KPQC 공모전 1 라운드 격자기반 알고리즘 안전성 분석

(2023-080) KPQC 공모전 격자기반 알고리즘 기반문제 안전성 분석 기술연구

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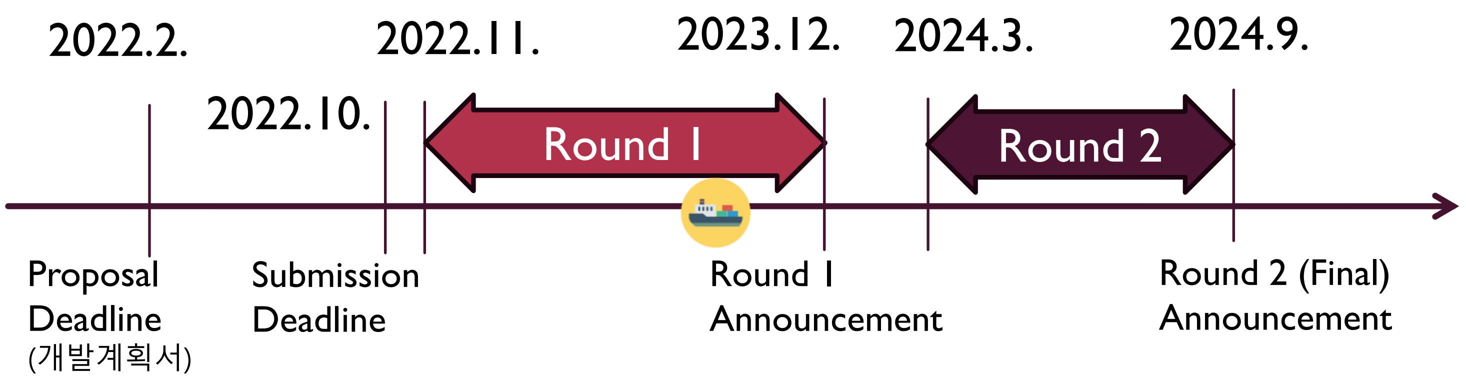
- CCA Attack for NTRU+
- May's Meet-LWE Attack Costs for Lattice-based KEMs
- Security Evaluation of {LWE, LWR}-based schemes Using Lattice Estimator

KPQC COMPETITION



- 16 algorithms in Round 1
  - 7 KEMs & 9 Signatures
- KpqC Bulletin : https://groups.google.com/g/kpqc-bulletin
  - Analysis reports
  - Benchmarks
  - Scheme Updates
  - Etc.

KPQC ROUND 1 –LATTICE-BASED SCHEMES

- Among Round 1 candidates, 3 KEMs and 5 signatures are lattice-based schemes.

Category	Name	Base problem	Note
KEM	NTRU+	NTRU/RLWE	• RLWE with binary secrets/ternary errors • Analysis Reported (6/14/23)
	SMAUG	MLWE/MLWR	• MLWE/MLWR with sparse secrets
	TiGER	RLWR/RLWE	• RLWR/RLWE with sparse secrets • Analysis Reported (7/9/23)
Signature	GCKSign	GCK	• Analysis Reported (1/14/23)
	HAETAE	MLWE/MSIS	
	NCC-Sign	RLWE/RSIS	
	Peregrine	NTRU/SIS	• Analysis Reported (1/6/23)
	SOLMAE	NTRU/SIS	

CCA ATTACK FOR NTRU+

RESULTS

Game OW-CCA	$O_{dec}(c)$
1: $(pk, sk) \leftarrow \mathsf{KeyGen}(1^\lambda)$	1: **if** $c = c^*$
2: $(K^*, c^*) \leftarrow \mathsf{Encaps}(pk)$	2: **return** $\perp$
3: $K' \leftarrow \mathcal{A}^{O_{dec}(\cdot)}(pk, c^*)$	3: **else return**
4: **return** $[K' = K^*]$	4: $K \leftarrow \mathsf{Decaps}(sk, c)$

***OW-CCA SECURITY GAME**

- Reported on 6/14/23, in the KpqC Bulletin
  - Analysis of NTRU+ (google.com), eprint: A Novel CCA Attack for NTRU+ KEM (iacr.org)
- For NTRU+ (CCA ver.), we can achieve the challenge encapsulated key K^*, and **win the CCA game** with 4 decapsulation queries in average**.**
  - **It breaks the OW-CCA security and hence NTRU+ is *not* IND-CCA secure.**
- It can be fixed by adding a verification process in the decoding algorithm("Inv") to check if the intermediate value $M + u_2$ (or the output) is binary and abort otherwise.
- We summarize some comments for the security proof, some of which introduced our attack.

NTRU

- Firstly suggested by Jeffrey Hoffstein, Jill Pipher, and Joseph H. Silverman in 1998.
  - refered as "grandfather of lattice-based encryption schemes"
- Simple and efficient
  - h : public key in R_q (h is typically set as a ratio of small polynomials g/f which are the secret keys)
  - Computes the ciphertext

$$c = m + h \cdot r \mod q$$

for small m, r

- Use the rings of the form $\mathrm{R}_q = Z_q[x]/(x^p - 1)$, where p is a prime and q is a power of 2
  - Not NTT-friendly

NTRUENCRYPT VS. RLWE-BASED ENCRYPTION

RLWE-based Enc	NTRUEncrypt
$pk = (a, b)$, • the uniform random a can be compressed with a random seed	$pk = h$
$ct = (c_1, c_2)$, • c_2 can be compressed so that only 2~3 bits for each component need to be output	$ct = c$
For flexible parameters, Module structure can be used for LWE (e.g., Kyber) • Can use smaller-degree power-of-2 rings e.g. $Z_q[x]/(x^{256}+1)$ • It does not increase pk sizes	Module approach doesn't work well • avoids power-of-2 rings because they are sparse (512, 1024, …)
Can use NTT • Highly parallelizable (with AVX implementation)	Cannot use NTT • Slow KeyGen • Other divide-and-conquer approaches such as Toom-cook, Karatsuba can be used
Decryption failure rates are dealt in the average cases • Message is an additive term in the decryption procedure	For correctness, $p(gr + mf) < q/2$ where m is a message, and g, r, f are small • Considers decryption error for worst-case messages, since an attacker might use "bad" messages to recover the secret key

NTTRU

- Vadim Lyubashevsky and Gregor Seiler. NTTRU: Truly fast NTRU using NTT. IACR Transactions on Cryptographic Hardware and Embedded Systems, 2019 (https://tches. iacr.org/index.php/TCHES/article/view/8293).

- In lattice-based schemes, typically the LWE dimension (ring dimension) $n = 7{\sim}800$ would be enough for 128-bit security.

- They show that NTT over the ring $Z_{7681}[x]/(x^{768} - x^{384} + 1)$ is as efficient as NTT over power-of-2 rings
  - Can be generalized for dimensions $2^k 3^\ell$ (n can be 576, 648, 768, 864, …)
  - q is larger than that in LWE-based enc (e.g. In Kyber, they used $q = 3329$)

Schemes	pk size	ct size	KG (cycles)	Enc (cycles)	Dec (cycles)
Kyber 512*	800	768	13K	17K	18K
Kyber 768*	1184	1088	25K	28K	30K
NTTRU**	1248	1248	6K	6K	8K

* Kyber Performance; taken from "Faster Lattice-Based KEMs via a Generic Fujisaki-Okamoto Transform Using Prefix Hashing (CCS'21)"

** Performance Taken from the NTTRU Paper

NTRU-A,B,C

- Julien Duman, Kathrin Hövelmanns, Eike Kiltz, Vadim Lyubashevsky, Gregor Seiler, and Dominique Unruh. A thorough treatment of highly-efficient NTRU instantiations. Public-Key Cryptography – PKC 2023 (https://eprint.iacr.org/2021/1352).

- They also use the NTT-friendly rings of the form $Z_q[x]/(x^n - x^{n/2} + 1)$

- They suggest transforms from PKE with small average-case correctness error into PKE' with small worst-case correctness error : NTRU-A, NTRU-B, NTRU-C
  - Smaller modulus such as $q = 3457$ for $n = 768$ are available

- E.g.

