - 2\,d(z_1)\,d(z_2)\ cos(\theta)} \ ,$$

$$cos(\theta) = sin(\delta_1)\,sin(\delta_2) + cos(\delta_1)\,cos(\delta_2)\ cos(\alpha_1 - \alpha_2) \ ,$$

α and δ are respectively their RA and DEC, d is the radial comoving distance

$$d(z) = \int_0^z c\frac{dz'}{H(z')}\ ; \ \ H(z) = H_0\sqrt{\Omega_m(1+z)^2 + \Omega_\Lambda}\ .$$

Interpretation:

Probability of finding an object (quasar) within a sphere of comoving radius r centered on another quasar.

If $C_2(r) \propto r^{D_2}$,

D_2 is defined to be the correlation dimension.

Minkowski - Bouligand dimension

- A multifractal is an extension of the concept of a fractal. It incorporates the possibility that the particle distribution in different density environments may exhibit a different scaling or self-similar behaviour.
- Multifractal analysis provides a continuous spectrum of generalized dimension, D_q.
- The quantity $C_2(r)$, is now generalized to,

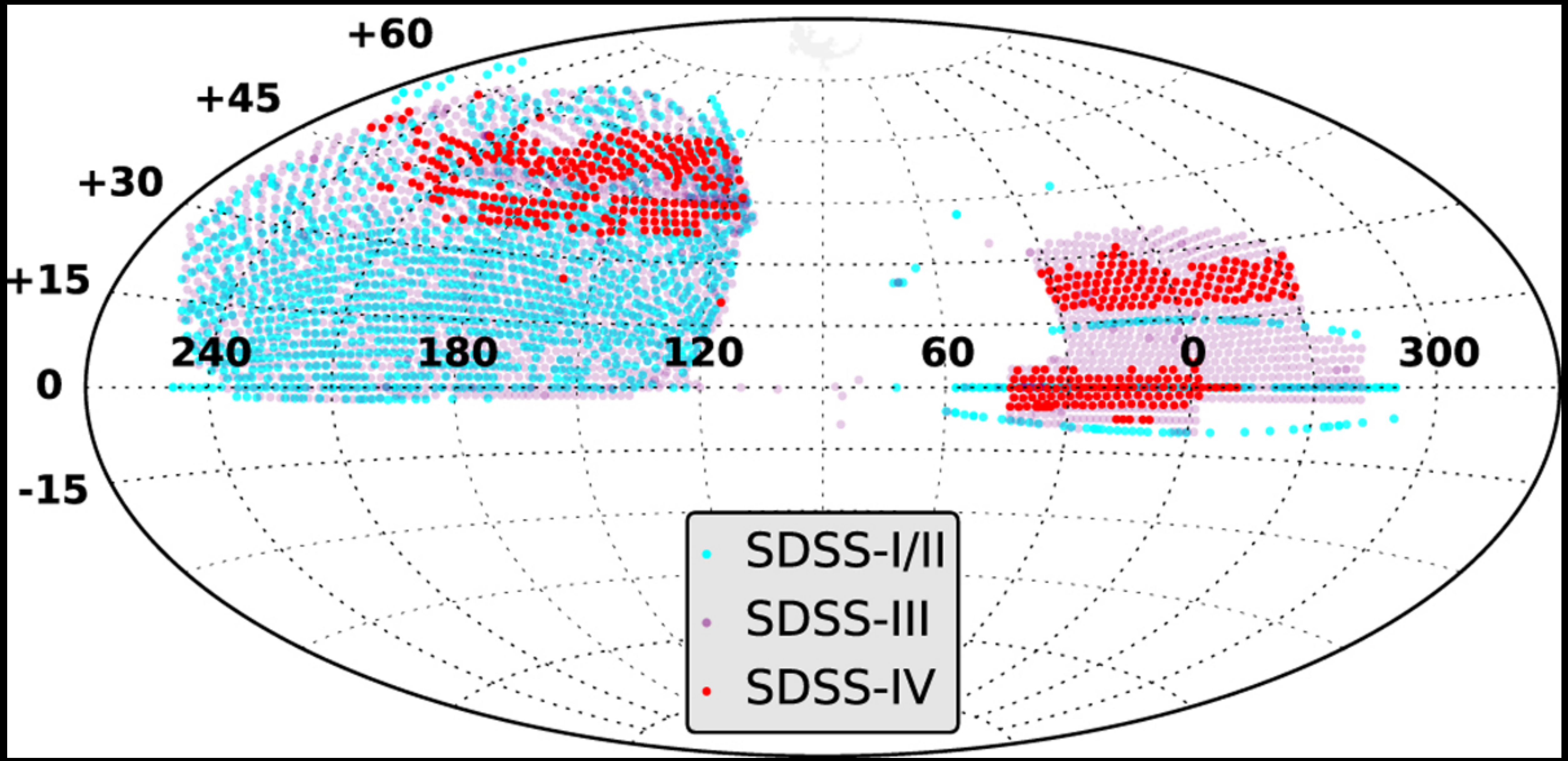
$$C_q(r) = \frac{1}{MN} \sum_{i=1}^{M} [n_i(< r)]^{q-1} \ ,$$

