Apparent distortion of the cosmic standard shape

- A Group Project of 'Cosmology Basics and a Special Topic'

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In this project we measure the shape of a class of objects in the universe whose true shape is known to be unchanging. Suppose an object at redshift z spans Δz in redshift and $\Delta\theta$ in angle. Its comoving sizes in radial and transverse directions are given by

$$\Delta r_{\parallel} = \frac{c}{H(z)}\Delta z,$$
$$\Delta r_{\perp} = (1+z)D_A(z)\Delta\theta$$

where H is the Hubble parameter and D_A the proper angular diameter distance. If our universe is geometrically flat and dominated by the dark energy with the constant equation of state parameter w, they are related with the cosmological parameters through the following equations

$$H(z) = H_0\sqrt{\Omega_m a^{-3} + (1-\Omega_m)a^{-3(1+w)}}$$
$$D_A(z) = \frac{c}{1+z}r(z) = \frac{c}{1+z}\int_0^z \frac{dz'}{H(z')},$$

where $a = 1/(1+z)$ is the cosmic scale factor, H_0 is the present value of the Hubble parameter, Ω_m is the present-day matter density parameter, and $r(z)$ is the comoving distance. This means that we must adopt a cosmology to convert the observational quantities to physical lengths.

Let's assume that the universe actually has $\Omega_m = 0.29$, and $w = -0.8$ (the true cosmology), but we adopt a set of wrong values of Ω_m and w (the wrong cosmology). This results in an apparent distortion of the shape of the objects, which is called the Alcock-Paczynski effect. The degree of distortion in shape or the distortion factor is

$$\frac{[\Delta r_{\parallel}/\Delta r_{\perp}]_{\text{wrong}}}{[\Delta r_{\parallel}/\Delta r_{\perp}]_{\text{true}}} = \frac{[D_A(z)H(z)]_{\text{true}}}{[D_A(z)H(z)]_{\text{wrong}}}.$$

One can constrain the cosmological parameters governing the expansion history of the universe by examining if the shape of the objects $\Delta r_{\parallel}/\Delta r_{\perp}$ changes across a range of redshift.

1. Please plot the distortion factor for various sets of cosmological parameters.
   (You can use the above equations.)
2. Extend the problem to the case where w evolves linearly in a as in
   $w(a) = w_0 + w_a(1-a)$,
   where w_0 is the current value of w and $w_a = -dw/da$. And plot the distortion factor for some choices of w_0 and w_a when $\Omega_m = 0.29$.