$\mathsf{Enc}'(pk, m \in \{0,1\}^\lambda)$	$\mathsf{Dec}'(sk, (c, u))$
01 $r \leftarrow \psi_{\mathcal{R}}$	03 $r := \mathsf{Dec}(sk, c)$
02 **return** $(\mathsf{Enc}(pk, r), \mathsf{F}(r) \oplus m)$	04 **return** $\mathsf{F}(r) \oplus u$

Correctness error does not have the term 'm'

	pk size	ct size
Kyber 512	800	768
Kyber 768	1184	1088
NTTRU	1248	1248
NTRU-A*	1152	1152

* Numbers Taken from the NTRU-A,B,C paper, when $n = 768$

SUMMARY ON NTRU+ KEM

- Security based on the **NTRU**, **RLWE** assumptions
  - RLWE here uses (random) binary secrets and ternary errors
- Uses **NTT-friendly rings**
  - $R_q = Z_q[x]/(f(x))$, where $f(x) = x^n - x^{n/2} + 1$ and $n = 2^i 3^j$ [1,2]
- Uses a new encoding named **SOTP** (Semi-generalized One Time Pad)
- In the CCA-secure KEM, they remove **re-encryption** in decapsulation by adjusting Fujisaki-Okamoto transform

- [1] Vadim Lyubashevsky and Gregor Seiler. NTTRU: Truly fast NTRU using NTT. IACR Transactions on Cryptographic Hardware and Embedded Systems, 2019 (https://tches. iacr.org/index.php/TCHES/article/view/8293).
- [2] Julien Duman, Kathrin Hövelmanns, Eike Kiltz, Vadim Lyubashevsky, Gregor Seiler, and Dominique Unruh. A thorough treatment of highly-efficient NTRU instantiations. Public-Key Cryptography – PKC 2023 (https://eprint.iacr.org/2021/1352).

Parameters	Security level	n	q	Sizes (Bytes)			Cycles(ref)			Cycles(AVX2)			Claimed Security (bits)	
				pk	ct	sk	Keygen	Encaps	Decaps	Keygen	Encaps	Decaps	Classical	Quantum
NTRU+576	1	576	3,457	864	864	1,728	321,405	110,754	163,277	17,440	14,307	12,445	115	104
NTRU+768	1	768	3,457	1,152	1,152	2,304	313,669	145,658	227,028	16,032	17,514	15,848	164	148
NTRU+864	3	864	3,457	1,296	1,296	2,592	339,912	169,634	262,017	14,068	19,293	17,671	188	171
NTRU+1152	5	1,152	3,457	1,728	1,728	3,456	905,131	230,448	348,076	42,993	25,592	24,063	264	240

CCA-NTRU+

- Use a new encoding **SOTP** defined by :
  - $m \in \{0,1\}^n, u = (u_1, u_2) \in \{0,1\}^{2n}$
    - $SOTP\big(m, u = (u_1, u_2)\big) \coloneqq (m \oplus u_1) - u_2 \in \{-1,0,1\}^n$
    - $Inv\big(M \in \{-1,0,1\}^n, u = (u_1, u_2)\big) \coloneqq (M + u_2) \oplus u_1 \in \{0,1\}^n$

$\underline{\mathsf{Gen}(1^\lambda)}$

- $\mathbf{f}', \mathbf{g} \leftarrow \psi_1^n$
- $\mathbf{f} = 3\mathbf{f}' + 1$
- **if** $\mathbf{f}, \mathbf{g}$ are not invertible in R_q
  - restart
- **return** $(pk, sk) = (\mathbf{h} = 3\mathbf{g}\mathbf{f}^{-1}, \mathbf{f})$

$\underline{\mathsf{Encap}(pk)}$

- $m \leftarrow \{0,1\}^n$
- $(\mathbf{r}, K) = \mathsf{H}(m)$
- $\mathbf{m} = \boxed{\mathsf{SOTP}(m, \mathsf{G}(\mathbf{r}))}$
- $\mathbf{c} = \mathbf{h}\mathbf{r} + \mathbf{m}$
- **return** $(\mathbf{c}, K)$

$\underline{\mathsf{Decap}(sk, \mathbf{c})}$

- $\mathbf{m} = (\mathbf{c}\mathbf{f} \mod {}^{\pm}q) \mod {}^{\pm}3$
- $\mathbf{r} = (\mathbf{c} - \mathbf{m})\mathbf{h}^{-1}$
- $m = \boxed{\mathsf{Inv}(\mathbf{m}, \mathsf{G}(\mathbf{r}))}$
- $(\mathbf{r}', K) = \mathsf{H}(m)$
- **if** $\mathbf{r} = \mathbf{r}'$
  - **return** K
- **else**
  - **return** $\perp$

Figure 15: CCA-NTRU+

CCA-NTRU+

- Use a new encoding **SOTP** defined by :
  - $m \in \{0,1\}^n, u = (u_1, u_2) \in \{0,1\}^{2n}$
    - $SOTP\big(m, u = (u_1, u_2)\big) := (m \oplus u_1) - u_2 \in \{-1,0,1\}^n$
    - $Inv\big(M \in \{-1,0,1\}^n, u = (u_1, u_2)\big) := (M + u_2) \oplus u_1 \in \{0,1\}^n$

Caution! $M + u_2$ has to be binary (if not, they make the result binary by computing &0x1)

$\underline{\mathsf{Gen}(1^\lambda)}$

1: $\mathbf{f}', \mathbf{g} \leftarrow \psi_1^n$
2: $\mathbf{f} = 3\mathbf{f}' + 1$
3: **if** $\mathbf{f}, \mathbf{g}$ are not invertible in R_q
4: restart
5: **return** $(pk, sk) = (\mathbf{h} = 3\mathbf{g}\mathbf{f}^{-1}, \mathbf{f})$

$\underline{\mathsf{Encap}(pk)}$

1: $m \leftarrow \{0,1\}^n$
2: $(\mathbf{r}, K) = \mathsf{H}(m)$
3: $\mathbf{m} = \boxed{\mathsf{SOTP}(m, \mathsf{G}(\mathbf{r}))}$
4: $\mathbf{c} = \mathbf{h}\mathbf{r} + \mathbf{m}$
5: **return** $(\mathbf{c}, K)$

$\underline{\mathsf{Decap}(sk, \mathbf{c})}$

1: $\mathbf{m} = (\mathbf{c}\mathbf{f} \mod {}^{\pm}q) \mod {}^{\pm}3$
2: $\mathbf{r} = (\mathbf{c} - \mathbf{m})\mathbf{h}^{-1}$
3: $m = \boxed{\mathsf{Inv}(\mathbf{m}, \mathsf{G}(\mathbf{r}))}$
4: $(\mathbf{r}', K) = \mathsf{H}(m)$
5: **if** $\mathbf{r} = \mathbf{r}'$
6: **return** K
7: **else**
8: **return** $\perp$

CCA-NTRU+

- Use a <u>new encoding</u> **SOTP** defined by :
  - $m \in \{0,1\}^n, u = (u_1, u_2) \in \{0,1\}^{2n}$
    - $SOTP(m, u = (u_1, u_2)) \coloneqq (m \oplus u_1) - u_2 \in \{-1,0,1\}^n$
    - $Inv(M \in \{-1,0,1\}^n, u = (u_1, u_2)) \coloneqq (M + u_2) \oplus u_1 \in \{0,1\}^n$

```
void poly_sotp_inv(unsigned char *msg, const poly *e, const unsigned char *buf)
{
    uint32_t t1, t2, t3;

    for(int i = 0; i < 2; i++)
    {
        for(int j = 0; j < 8; j++)
        {
            t1 = load32_littleendian
            t2 = load32_littleendian
            t3 = 0;

            for (int k = 0; k < 2; k++)
            {
                for(int l = 0; l < 16; l++)
                {
                    t3 ^= (((e->coeffs[256*i + 16*l + 2*j + k] + t2)^t1) & 0x1) << (l+16*k);

                    t1 >>= 1;
                    t2 >>= 1;
                }
            }

            msg[32*i + 4*j    ] = t3;
            msg[32*i + 4*j + 1] = t3 >> 8;
            msg[32*i + 4*j + 2] = t3 >> 16;
```

Algorithm 5 Inv

Require: Polynomial $y \in R_q$

Require: Byte array $B = (b_0, b_1, \cdots, b_{n/4-1})$

Ensure: Message Byte array $m = (m_0, m_1, \cdots, m_{31})$

1: $(\beta_0, \cdots, \beta_{2n-1}) \coloneqq \mathsf{BytesToBits}(B)$

2: **for** i from 0 to $n-1$ **do**

3: $\quad m_i \coloneqq ((f_i + \beta_{i+n} \,\&1) \oplus \beta_i$

$\quad m = \mathsf{BitsToBytes}((m_0, \cdots, m_{n-1}))$

4: **return** m

Caution! $M + u_2$ has to be binary (if not, they make the result binary by computing &0x1)