$$D_q = \frac{1}{q-1} \frac{d \, log \, C_q(r)}{d \, log r} ,$$

- The correlation dimension corresponds to the q=2.
- Other ($\neq$ 2) values of q are related to scaling of higher order correlation functions.
- For a monofractal D_q is constant, independent of q.
- +q values characterizes the scaling behavior in high density regions, while -q values gives more weightage to underdense regions.

Observational Data

- SDSS DR14 quasars of extended Baryon acoustic oscillation survey (eBOSS) .
- The survey covers sky area of about 2000 deg^2 consisting of about 144 046 quasars in redshift interval of $0.8 < z < 2.25$.
- DR14 is divided into two regions in the sky, north and south galactic caps.



Distribution on the sky of the SDSS-DR14/eBOSS spectroscopy in J2000 equatorial coordinates (expressed in decimal degrees). Cyan dots correspond to the 1462 plates observed as part of SDSS-I/II. The purple area indicates the 2587 plates observed as part of SDSS-III/BOSS. The red area represents to the 496 new plates observed as part of SDSS-IV (i.e. with MJD ≥ 56 898).

Redshift interval	No of quasars
0.80 - 1.17	19167
1.25 - 1.48	19135
1.56 - 1.82	19198
1.90 -2.25	19224

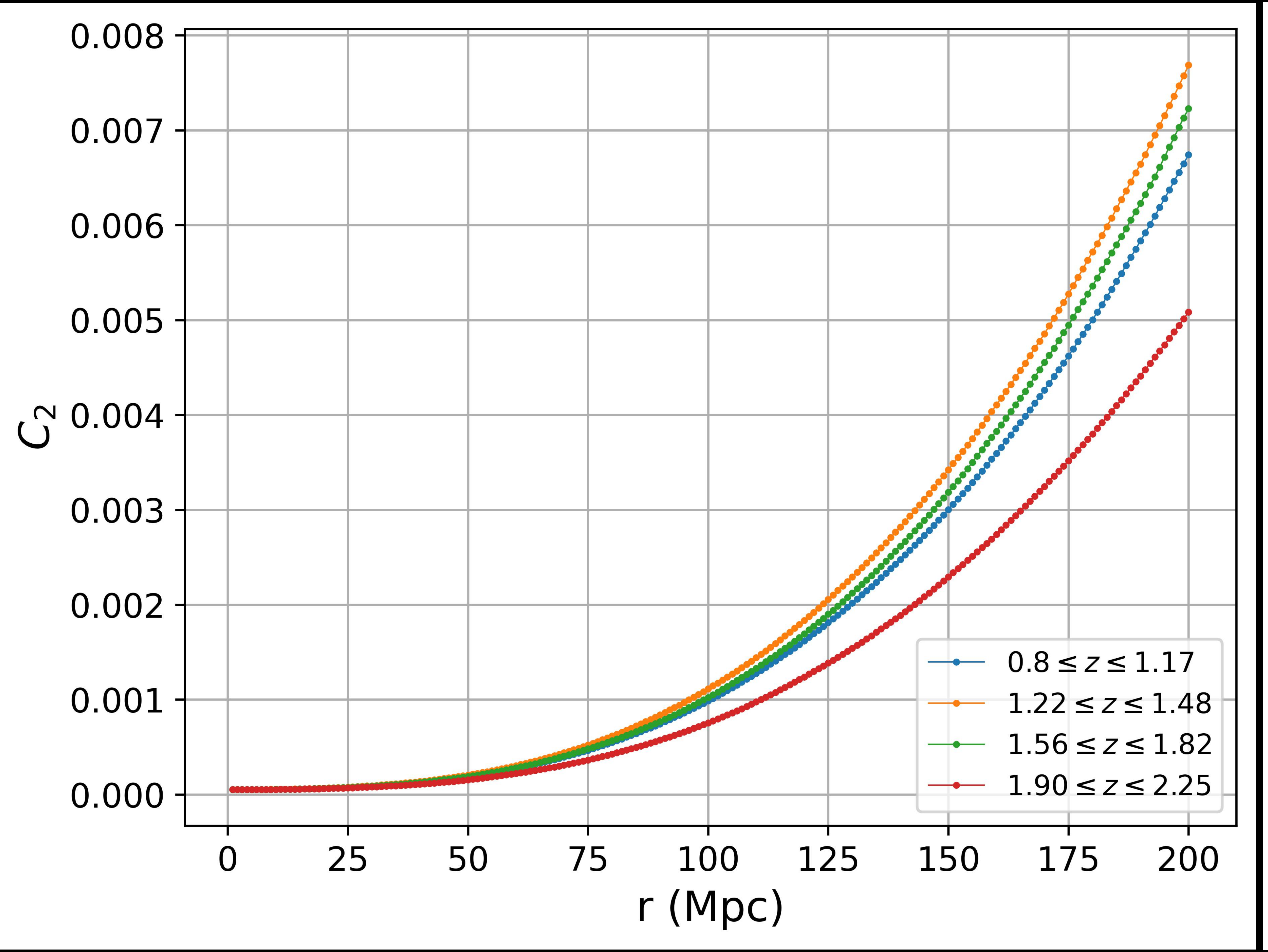
- Split the sample into four redshift bins, as presented in table on left, of width 0.26 < z < 0.37.
- The mean redshift of the bins are $\bar{z}$= 0.985, 1.35, 1.690, 2.075.
- The number of quasars in each redshift bin is $N_q \geq 19{,}000$, thus providing good statistical performance for the analysis.
- In order to avoid correlation between neighbouring bins, non-contiguous redshift bins are chosen.

