The 29th KIAS Combinatorics Workshop

Shilla Stay Haeundae
Busan, Korea
December 26–28, 2023

Contents

1 General Information

Title The 29th KIAS Combinatorics Workshop

Date December 26–28, 2023

Venue Shilla Stay Haeundae, Busan

Homepage `http://events.kias.re.kr/h/combinatorics/`

Invited Speakers

Linda Cook (IBS, DIMAG)

Yibo Gao (Peking University)

Cheolwon Heo (KIAS)

JiSun Huh (Ajou University)

Jinha Kim (Chonnam National University)

Seog-Jin Kim (Konkuk University)

Dong-il Lee (Seoul Women's University)

Jaeseong Oh (Yonsei University)

Se-jin Oh (Sungkyunkwan University)

Jeong Rye Park (Kyungpook National University)

Sebastian Wiederrecht (IBS, DIMAG)

Organizers

Jaehoon Kim (KAIST)

Jang Soo Kim (Sungkyunkwan University)

Jeong Han Kim (KIAS)

Seog-Jin Kim (Konkuk University)

Young Soo Kwon (Yeungnam University)

Sang June Lee (Kyung Hee University)

Jongyook Park (Kyungpook National University)

Seunghyun Seo (Kangwon National University)

2 Schedule and Abstracts

1st Day: December 26 (Tuesday)

14:30 - 15:00 Registration and Opening

Session A Chair: Jang Soo Kim

15:00 - 15:40 **Yibo Gao**. *Symmetric structures in the strong Bruhat order*

15:50 - 16:30 **Sebastian Wiederrecht**. *Bipartite Treewidth*

16:30 - 16:50 Coffee break

16:50 - 17:30 **Cheolwon Heo**. *Small separations in pinch-graphic matroids*

18:00 - Dinner

2nd Day: December 27 (Wednesday)

Session B Chair: Seunghyun Seo

09:20 - 10:00 **Jinha Kim**. *An Eisenbud–Goto type inequality for Stanley–Reisner ideals and simplicial complexes*

10:00 - 10:30 Coffee break

10:30 - 11:10 **Linda Cook**. *On polynomial degree-boundedness*

11:20 - 12:00 **Dong-il Lee**. *Groebner–Shirshov bases for Temperley–Lieb algebras*

12:00 - 14:00 Lunch

Session C Chair: Jongyook Park

14:00 - 14:40 **Se-jin Oh**. *Valued quiver and quantum cluster algebra of non skew-symmetric type*

14:50 - 15:30 **Jeong Rye Park**. *Distance Matrices with three distinct eigenvalues*

15:30 - 15:50 Coffee break

15:50 - 17:30 Free discussion

18:00 - Banquet

3rd Day: December 28 (Thursday)

Session D Chair: Young Soo Kwon

09:30 - 10:10 **JiSun Huh**. *Schröder paths without triple descents and related topics*

10:20 - 11:00 **Jaeseong Oh**. *Rook Theory and α-Chromatic Symmetric Functions*

11:10 - 11:50 **Seog-Jin Kim**. *Tight upper bound on the clique size in the square of 2-degenerate graphs*

11:50 - 12:00 Closing

12:00 - Lunch

Speaker: Linda Cook

Affiliation: IBS, DIMAG

Title: On polynomial degree-boundedness

Abstract

We prove a conjecture of Bonamy, Bousquet, Pilipczuk, Rzążewski, Thomassé, and Walczak, that for every graph H, there is a polynomial p such that for every positive integer s, every graph of average degree at least $p(s)$ contains either $K_{s,s}$ as a subgraph or contains an induced subdivision of H. This improves upon a result of Kühn and Osthus from 2004 who proved it for graphs whose average degree is at least triply exponential in s and a recent result of Du, Girão, Hunter, McCarty and Scott for graphs with average degree at least singly exponential in s.

As an application, we prove that the class of graphs that do not contain an induced subdivision of $K_{s,t}$ is polynomially χ-bounded. In the case of $K_{2,3}$, this is the class of theta-free graphs, and answers a question of Davies. Along the way, we also answer a recent question of McCarty, by showing that if $\mathcal{G}$ is a hereditary class of graphs for which there is a polynomial p such that every *bipartite* $K_{s,s}$-free graph in $\mathcal{G}$ has average degree at most $p(s)$, then more generally, there is a polynomial p' such that every $K_{s,s}$-free graph in $\mathcal{G}$ has average degree at most $p'(s)$. Our main new tool is an induced variant of the Kővári-Sós-Turán theorem, which we find to be of independent interest.