$\mathsf{Gen}(1^\lambda)$

1: $\mathbf{f}', \mathbf{g} \leftarrow \psi_1^n$

2: $\mathbf{f} = 3\mathbf{f}' + 1$

3: **if** $\mathbf{f}, \mathbf{g}$ are not invertible in R_q

4: $\quad$ restart

5: **return** $(pk, sk) = (\mathbf{h} = 3\mathbf{g}\mathbf{f}^{-1}, \mathbf{f})$

$\mathsf{Encap}(pk)$

1: $m \leftarrow \{0,1\}^n$

2: $(\mathbf{r}, K) = \mathsf{H}(m)$

3: $\mathbf{m} = \mathsf{SOTP}(m, \mathsf{G}(\mathbf{r}))$

4: $\mathbf{c} = \mathbf{hr} + \mathbf{m}$

5: **return** $(\mathbf{c}, K)$

$\mathsf{Decap}(sk, \mathbf{c})$

1: $\mathbf{m} = (\mathbf{cf} \mod {}^{\pm}q) \mod {}^{\pm}3$

2: $\mathbf{r} = (\mathbf{c} - \mathbf{m})\mathbf{h}^{-1}$

3: $m = \mathsf{Inv}(\mathbf{m}, \mathsf{G}(\mathbf{r}))$

4: $(\mathbf{r}', K) = \mathsf{H}(m)$

5: **if** $\mathbf{r} = \mathbf{r}'$

6: $\quad$ **return** K

7: **else**

8: $\quad$ **return** $\perp$

ATTACK FOR CCA-NTRU+

- **Step 1.** find an example of malicious $M' \in \{-1,0,1\}^n$ such that an intermediate value $M' + u_2$ in $Inv(M', u)$ is non-binary ;
  - Example (n=4):

Suppose m=(1,0,1,1), $\mathrm{G}(r) = u$ =(1,1,0,1,1,0,1,0)

$SOTP(m, G(r))$ = $(m \oplus u_1) - u_2$
= ((1,0,1,1)⊕(1,1,0,1)) − (1,0,1,0)
= (0,1,1,0) − (1,0,1,0)
= (-1,1,0,0) := M

Let $M' \coloneqq M + (2, 0,0,0) = (1, 1,0,0)$. Then,
$Inv\big(M', G(r)\big) = (M' + u_2) \oplus u_1$
= ((1,1,0,0)+(1,0,1,0)) ⊕ (1,1,0,1)
=(2,1,1,0) ⊕ (1,1,0,1)
=(3,0,1,1) → (1,0,1,1) = m

&0x1

ATTACK FOR CCA-NTRU+

- **Step 2.** use Step 1 to construct a malicious ciphertext c' from a challenge ciphertext $c^* = h \cdot r^* + M^*$ in the CCA security game ;

  - Assume $c^* = h \cdot r^* + M^*$

    where m^*=(1,0,1,1), $G(r^*) = u$ =(1,1,0,1,1,0,1,0), M^*= $SOTP(m^*, G(r^*))$

  - We set $c' \coloneqq c^* + (2,0,0,0) = h \cdot r^* + M'$

    then $Decaps(sk, c')$ successfully produces the secret key K^* which is a decapsulation result of c^* ($\because$ $Inv\left(M', G(r^*)\right) = m^*$).

    - i.e., we can ask decapsulation oracle to achieve K^*

  - But, in the CCA security game, since we don't know both M^* and $G(r^*)$ used in the challenge ciphertext, we need to guess :

    - the 0-th bits of M^* and $G(r^*)$ should be one of the four cases.

	M^* [0]	$G(r^*)$[0]	$M^* + G(r^*)$ [0]
i)	1	0	1
i)	0	1	1
i)	0	0	0
ii)	-1	1	0

- When i) happens and we add (2,0,0,0) to c^*, <u>decapsulation fails</u> (since it produces different r)
- ii) happens with probability ¼

ATTACK FOR CCA-NTRU+

- **Step 2.** use Step 1 to construct a malicious ciphertext c' from a challenge ciphertext $c^* = h \cdot r^* + M^*$ in the CCA security game ;
  - Assume $c^* = h \cdot r^* + M^*$

    where m^*=(1,0,1,1), $G(r^*) = u$ =(1,1,0,1,1,0,1,0), M^*= $SOTP(m^*, G(r^*))$
  - We set $c' \coloneqq c^* + (2,0,0,0) = h \cdot r^* + M'$

    then $Decaps(sk, c')$ successfully produces the secret key K^* which is a decapsulation result of c^* ($\because$ $Inv(M', G(r^*)) = m^*$).
    - i.e., we can ask decapsulation oracle to achieve K^*
  - But, in the CCA security game, since we don't know both M^* and $G(r^*)$ used in the challenge ciphertext, we need to guess with probability 1/4 for each component
  - With 4 decapsulation queries in average, we can achieve K^*, and **win the CCA game**

OW-CCA SECURITY GAME & ATTACK ALGORITHM

Game OW-CCA	$\mathcal{O}_{dec}(c)$
$(pk, sk) \leftarrow \mathsf{KeyGen}(1^\lambda)$	**if** $c = c^*$
$(K^*, c^*) \leftarrow \mathsf{Encaps}(pk)$	**return** $\perp$
$K' \leftarrow \mathcal{A}^{\mathcal{O}_{dec}(\cdot)}(pk, c^*)$	**else return**
return $[K' = K^*]$	$K \leftarrow \mathsf{Decaps}(sk, c)$

***OW-CCA SECURITY GAME**

Algorithm 1 Pseudocode for our attack algorithm

Require: a challenge ciphertext $c^* \in \mathcal{R}_q$
Ensure: a secret key $K \in \{0,1\}^{2\lambda}$
  for $i \in \{1, \cdots, n\}$ **do**
    $c' \leftarrow c^* + 2 \cdot e_i$ (Note that $c' \neq c^*$)
    Send c' to the decapsulation oracle $\mathcal{O}_{dec}$
    if $\mathcal{O}_{dec}$ outputs K' **then**
      Output K' as a decapsulation for c^*
      break;
    end if
  end for

***ATTACK ALGORITHM**

COMMENTS FOR THE SECURITY PROOF

[FO transform without Re-Encryption (Lemma 5 in NTRU+ paper)]

$\underline{\mathsf{Gen}(1^\lambda)}$

$(pk, sk) := \mathsf{Gen}'(1^\lambda)$

return (pk, sk)

$\underline{\mathsf{Encap}(pk)}$

$m \leftarrow \mathcal{M}$

$(r, K) := \mathsf{H}(m)$

$c := \mathsf{Enc}'(pk, m; r)$

- $M := \mathsf{SOTP}(m, \mathsf{G}(r))$
- $c := \mathsf{Enc}(pk, M; r)$

return (K, c)

$\underline{\mathsf{Decap}(sk, c)}$

$m' := \mathsf{Dec}'(sk, c)$

- $M' = \mathsf{Dec}(sk, c)$
- $r' = \mathsf{RRec}(pk, M', c)$
- $m' = \mathsf{Inv}(M', \mathsf{G}(r'))$

$(r'', K') := \mathsf{H}(m')$

if $m' = \perp$ or $\boxed{c \neq \mathsf{Enc}'(pk, m'; r'')}$

  return $\perp$

else

  return K'

Fig. 9. $\mathsf{KEM} = \mathsf{FO}^{\perp}[\mathsf{PKE}', \mathsf{H}]$.

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$m \leftarrow \mathcal{M}$

$(r, K) := \mathsf{H}(m)$

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- $M := \mathsf{SOTP}(m, \mathsf{G}(r))$
- $c := \mathsf{Enc}(pk, M; r)$

return (K, c)

$\underline{\mathsf{Decap}(sk, c)}$

$m' := \mathsf{Dec}'(sk, c)$

- $M' = \mathsf{Dec}(sk, c)$
- $r' = \mathsf{RRec}(pk, M', c)$
- $m' = \mathsf{Inv}(M', \mathsf{G}(r'))$

$(r'', K') := \mathsf{H}(m')$

if $m' = \perp$ or $\boxed{r' \neq r''}$

  return $\perp$

else

  return K'

Fig. 10. $\mathsf{KEM} = \overline{\mathsf{FO}}^{\perp}[\mathsf{PKE}', \mathsf{H}]$.