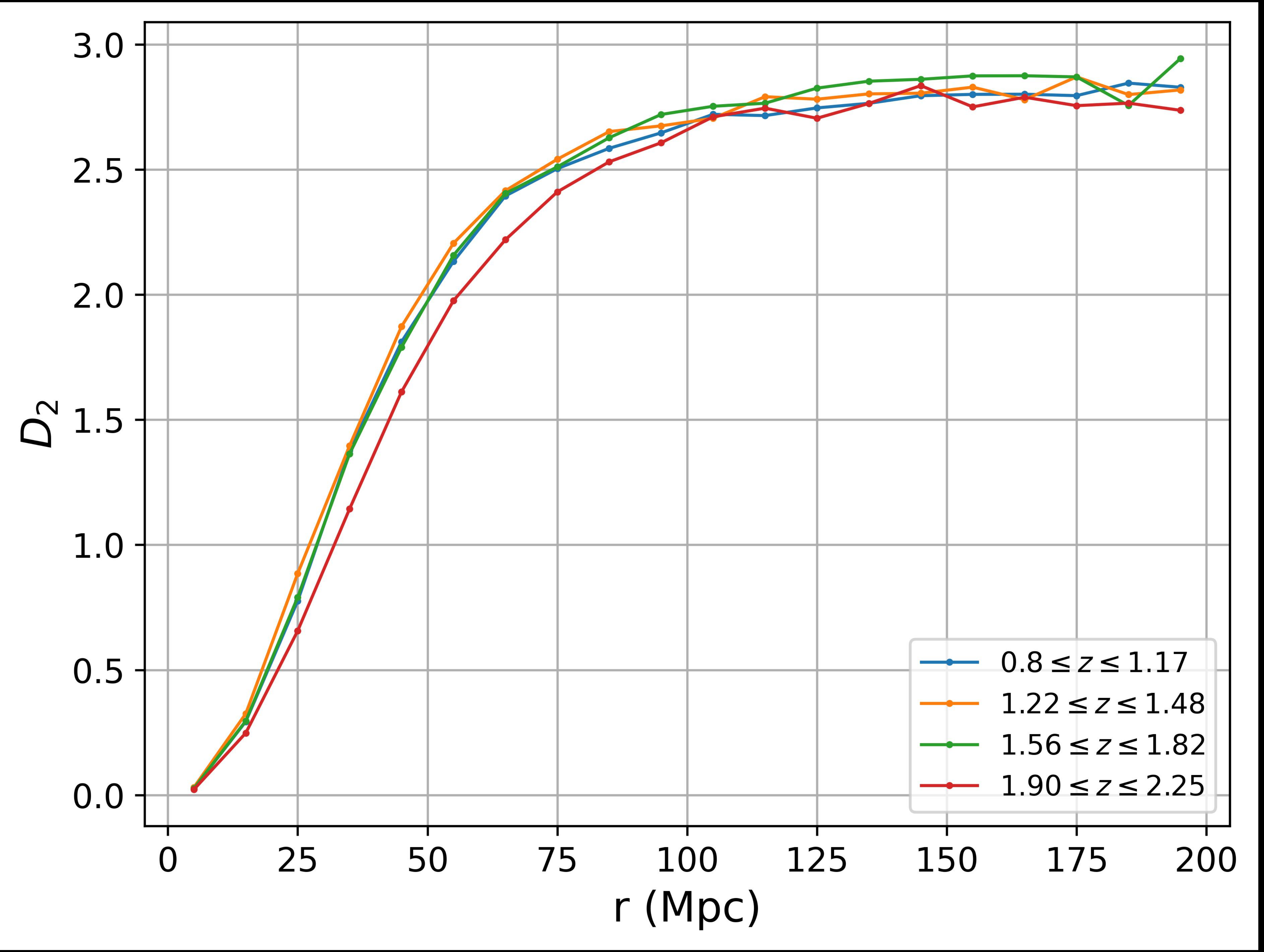
Analysis & Results:

- We computed n_i(r), with radius r increased starting from 1 Mpc h^{-1} to 200 h^{-1} Mpc for the DR14 eBOSS quasars.
- The values of n_i(r) determined using different quasars as centres were then averaged to determine C_q(r) for q = -5, -3, -2, 0, 2, 3, 5.
- It should be noted that the number of centres falls with increasing r.
- The large-scale behaviour of C_q(r) was analysed to determine the range of length-scales where it exhibits a scaling behaviour and to identify the scaling exponent D_q as a function of q.
- We also use 20 random catalogues (provided by SDSS team) that are generated by a Poisson distribution with the same geometry and completeness as the DR14 and, computed C_q(r) and hence D_q for these also.

Analysis & Results:

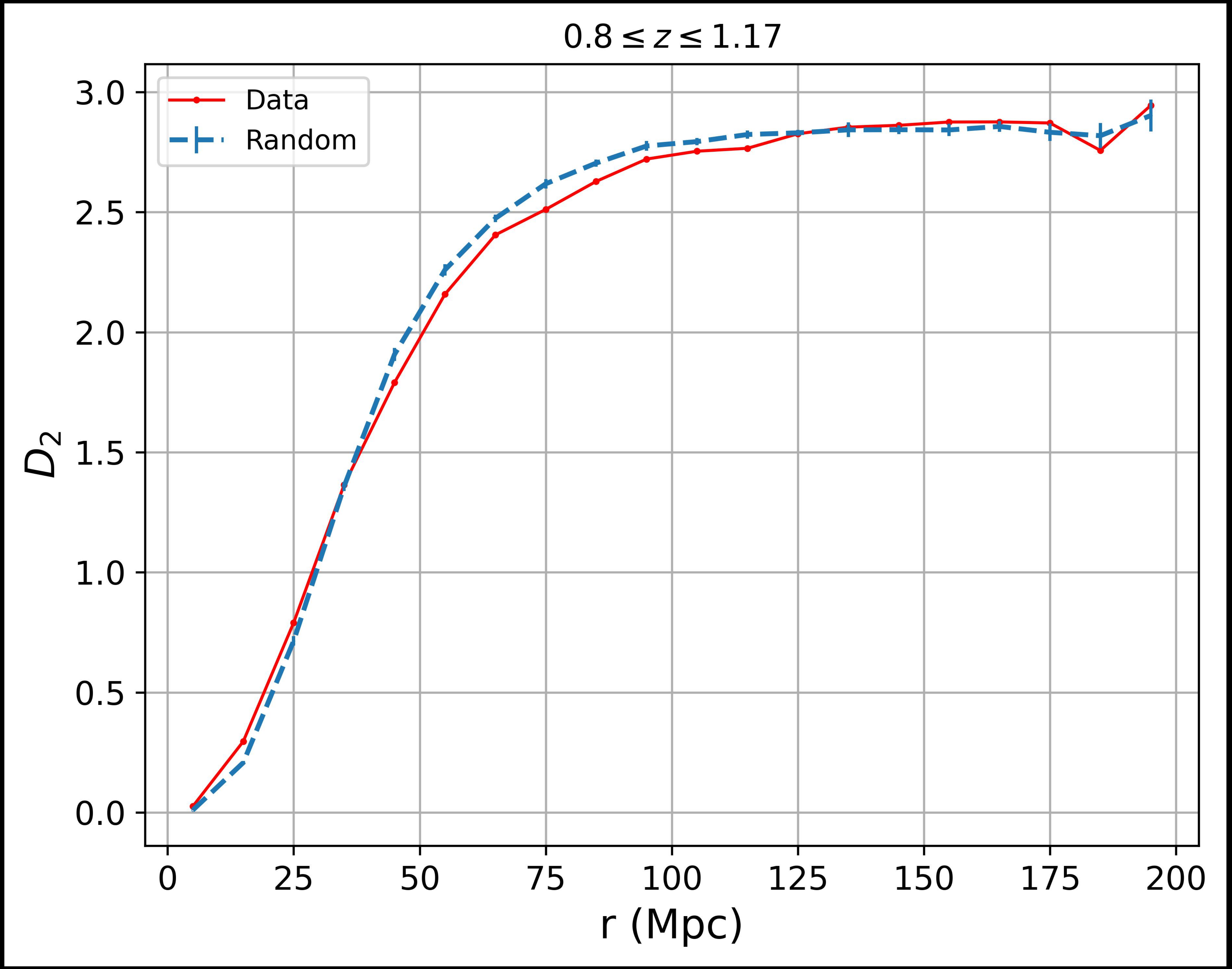
$$D_2 = \frac{dlogC_2}{dlogr}$$

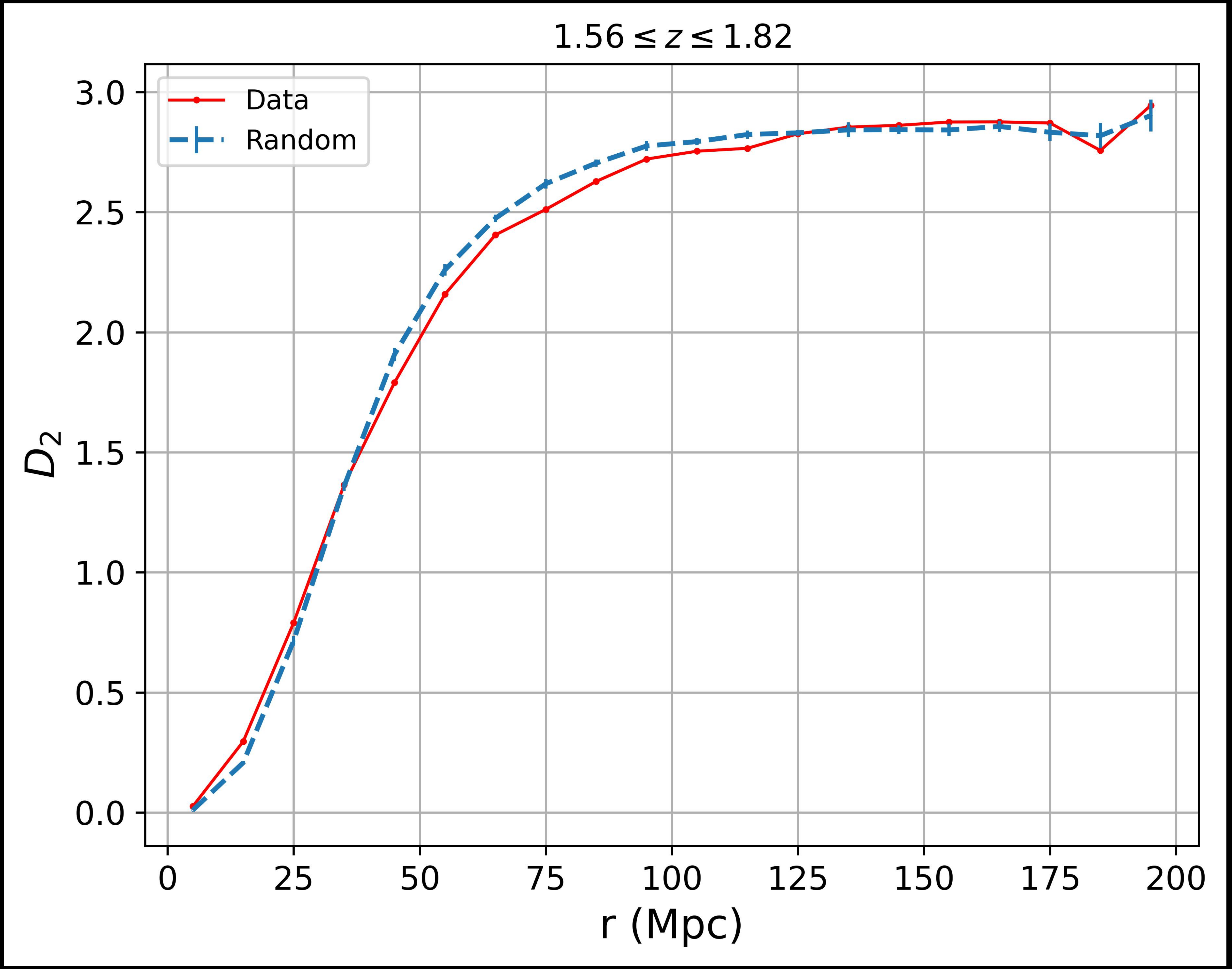




$$C_2(r) = \frac{1}{MN} \sum_{i=1}^{M} n_i(r)$$

Correlation dimension, q=2





Correlation dimension, q=2

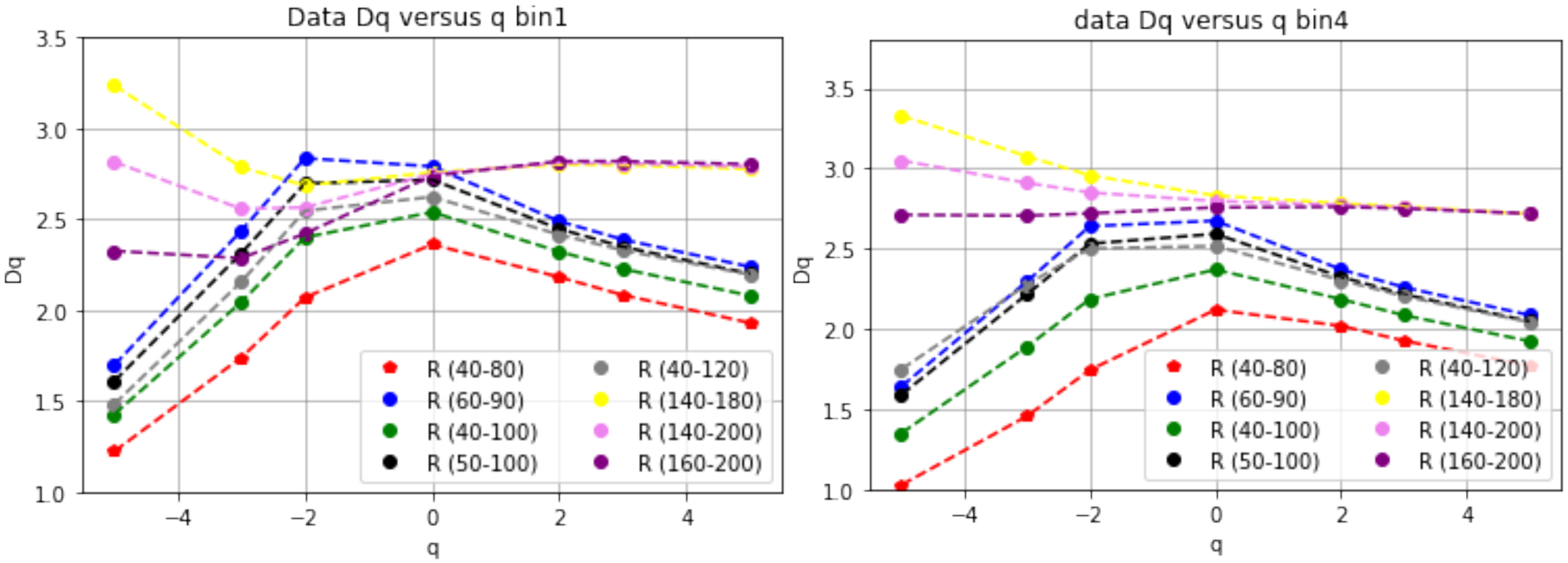
How do we obtain homogeneity scale (R_H) from this ?