Joint with: Romain Bourneuf (ENS de Lyon), Matija Bucić (Princeton), James Davies (Cambridge)

Speaker: Cheolwon Heo

Affiliation: KIAS

Title: Small separations in pinch-graphic matroids

Abstract

A k-separation of a matroid M is a partition (X, Y) of $E(M)$ such that $r(X)+r(Y)-r(M) < k$ and $|X|, |Y| \geq k$. Even-cycle matroids are elementary lifts of graphic matroids. An even-cycle matroid is pinch-graphic if it has a signed-graph representation with a blocking pair. In this talk, we analyze the structure of 1-, 2-, and 3-separations in these matroids.

This is joint work with Bertrand Guenin.

Speaker: JiSun Huh

Affiliation: Ajou University

Title: Schröder paths without triple descents and related topics

Abstract

A Schröder path of semilength n is a lattice path from $(0,0)$ to $(2n,0)$ that does not go below the x-axis and consists of up steps $u = (1,1)$, down steps $d = (1,-1)$, and horizontal steps $h = (2,0)$. For a Schröder path P, three consecutive steps ddd is called a *triple descent*. A sequence $e = (e_1, e_2, \ldots, e_n)$ is called an *inversion sequence of length* n if $0 \leq e_i < i$ for all $i \in [n]$. We say that an inversion sequence $e = (e_1, e_2, \ldots, e_n)$ *avoids* the patterns 101 and 102 if there are no indices $i < j < k$ such that $e_j < e_i \leq e_k$.

In this talk, we introduce an $\mathcal{F}$-path and construct bijections between the set of F-paths of length n and each of the following sets: Schröder paths of semilength n without triple descent; $(4123, 4132, 4213)$-avoiding permutations on $[n+1]$; $(101, 102)$-avoiding inversion sequences of length $n+1$. We also count the number of F-paths, $(4123, 4132, 4213)$-avoiding permutations, and $(101, 102)$-avoiding inversion sequences with respect to some statistics by counting Schröder paths without triple descent.

This is joint work with Sangwook Kim, Seunghyun Seo, and Heesung Shin.

Speaker: Yibo Gao

Affiliation: Peking University

Title: Symmetric structures in the strong Bruhat order

Abstract

The Bruhat order encodes algebraic and topological information of Schubert varieties in the flag manifold and possesses rich combinatorial properties. In this talk, we discuss three interrelated stories on smooth permutations: self-dual Bruhat intervals, vertex-transitive permutations and a conjecture by Billey-Postnikov. This is joint work with Christian Gaetz.

Speaker: Jinha Kim

Affiliation: Chonnam National University

Title: An Eisenbud–Goto type inequality for Stanley–Reisner ideals and simplicial complexes

Abstract

The Leray number of an abstract simplicial complex is the minimal integer d where its induced subcomplexes have trivial homology groups in dimension d or greater. We give an upper bound on the Leray number of a complex in terms of how the facets are attached to each other. We also describe the structure of complexes for the equality of the bound that we found. Through the Stanley–Reisner correspondence, our results give an Eisenbud–Goto type inequality for any square-free monomial ideals.

This is joint work with Jaewoo Jung, Minki Kim, and Yeongrak Kim.

Speaker: Seog-Jin Kim

Affiliation: Konkuk University

Title: Tight upper bound on the clique size in the square of 2-degenerate graphs

Abstract

The *square* of a graph G, denoted G^2, has the same vertex set as G and has an edge between two vertices if the distance between them in G is at most 2. In general, $\Delta(G)+1 \leq \chi(G^2) \leq \Delta(G)^2+1$ for every graph G. Charpentier (2014) asked whether $\chi(G^2) \leq 2\Delta(G)$ if $mad(G) < 4$. But Hocquard, Kim, and Pierron (2019) answered his question negatively. For every even value of $\Delta(G)$, they constructed a 2-degenerate graph G such that $\omega(G^2) = \frac{5}{2}\Delta(G)$. Note that if G is a 2-degenerate graph, then $mad(G) < 4$. Thus, we have that

$$\frac{5}{2}\Delta(G) \leq \max\{\chi(G^2) : G \text{ is a 2-degenerate graph}\} \leq 3\Delta(G)+1.$$