COMMENTS FOR THE SECURITY PROOF

[FO transform without Re-Encryption (Lemma 5 in NTRU+ paper)]

- To show: For input ciphertext c,

$$c = Enc'(pk, m'; r'') \text{ if and only if } r' = r''$$

$\mathsf{Gen}(1^\lambda)$

$(pk, sk) := \mathsf{Gen}'(1^\lambda)$
return (pk, sk)

$\mathsf{Encap}(pk)$

$m \leftarrow \mathcal{M}$
$(r, K) := \mathsf{H}(m)$
$c := \mathsf{Enc}'(pk, m; r)$
- $M := \mathsf{SOTP}(m, \mathsf{G}(r))$
- $c := \mathsf{Enc}(pk, M; r)$

return (K, c)

$\mathsf{Decap}(sk, c)$

$m' := \mathsf{Dec}'(sk, c)$
- $M' = \mathsf{Dec}(sk, c)$
- $r' = \mathsf{RRec}(pk, M', c)$
- $m' = \mathsf{Inv}(M', \mathsf{G}(r'))$

$(r'', K') := \mathsf{H}(m')$
if $m' = \perp$ or $\boxed{c \neq \mathsf{Enc}'(pk, m'; r'')}$
  return $\perp$
else
  return K'

Fig. 9. $\mathsf{KEM} = \mathsf{FO}^{\perp}[\mathsf{PKE}', \mathsf{H}]$.

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$(pk, sk) := \mathsf{Gen}'(1^\lambda)$
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$m \leftarrow \mathcal{M}$
$(r, K) := \mathsf{H}(m)$
$c := \mathsf{Enc}'(pk, m; r)$
- $M := \mathsf{SOTP}(m, \mathsf{G}(r))$
- $c := \mathsf{Enc}(pk, M; r)$

return (K, c)

$\mathsf{Decap}(sk, c)$

$m' := \mathsf{Dec}'(sk, c)$
- $M' = \mathsf{Dec}(sk, c)$
- $r' = \mathsf{RRec}(pk, M', c)$
- $m' = \mathsf{Inv}(M', \mathsf{G}(r'))$

$(r'', K') := \mathsf{H}(m')$
if $m' = \perp$ or $\boxed{r' \neq r''}$
  return $\perp$
else
  return K'

Fig. 10. $\mathsf{KEM} = \overline{\mathsf{FO}}^{\perp}[\mathsf{PKE}', \mathsf{H}]$.

COMMENTS FOR THE SECURITY PROOF

[FO transform without Re-Encryption (Lemma 5 in NTRU+ paper)]

- To show: For input ciphertext c,

$$c = Enc'(pk, m'; r'') \text{ if and only if } r' = r''$$

- $(\rightarrow)$ Assume $c = Enc'(pk, m'; r'')$ in Decaps.

Because PKE' is injective, the pair (m, r) used in Encaps is the same as (m', r'')

$\vdots$

$\mathsf{Gen}(1^\lambda)$

$(pk, sk) := \mathsf{Gen}'(1^\lambda)$
return (pk, sk)

$\mathsf{Encap}(pk)$

$m \leftarrow \mathcal{M}$
$(r, K) := \mathsf{H}(m)$
$c := \mathsf{Enc}'(pk, m; r)$
- $M := \mathsf{SOTP}(m, \mathsf{G}(r))$
- $c := \mathsf{Enc}(pk, M; r)$
return (K, c)

$\mathsf{Decap}(sk, c)$

$m' := \mathsf{Dec}'(sk, c)$
- $M' = \mathsf{Dec}(sk, c)$
- $r' = \mathsf{RRec}(pk, M', c)$
- $m' = \mathsf{Inv}(M', \mathsf{G}(r'))$
$(r'', K') := \mathsf{H}(m')$
if $m' = \perp$ or $\boxed{c \neq \mathsf{Enc}'(pk, m'; r'')}$
  return $\perp$
else
  return K'

Fig. 9. $\mathsf{KEM} = \mathsf{FO}^{\perp}[\mathsf{PKE}', \mathsf{H}]$.

$\mathsf{Gen}(1^\lambda)$

$(pk, sk) := \mathsf{Gen}'(1^\lambda)$
return (pk, sk)

$\mathsf{Encap}(pk)$

$m \leftarrow \mathcal{M}$
$(r, K) := \mathsf{H}(m)$
$c := \mathsf{Enc}'(pk, m; r)$
- $M := \mathsf{SOTP}(m, \mathsf{G}(r))$
- $c := \mathsf{Enc}(pk, M; r)$
return (K, c)

$\mathsf{Decap}(sk, c)$

$m' := \mathsf{Dec}'(sk, c)$
- $M' = \mathsf{Dec}(sk, c)$
- $r' = \mathsf{RRec}(pk, M', c)$
- $m' = \mathsf{Inv}(M', \mathsf{G}(r'))$
$(r'', K') := \mathsf{H}(m')$
if $m' = \perp$ or $\boxed{r' \neq r''}$
  return $\perp$
else
  return K'

Fig. 10. $\mathsf{KEM} = \overline{\mathsf{FO}}^{\perp}[\mathsf{PKE}', \mathsf{H}]$.

COMMENTS FOR THE SECURITY PROOF

[FO transform without Re-Encryption (Lemma 5 in NTRU+ paper)]

- To show: For input ciphertext c,

$$c = Enc'(pk, m'; r'') \text{ if and only if } r' = r''$$

- $(\rightarrow)$ Assume $c = Enc'(pk, m'; r'')$ in Decaps.

Because PKE' is injective, the pair (m, r) used in Encaps is the same as (m', r'')

⋮

They assumed that c (input of Decaps) is an output of Encaps,
i.e., $c = Encaps(m, r)$ for some (m, r).
But, there is no guarantee that $c = Encaps(m, r)$ for some $m \in M, r \in R$

$\mathsf{Gen}(1^\lambda)$
- $(pk, sk) := \mathsf{Gen}'(1^\lambda)$
- **return** (pk, sk)

$\mathsf{Encap}(pk)$
- $m \leftarrow \mathcal{M}$
- $(r, K) := \mathsf{H}(m)$
- $c := \mathsf{Enc}'(pk, m; r)$
  - $M := \mathsf{SOTP}(m, \mathsf{G}(r))$
  - $c := \mathsf{Enc}(pk, M; r)$
- **return** (K, c)

$\mathsf{Decap}(sk, c)$
- $m' := \mathsf{Dec}'(sk, c)$
  - $M' = \mathsf{Dec}(sk, c)$
  - $r' = \mathsf{RRec}(pk, M', c)$
  - $m' = \mathsf{Inv}(M', \mathsf{G}(r'))$
- $(r'', K') := \mathsf{H}(m')$
- **if** $m' = \perp$ or $c \neq \mathsf{Enc}'(pk, m'; r'')$
  - **return** $\perp$
- **else**
  - **return** K'

Fig. 9. $\mathsf{KEM} = \mathsf{FO}^{\perp}[\mathsf{PKE}', \mathsf{H}]$.

$\mathsf{Gen}(1^\lambda)$
- $(pk, sk) := \mathsf{Gen}'(1^\lambda)$
- **return** (pk, sk)

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- $m \leftarrow \mathcal{M}$
- $(r, K) := \mathsf{H}(m)$
- $c := \mathsf{Enc}'(pk, m; r)$
  - $M := \mathsf{SOTP}(m, \mathsf{G}(r))$
  - $c := \mathsf{Enc}(pk, M; r)$
- **return** (K, c)

$\mathsf{Decap}(sk, c)$
- $m' := \mathsf{Dec}'(sk, c)$
  - $M' = \mathsf{Dec}(sk, c)$
  - $r' = \mathsf{RRec}(pk, M', c)$
  - $m' = \mathsf{Inv}(M', \mathsf{G}(r'))$
- $(r'', K') := \mathsf{H}(m')$
- **if** $m' = \perp$ or $r' \neq r''$
  - **return** $\perp$
- **else**
  - **return** K'

Fig. 10. $\mathsf{KEM} = \overline{\mathsf{FO}}^{\perp}[\mathsf{PKE}', \mathsf{H}]$.