Since there is only a gradual approach to homogeneity, such a definition may be arbitrary

- One way to define the ‘scale of homogeneity’, the scale where the data become consistent with homogeneity within 1σ.

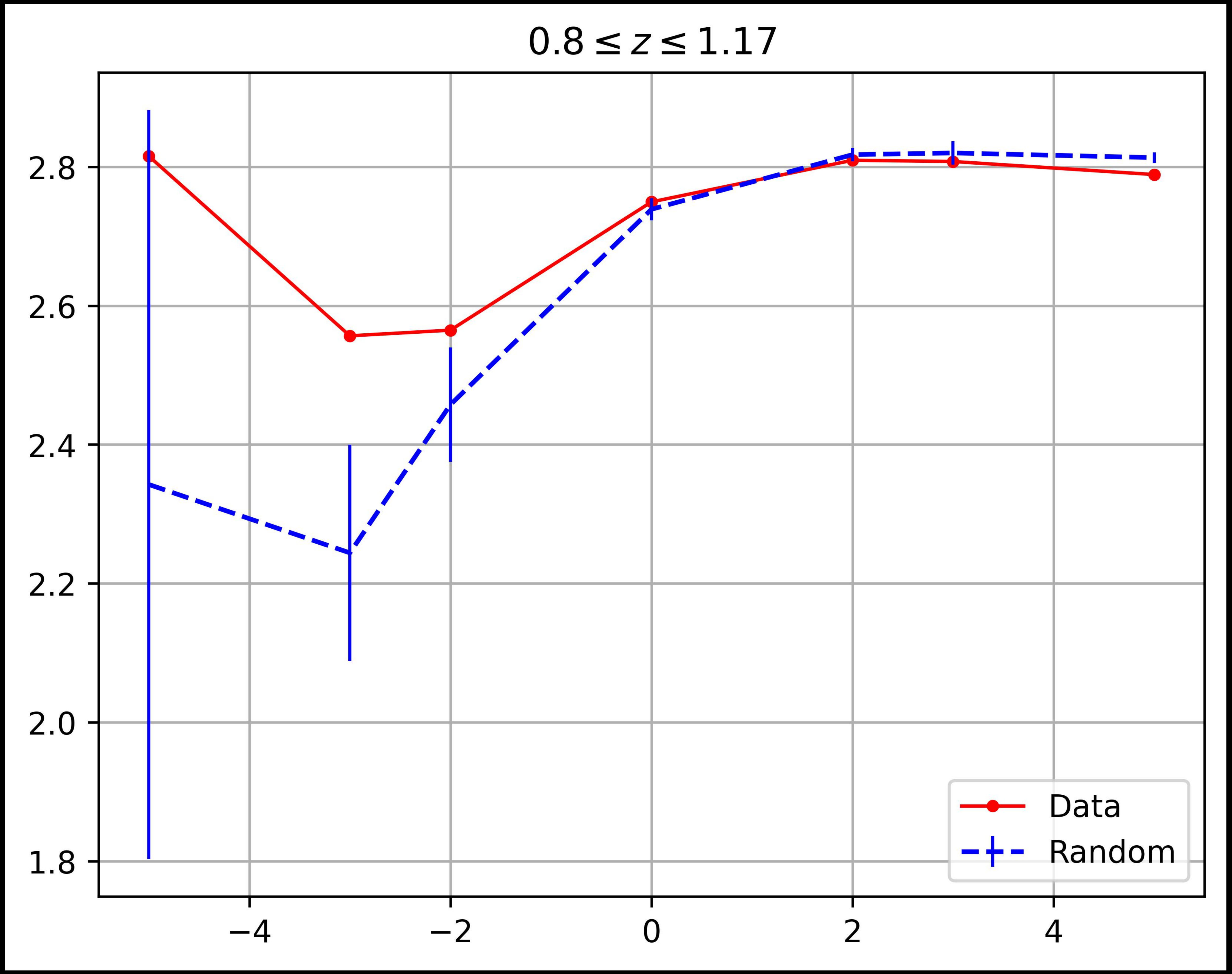
- Another way, to fit a smooth, model-independent polynomial to the data, and find the scale at which this intercepts a chosen value, or ‘threshold’, close to homogeneity. This scale is then defined as the homogeneity scale R_H .