So, it was naturally asked whether there exists a constant D_0 such that $\chi(G^2) \leq \frac{5}{2}\Delta(G)$ if G is a 2-degenerate graph with $\Delta(G) \geq D_0$. Recently Cranston and Yu (2023) showed that $\omega(G^2) \leq \frac{5}{2}\Delta(G)+72$ if G is a 2-degenerate graph, and $\omega(G^2) \leq \frac{5}{2}\Delta(G)+60$ if G is a 2-degenerate graph with $\Delta(G) \geq 1729$. We show that there exists a constant D_0 such that $\omega(G^2) \leq \frac{5}{2}\Delta(G)$ if G is a 2-degenerate graph with $\Delta(G) \geq D_0$. This upper bound on $\omega(G^2)$ is tight. This is joint work with Xiaopan Lian (Nankai University).

Speaker: Dong-il Lee

Affiliation: Seoul Women's University

Title: Groebner–Shirshov bases for Temperley–Lieb algebras

Abstract

For Temperley–Lieb algebras of types B and D, we construct their Groebner–Shirshov bases and the corresponding standard monomials, which give another combinatorial interpretation for the fully commutative elements.

Speaker: Jaeseong Oh

Affiliation: Yonsei University

Title: Rook Theory and α-Chromatic Symmetric Functions

Abstract

We offer a gentle introduction to rook theory, with a special emphasis on the Goldman-Joichi-White formula, and its q-analogues studied by Garsia and Remmel. Furthermore, we present a novel extension of Stanley's chromatic symmetric function with an additional parameter α. We explore the relationship between α-chromatic symmetric functions and rook theory, as well as their relevance to various other topics in symmetric functionology. This is based on ongoing work with Jim Haglund and Meesue Yoo.

Speaker: Se-jin Oh

Affiliation: Sungkyunkwan University

Title: Valued quiver and quantum cluster algebra of non skew-symmetric type

Abstract

In this talk, I will introduce the notion of valued quiver as a realization of exchange matrix of a quantum cluster algebra. Using the quantum exchange relation described by valued quiver, I will discuss the quantum folded T-system which originated from mathematical physics and representation theory. These works are based on the joint works with Jang and Lee.

Speaker: Jeong Rye Park

Affiliation: Kyungpook National University

Title: Distance Matrices with three distinct eigenvalues

Abstract

Let G be a connected graph with n vertices. The distance between two vertices v_i and v_j, denoted by $d(v_i, v_j)$, is the length of the shortest path from v_i to v_j. The concept of distances in a graph has been used in applications much longer. For example, the Wiener index, a graph parameter readily computed from the distances of a graph, was introduced in 1947 in chemical graph theory by chemist Wiener. It was used to determine boiling points. The distance matrix $D(G)$ is defined as the symmetric $n \times n$-matrix indexed by the vertices of a graph G such that $D(G)_{ij} = d(v_i, v_j)$. It was introduced by Graham and Pollack in 1971. They established a relationship between the number of negative eigenvalues of the distance matrix and the addressing problem in data communication systems. Since then, there has been extensive research on the distance matrices of graphs. Several mathematicians were interested in studying the eigenvalue properties of the distance matrix of a connected graph. The distance eigenvalues are closely linked to the structure of a graph. This problem can be considered into two types. One is determining graphs with some prescribed distance eigenvalues. The other is determining graphs with some distinct distance eigenvalues but without given values. In this talk, we focus on connected graphs with three distinct distance eigenvalues. This is joint work with J. Koolen(USTC) and J Park(KNU).

Speaker: Sebastian Wiederrecht

Affiliation: IBS, DIMAG

Title: Bipartite Treewidth

Abstract

We generalize the theory of packing and covering cycles with divisibility conditions on their length to more complicated structures. That is, we consider undirected graphs whose edges are labeled with the elements of some finite abelian group together with the concept of "parity minors" which is a variant of minors that ensures that a cycle in the minor has total weight zero if and only if the corresponding cycle in the minor model does. As a first step we provide a duality theorem between the existence of certain grid-like parity minors or a tree decomposition where every bag B comes equipped with a small set S - the size of S determines the "width" - such that every non-zero cycle which interacts non-trivially with the vertices in B must also contain a vertex in S.

This is joint work with Rutger Campbell, Kevin Hendrey, and Pascal Gollin.