COMMENTS FOR THE SECURITY PROOF

[FO transform without Re-Encryption (Lemma 5 in NTRU+ paper)]

- To show: For input ciphertext c,

$$c = Enc'(pk, m'; r'') \text{ if and only if } r' = r''$$

- ($\leftarrow$) Assume $r' = r''$ in Decaps.

Because SOTP is rigid, $m' = Inv(M', G(r'))$ implies $M' = SOTP(m', G(r'))$, and thus $M' = SOTP(m', G(r''))$

⋮

$\mathsf{Gen}(1^\lambda)$

$(pk, sk) := \mathsf{Gen}'(1^\lambda)$
return (pk, sk)

$\mathsf{Encap}(pk)$

$m \leftarrow \mathcal{M}$
$(r, K) := \mathsf{H}(m)$
$c := \mathsf{Enc}'(pk, m; r)$
- $M := \mathsf{SOTP}(m, \mathsf{G}(r))$
- $c := \mathsf{Enc}(pk, M; r)$

return (K, c)

$\mathsf{Decap}(sk, c)$

$m' := \mathsf{Dec}'(sk, c)$
- $M' = \mathsf{Dec}(sk, c)$
- $r' = \mathsf{RRec}(pk, M', c)$
- $m' = \mathsf{Inv}(M', \mathsf{G}(r'))$

$(r'', K') := \mathsf{H}(m')$
if $m' = \perp$ or $c \neq \mathsf{Enc}'(pk, m'; r'')$
  return $\perp$
else
  return K'

Fig. 9. $\mathsf{KEM} = \mathsf{FO}^{\perp}[\mathsf{PKE}', \mathsf{H}]$.

$\mathsf{Gen}(1^\lambda)$

$(pk, sk) := \mathsf{Gen}'(1^\lambda)$
return (pk, sk)

$\mathsf{Encap}(pk)$

$m \leftarrow \mathcal{M}$
$(r, K) := \mathsf{H}(m)$
$c := \mathsf{Enc}'(pk, m; r)$
- $M := \mathsf{SOTP}(m, \mathsf{G}(r))$
- $c := \mathsf{Enc}(pk, M; r)$

return (K, c)

$\mathsf{Decap}(sk, c)$

$m' := \mathsf{Dec}'(sk, c)$
- $M' = \mathsf{Dec}(sk, c)$
- $r' = \mathsf{RRec}(pk, M', c)$
- $m' = \mathsf{Inv}(M', \mathsf{G}(r'))$

$(r'', K') := \mathsf{H}(m')$
if $m' = \perp$ or $r' \neq r''$
  return $\perp$
else
  return K'

Fig. 10. $\mathsf{KEM} = \overline{\mathsf{FO}}^{\perp}[\mathsf{PKE}', \mathsf{H}]$.

COMMENTS FOR THE SECURITY PROOF

[FO transform without Re-Encryption (Lemma 5 in NTRU+ paper)]

- To show: For input ciphertext c,

$$c = Enc'(pk, m'; r'') \text{ if and only if } r' = r''$$

- ($\leftarrow$) Assume $r' = r''$ in Decaps.

Because SOTP is rigid, $m' = Inv(M', G(r'))$ implies $M' = SOTP(m', G(r'))$, and thus $M' = SOTP(m', G(r''))$

⋮

They assumed that $M' = Dec(sk, c)$ is an output of SOTP w.r.t. $u = G(r')$, i.e., $M' = SOTP(m, G(r'))$ for some m.
But, there is no guarantee that $M' \in \{SOTP(m, G(r')) | m \in M\}$ as shown in our attack.

(Recall) Rigidity of SOTP ;
For all $u \in U$, and $y \in Y$ encoded with respect to u, it holds that $SOTP(Inv(y, u), u) = y$

$\mathsf{Gen}(1^\lambda)$
- $(pk, sk) := \mathsf{Gen}'(1^\lambda)$
- **return** (pk, sk)

$\mathsf{Encap}(pk)$
- $m \leftarrow \mathcal{M}$
- $(r, K) := \mathsf{H}(m)$
- $c := \mathsf{Enc}'(pk, m; r)$
  - $M := \mathsf{SOTP}(m, \mathsf{G}(r))$
  - $c := \mathsf{Enc}(pk, M; r)$
- **return** (K, c)

$\mathsf{Decap}(sk, c)$
- $m' := \mathsf{Dec}'(sk, c)$
  - $M' = \mathsf{Dec}(sk, c)$
  - $r' = \mathsf{RRec}(pk, M', c)$
  - $m' = \mathsf{Inv}(M', \mathsf{G}(r'))$
- $(r'', K') := \mathsf{H}(m')$
- **if** $m' = \perp$ or $c \neq \mathsf{Enc}'(pk, m'; r'')$
- **return** $\perp$
- **else**
- **return** K'

Fig. 9. $\mathsf{KEM} = \mathsf{FO}^{\perp}[\mathsf{PKE}', \mathsf{H}]$.

$\mathsf{Gen}(1^\lambda)$
- $(pk, sk) := \mathsf{Gen}'(1^\lambda)$
- **return** (pk, sk)

$\mathsf{Encap}(pk)$
- $m \leftarrow \mathcal{M}$
- $(r, K) := \mathsf{H}(m)$
- $c := \mathsf{Enc}'(pk, m; r)$
  - $M := \mathsf{SOTP}(m, \mathsf{G}(r))$
  - $c := \mathsf{Enc}(pk, M; r)$
- **return** (K, c)

$\mathsf{Decap}(sk, c)$
- $m' := \mathsf{Dec}'(sk, c)$
  - $M' = \mathsf{Dec}(sk, c)$
  - $r' = \mathsf{RRec}(pk, M', c)$
  - $m' = \mathsf{Inv}(M', \mathsf{G}(r'))$
- $(r'', K') := \mathsf{H}(m')$
- **if** $m' = \perp$ or $r' \neq r''$
- **return** $\perp$
- **else**
- **return** K'

Fig. 10. $\mathsf{KEM} = \overline{\mathsf{FO}}^{\perp}[\mathsf{PKE}', \mathsf{H}]$.

ABOUT NTRU+ VERSION 1.1

- On 9/16/23, NTRU+ ver. 1.1 has been released
  - We checked that the attack strategy does not work for the updated algorithm.

Algorithm 7 Inv

Require: Polynomial $\mathbf{f} \in R_q$
Require: Byte array $B = (b_0, b_1, \cdots, b_{n/4-1})$
Ensure: Message Byte array $m = (m_0, m_1, \cdots, m_{31})$

$(\beta_0, \cdots, \beta_{n-1}) :=$ BytesToBits$((b_0, \cdots, b_{n/8-1}))$
$(\beta_n, \cdots, \beta_{2n-1}) :=$ BytesToBits$((b_{n/8}, \cdots, b_{n/4-1}))$
for i from 0 to $n-1$ **do**
  if $f_i + \beta_{i+n} \notin \{0, 1\}$, **return** $\perp$
  $m_i := ((f_i + \beta_{i+n})\&1) \oplus \beta_i$
$m =$ BitsToBytes$((m_0, \cdots, m_{n-1}))$
return m

- Security proofs should be revised also, as pointed in this presentation

MEET-LWE ATTACK COSTS FOR LATTICE-BASED KEMS

- [May21] May, Alexander. "How to meet ternary LWE keys." *Advances in Cryptology–CRYPTO 2021: 41st Annual International Cryptology Conference, CRYPTO 2021, Virtual Event, August 16–20, 2021, Proceedings, Part II 41*. Springer International Publishing, 2021.