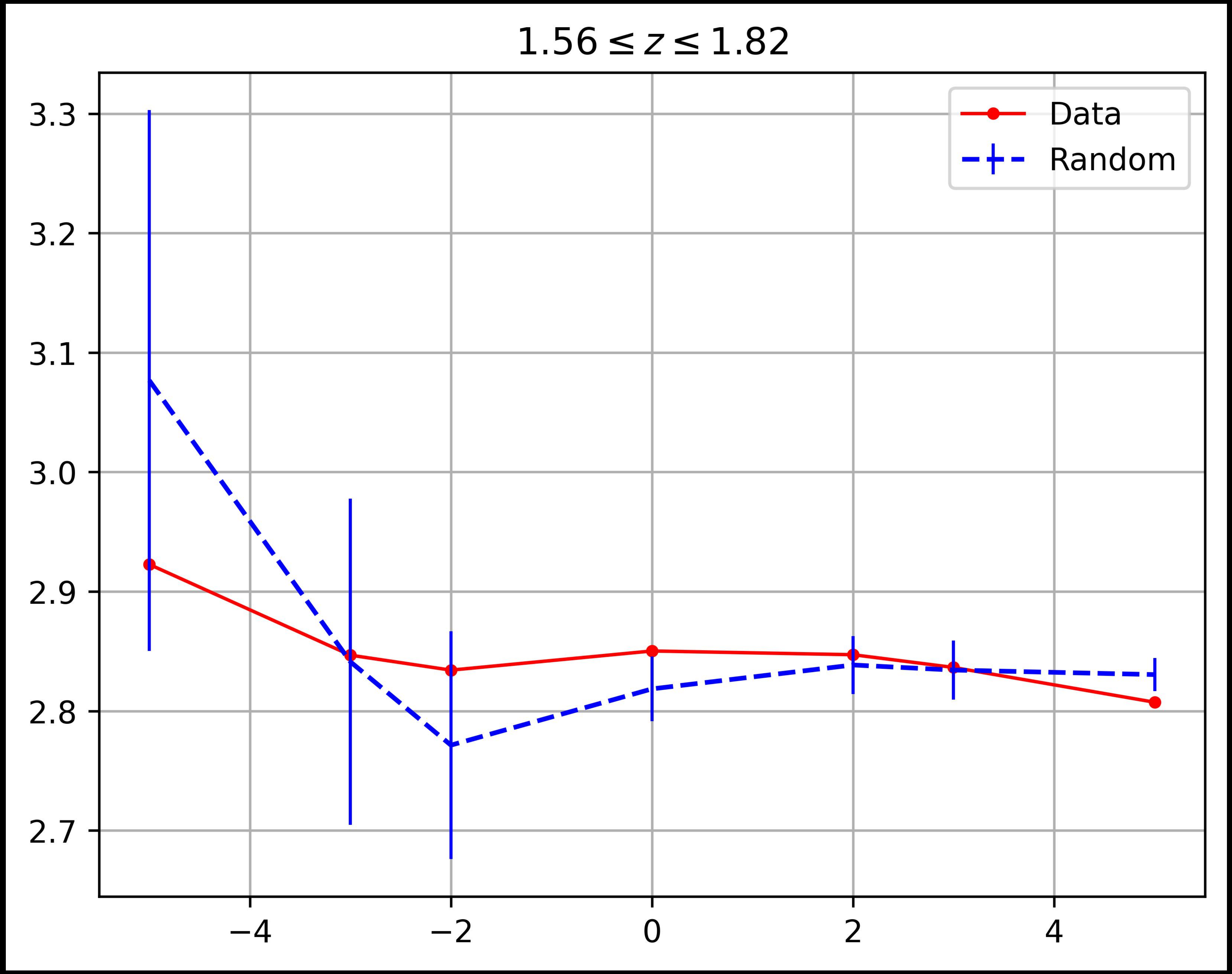
Another way ?



This figure shows the spectrum of generalized dimensions D_q as a function of *q for the* actual data across different length scales

D_q vs q plots in R 140 Mpc h^{-1}





Summary & Ongoing work

- We perform multifractal analysis using the counts-in-spheres approach, $n_i(< r)$, and using its logarithmic derivative –obtained the spectrum of Minkowski Bouligand fractal dimension D_q.
- We are trying to define another way to define (transition to) homogeneity scale R_H using the multi-fractal approach.
- We need to determine quasar bias at different redshifts and correct our values D_q in order to compare our results with theoretical expectations.
- This is important because, if we know the redshift dependence of the scale of homogeneity, it can be used as standard ruler, which are very important in cosmology.

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Thank you for you attention!