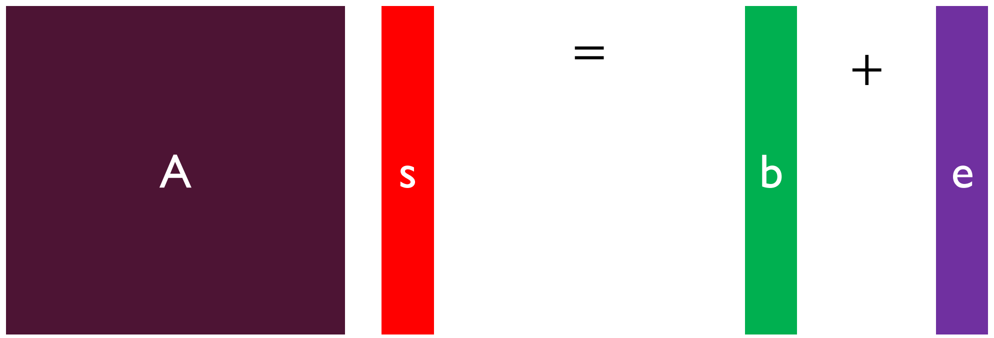
- Some parts of the slides introducing Meet LWE idea are taken from May's slide in Crypto 2021 ((40) How to Meet Ternary LWE Keys – YouTube)

TERNARY LWE PROBLEM

[Ternary LWE problem]

Given; $A \in Z_q^{n \times n}, b \in Z_q^n$ such that $A \cdot s = b + e$ for $\boldsymbol{s}, \boldsymbol{e} \in \{\boldsymbol{0}, \pm\boldsymbol{1}\}^{\boldsymbol{n}}$,

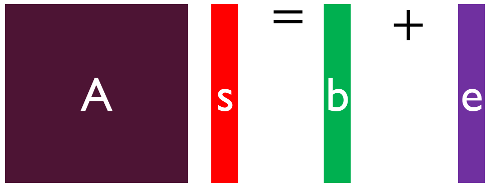
Find; $s \in \{0, \pm 1\}^n$



- Asymptotically, Brute Force < Odlyzko's MitM < Meet LWE
- Meet LWE can be extended to (fixed size of) small errors, so it is applicable to all 3 Lattice-based KEMs
  - SMAUG, TiGER use sparse secrets
  - NTRU+ uses the ternary LWE problem (they use sparse ternary or binary secrets)

BRUTE FORCE ATTACK FOR TERNARY LWE

- Equation : $A \cdot s = b + e \;\; mod \; q$



[Brute Force]
- Input : $A \in Z_q^{n \times n}, b \in Z_q^n$
- For all $s \in \{0, \pm 1\}^n$:
  - If $A \cdot s - b \in \{0, \pm 1\}^n$ then output s

- $S = 3^n$; search space size for ternary keys
- Running time is $T = S$

ODLYZKO'S MITM

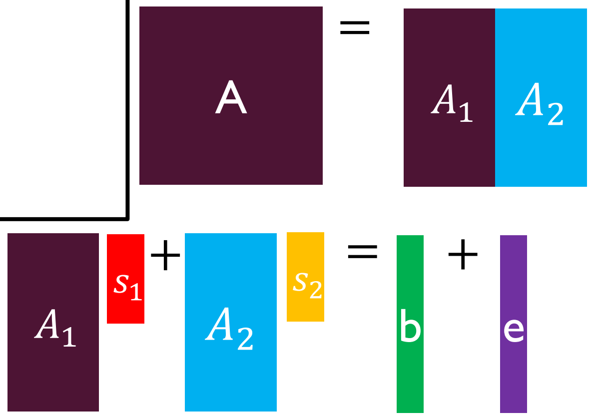
- Equation : $A_1 \cdot s_1 = -A_2 \cdot s_2 + b + e \ \ mod\ q$

  i.e. $A_1 \cdot s_1 \approx -A_2 \cdot s_2 + b \ \ mod\ q$

[Odlyzko's MitM]

- Input : $A = (A_1|A_2) \in Z_q^{n \times n}, b \in Z_q^n$
- For all $s_1 \in \{0, \pm 1\}^{n/2}$:
  - Construct L_1 with entries $(s_1, h(A_1 s_1))$
- For all $s_2 \in \{0, \pm 1\}^{n/2}$:
  - Construct L_2 with entries $(s_2, h(-A_2 s_2 + b))$
- Output $(s_1|s_2)$ with $h(A_1 s_1) = h(-A_2 s_2 + b)$

$*$ h: locality sensitive hash



- $S = 3^n$; search space size for ternary keys
- Running time is $T = 3^{n/2} = S^{1/2}$ with same memory

REPRESENTATIONS (HOWGRAVE-GRAHAM, JOUX '10)

- **Idea** ; $s \coloneqq s_1 + s_2$ for $s_1, s_2 \in \{0, \pm 1\}^n$
  - Allows redundancy

- (1,0,1,-1,0) = (1,0,0,-1,0) + (0,0,1,0,0)

  = (1,0,1,0,0) + (0,0,0,-1,0)

  = (0,0,1,0,0) + (1,0,0,-1,0)

  = (0,0,1,-1,0) + (1,0,0,0,0)

	1	0	-1
REP-0	• 1+0 • 0+1	-	• (-1)+0 • 0+(-1)
REP-1	• 1+0 • 0+1	• 1+(-1) • (-1)+1	• (-1)+0 • 0+(-1)
REP-2	• 1+0 • 0+1 • 2+(-1) • (-1)+2	• 1+(-1) • (-1)+1 • 2+(-2) • (-2)+2	• (-1)+0 • 0+(-1) • 1+(-2) • (-2)+1

MEET LWE ATTACK [MAY21]

- Equation : $A \cdot s_1 = -A \cdot s_2 + b + e \;\; mod\ q$

  i.e. $A \cdot s_1 \approx -A \cdot s_2 + b \;\; mod\ q$

[Meet LWE (high-level idea)]
- Input : $A \in Z_q^{n \times n}, b \in Z_q^n$
- Choose representation REP-0, REP-1, REP-2
- Guess r coordinates of e (say $\underline{e_r}$)
- For s_1, construct L_1 with entries (s_1, As_1)
- For s_2, construct L_2 with entries $(s_2, -As_2 + b)$
- Output $\mathrm{s}_1 + \mathrm{s}_2$ s.t.
  - $\pi_r(As_1) = \pi_r(-As_2 + b) + e_r$
  - $h(As_1) = h(-As_2 + b)$ for $\mathrm{n} - r$ coordinates

- By using representations $\mathrm{s} = s_1 + s_2$, **the number of solutions ($:= R$) increases**
- We can reduce the list (L_1, L_2) sizes with a factor of R (by guessing r coordinates of e), expecting at least one solution exists
  - This strategy can be recursively applied to s_1, s_2, respectively (lists can be obtained by tree-based construction)
- Run-time $T = T_g \cdot T_\ell$ where T_g; guessing complexity, T_ℓ; list construction complexity

ATTACK COMPLEXITIES OF MEET LWE

```
****************
 Meet-LWE Rep-0
****************
SMAUG128: time= 230.9 = 217.0 + 13.9, memory= 217.0
SMAUG192: time= 294.8 = 278.5 + 16.3, memory= 278.5
SMAUG256: time= 368.2 = 350.8 + 17.4, memory= 350.8
****************
 Meet-LWE Rep-1
****************
SMAUG128: time= 183.3 = 152.0 + 31.3, memory= 152.0 wi
SMAUG192: time= 233.1 = 192.4 + 40.6, memory= 192.4 wi
SMAUG256: time= 303.3 = 254.5 + 48.8, memory= 254.5 wi
****************
 Meet-LWE Rep-2
****************
SMAUG128: time= 176.4 = 147.4 + 29.0, memory= 147.4 wi
SMAUG192: time= 229.9 = 199.7 + 30.2, memory= 199.7 wi
SMAUG256: time= 296.5 = 253.6 + 43.0, memory= 253.6 wi
```

```
****************
 Meet-LWE Rep-0
****************
NTRU+576: time= 340.7 = 322.5 + 18.2, memory= 322.5
NTRU+768: time= 454.8 = 430.3 + 24.6, memory= 430.3
NTRU+864: time= 512.7 = 484.2 + 28.5, memory= 484.2
NTRU+1152: time= 683.8 = 645.8 + 38.0, memory= 645.8
****************
 Meet-LWE Rep-1
****************
NTRU+576: time= 269.4 = 237.7 + 31.7, memory= 237.7 with (1
NTRU+768: time= 361.7 = 320.5 + 41.2, memory= 320.5 with (2
NTRU+864: time= 411.4 = 366.2 + 45.2, memory= 366.2 with (2
NTRU+1152: time= 569.3 = 513.0 + 56.3, memory= 513.0 with (
****************
 Meet-LWE Rep-2
****************
NTRU+576: time= 263.3 = 227.6 + 35.7, memory= 227.6 with (2
NTRU+768: time= 349.1 = 302.3 + 46.8, memory= 302.3 with (2
NTRU+864: time= 391.7 = 338.6 + 53.1, memory= 338.6 with (2
NTRU+1152: time= 518.6 = 448.1 + 70.5, memory= 448.1 with (
```

```
****************
 Meet-LWE Rep-0
****************
TiGER128: time= 225.3 = 213.4 + 11.9, memory= 213.4
TiGER192: time= 220.3 = 209.9 + 10.4, memory= 209.9
TiGER256: time= 387.7 = 369.5 + 18.2, memory= 369.5
****************
 Meet-LWE Rep-1
****************
TiGER128: time= 175.5 = 150.2 + 25.4, memory= 150.2 with (11, 2, 1)
TiGER192: time= 194.0 = 164.9 + 29.0, memory= 164.9 with (8, 1, 1)
TiGER256: time= 309.5 = 269.1 + 40.4, memory= 269.1 with (16, 1, 1)
****************
 Meet-LWE Rep-2
****************
TiGER128: time= 167.0 = 141.7 + 25.4, memory= 141.7 with (11, 0, 2, 0)
TiGER192: time= 170.1 = 141.1 + 29.0, memory= 141.1 with (8, 0, 1, 0)
TiGER256: time= 298.5 = 258.1 + 40.4, memory= 258.1 with (16, 0, 3, 0)
```

- **We slightly modified Meet-LWE algorithm for non-ternary errors**
- **TiGER192** parameter is **vulnerable to Meet-LWE attack**
  - In their analysis, the claimed log complexity against best (quantum) attack was **192**, but it is dropped to **170.1** (Note. it is a classical attack)
  - (Recommendation) They need to increase h_s, h_r to fix it
- The other parameter sets of 3 lattice-based KEMs are fine

OUR EXPERIMENT

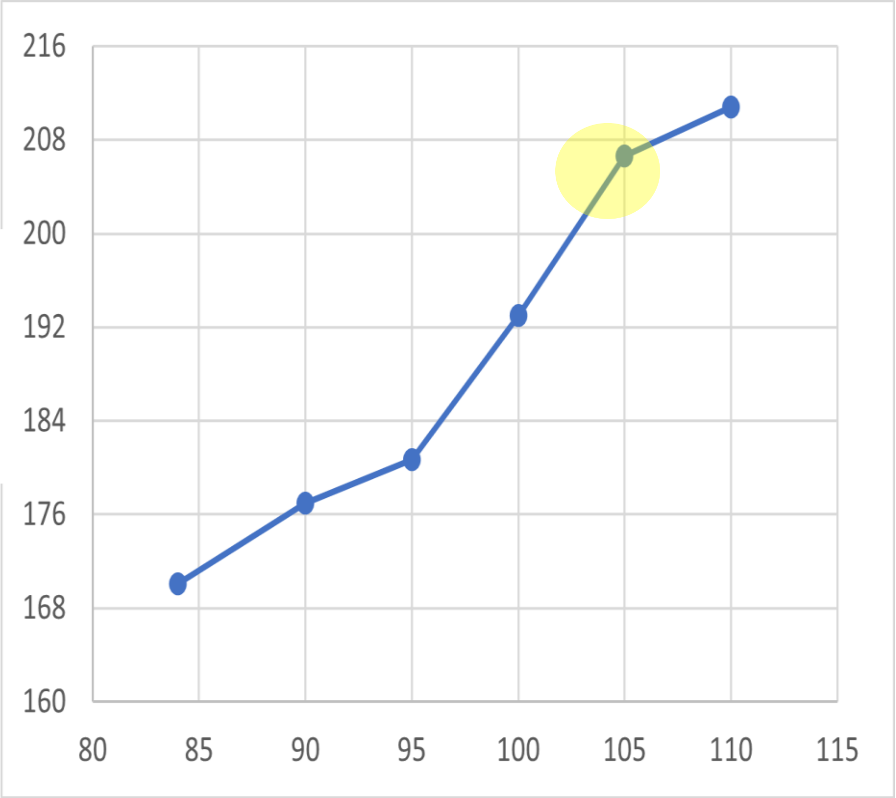
```
from math import log, floor, sqrt, log2 #, prod
from scipy.special import gammaln
import numpy as np

def log2_multinom(c):
    return (gammaln(c.sum()+1) - gammaln(c+1).sum()) / log(2)

def meet_lwe_rep0(n, q, w, B):
    n2 = floor(n/2)

    w2 = floor(w/2)
    w4 = floor(w/4)
    w8 = floor(w/8)


    # Compute log_2 of L^(1) = S^(1) / R^(1),
    # where S^(1) = (n choose w/4, w/4, n-w/2) and
    # R^(1) = (w/2 choose w/4, w/4).
    logS1 = log2_multinom(np.array([w4, w2-w4, n-w2]))
    logR1 = 2*log2_multinom(np.array([w4, w2-w4]))
    logL1 = logS1 - logR1

    # Compute log_2 of L^(2) = S^(2),
    # where S^(2) = n/2 choose w/8, w/9, n/2-w/4
    logL2 = log2_multinom(np.array([w8, w4-w8, n-n2-w4]))
```

```
    # Compute T_l for the list construction,
    # where T_l = max(L^(1), L^(2))
    logT_l = max(logL1, logL2)

    # Compute T_g for the guessing,
    # where T_g = 3^(r/2) = 3^( log_q(R^(1))/2 )
    # which leads to log_2 (T_g) = 0.5*log_2(R^(1))*log_2(3)/log_2(q)
    logT_g = 0.5*floor(logR1/log2(q))*log2(B)

    return (logT_l+logT_g, logT_l, logT_g, logT_l)
```

- Use **Python code** to compute the Meet LWE attack costs for Rep-0, Rep-1, and Rep-2, respectively
  - We utilized the python code modified from an open source "Meet_LWE.py" in SMAUG v1.0 helper scripts, by extending it into the non-ternary error cases
- **B**: error parameter for LWE

ATTACK COMPLEXITIES FOR VARIOUS PARAMETERS

$Y =$ Meet-LWE Time Complexity



$X = h_s$(Hamming weight parameter of LWR's secret key sk.)

- Meet LWE Complexity for the LWR instance in TiGER when increasing h_s (hamming weight of LWR's secret key sk.)

- (Recommendation) They need to increase h_s to be over **104** to achieve **200-bit classical security as claimed in TiGER** against the Meet-LWE attack. ($104 \leq h_s$)

SECURITY EVALUATION OF {LWE, LWR}-BASED SCHEMES USING LATTICE ESTIMATOR

- Lattice Estimator — Lattice Estimator 0.1 documentation (lattice-estimator.readthedocs.io)
- Albrecht, Martin R., Rachel Player, and Sam Scott. "On the concrete hardness of learning with errors." *Journal of Mathematical Cryptology* 9.3 (2015): 169-203.

GOAL

- Better understanding for the security estimation of KpqC Round 1 candidates
  - Analysis reports for the respective attacks
- Estimate the security for all the LWE/LWR based schemes {NTRU+, SMAUG, TiGER, HAETAE, NCC-Sign}

METHODS

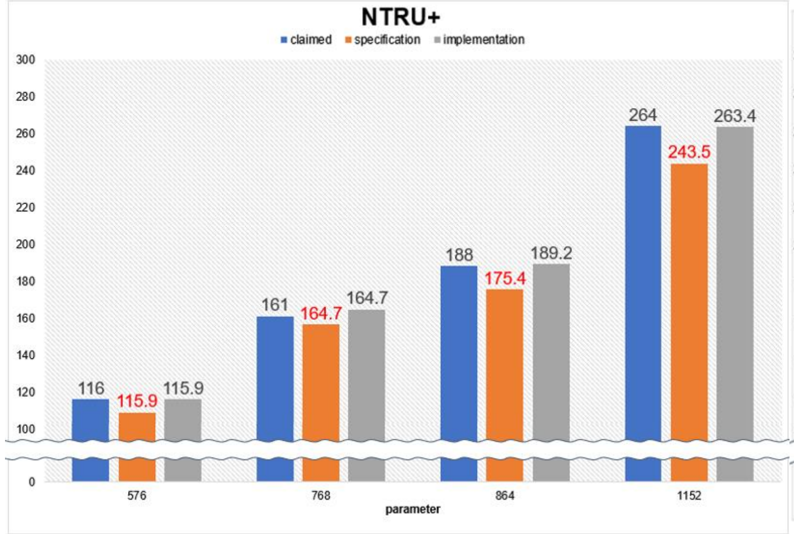
- **Lattice estimator**
  - For LWE/LWR security analysis, M. Albrecht's Lattice Estimator (Lattice Estimator — Lattice Estimator 0.1 documentation (lattice-estimator.readthedocs.io)) is used. Lattice Estimator is a Sage open source that calculates the attack complexities and additional parameters required for attack by taking LWE/LWR parameters as input values.

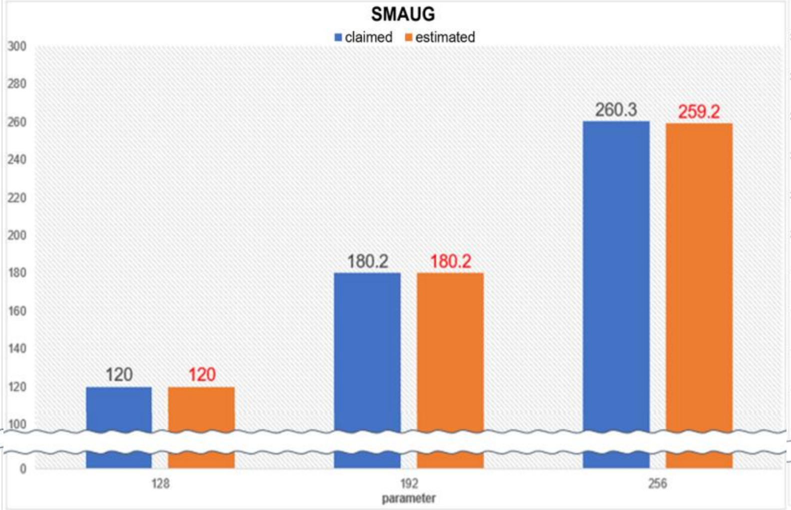
- **The BKZ Algorithm Complexity – Core-SVP model**
  - The principle of the BKZ algorithm is to repeatedly apply the SVP solver, an algorithm that finds the shortest vector, for a sub-lattice of dimension (β) smaller than that of a given lattice.
  - The Core-SVP model from the NewHope paper (USENIX'16) is a model for estimating the time complexity of the BKZ algorithm. The classical security in bits is estimated as $2^{c \cdot \beta}$ using $c = 0.292$, and the quantum security (bit) can be also estimated by calculating the classical security (bit) $\times\, c_q/0.292$ in the Core-SVP model.

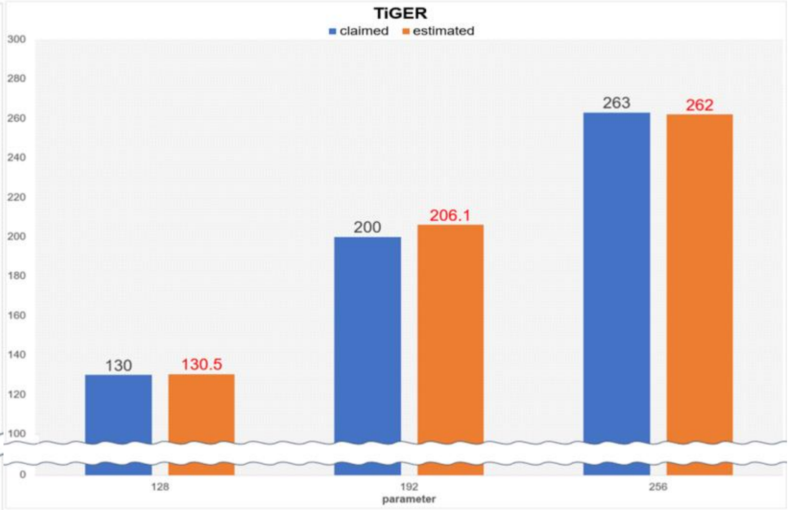
	Classical	Quantum[1]
c	0.292	0.257
T	$2^{0.292\beta}$	$2^{0.257\beta}$

[1] Chailloux, A., Loyer, J. Lattice Sieving via Quantum Random Walks. ASIACRYPT 2021

RESULTS - KEMS



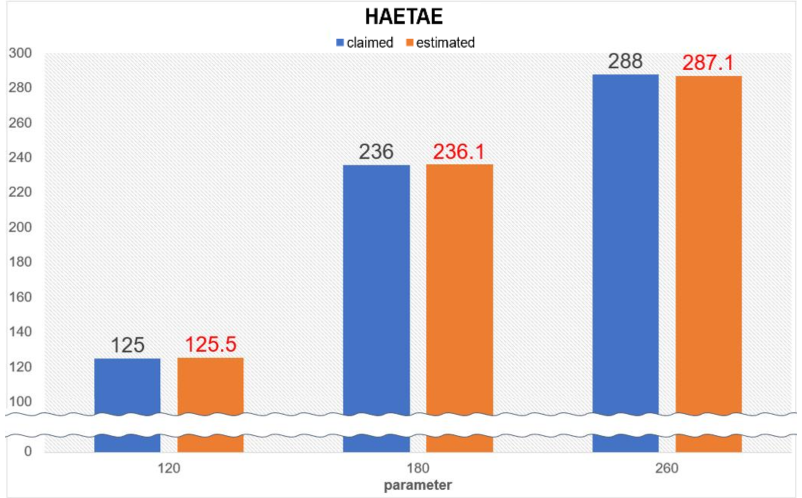


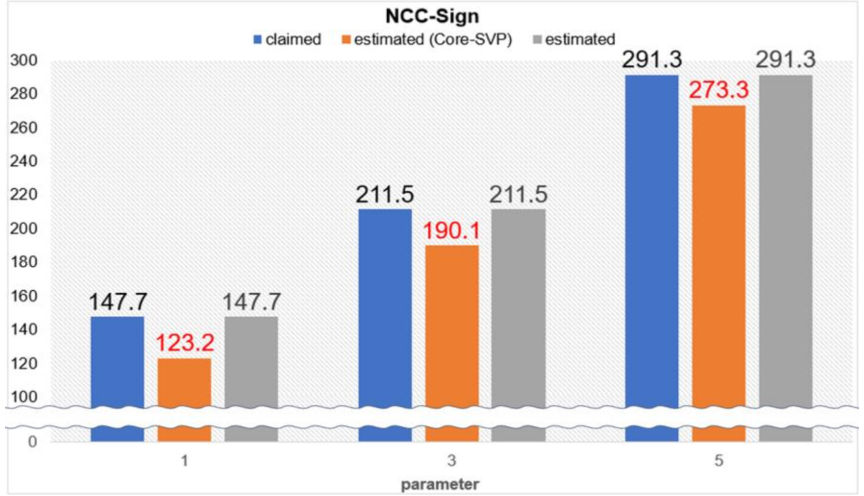


Notes.

- NTRU+ in its specification uses the binary secrets for LWE (Algorithm 6, 9 in the specification), while it uses the centered binomial distribution for the LWE secrets in the implementation. So, we present evaluations for both.
- Estimated security for SMAUG-256, TiGER-256 ; 1-bit lower than the proposed security

RESULTS - SIGNATURES





Notes.

- NCC-Sign proposed the security without core-SVP model, so we presented the security evaluation with and without the Core-SVP model.

SUMMARY

- CCA-NTRU+ can be attacked since their decoding method(the Inv algorithm) does not check if the intermediate value is binary
  - Can be fixed if they check if the intermediate value is binary, and abort otherwise.

- We evaluate the concrete security of 3 lattice-based KEMs against Meet LWE attack
  - TiGER needs to take into account Meet LWE attack for their TiGER192 parameter set
  - Can be fixed by increasing h_s, h_r
  - TiGER updated the parameter sets : now secure against Meet-LWE attack

Table 1: The detail parameters for each security level

parameters	security level	n	q	p	k_1	k_2	h_s	h_r	h_e	d	f
TiGER128	AES128	512	256	128	64	64	160	128	32	128	3
TiGER192	AES192	1024	256	64	64	4	84	84	84	256	5
TiGER256	AES256	1024	256	128	128	4	198	198	32	256	5

→

Table 1: The detail parameters for each security level

parameters	security level	n	q	p	k_1	k_2	h_s	h_r	h_e	d	f
TiGER128	AES128	512	256	128	64	16	142	110	32	128	3
TiGER192	AES192	1024	256	128	64	4	132	132	32	256	5
TiGER256	AES256	1024	256	128	128	4	196	196	32	256	5

- We estimated the security of all the {LWE, LWR}-based schemes using Lattice estimator and verified the (most of) claims in the proposals of {NTRU+, SMAUG, TiGER, HAETAE, NCC-Sign}

THANK YOU!

ANY QUESTIONS OR COMMENTS?

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