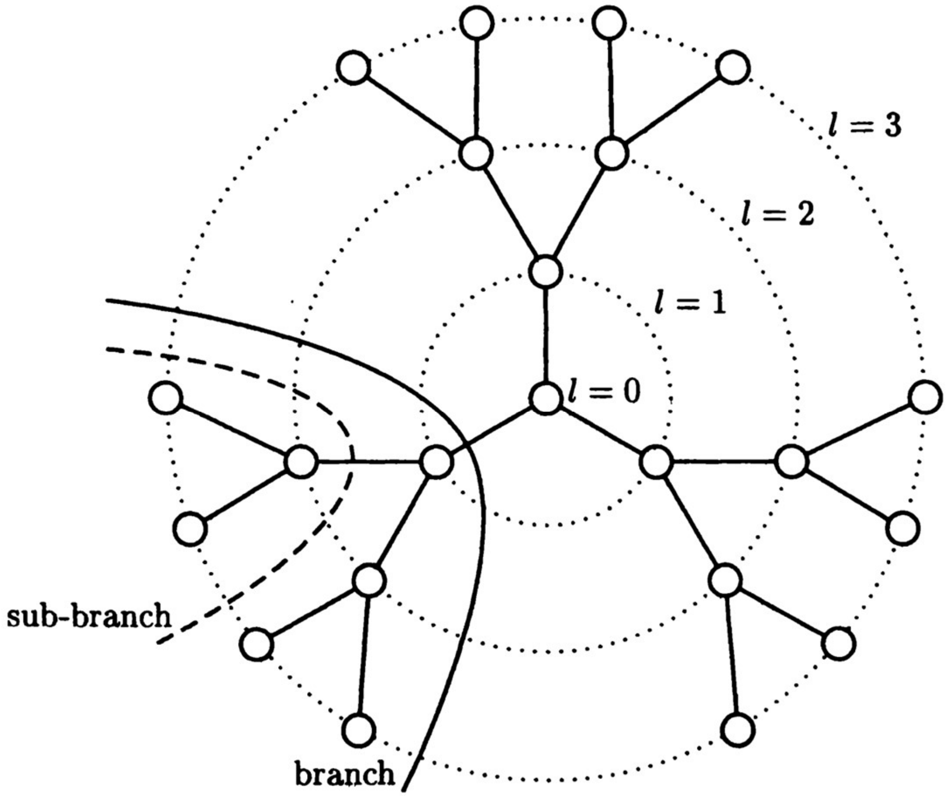
Problem 1. Bond percolation on the Bethe lattice

A Cayley tree with coordination number $z = 3$ and $n = 4$ generations is depicted.

When, $n \to \infty$,
the Cayley tree becomes the Bethe lattice.

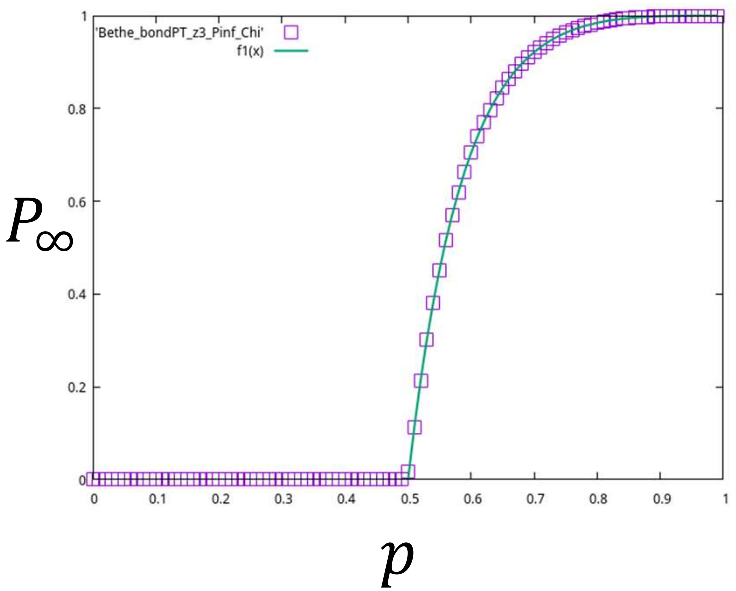
(1) Estimate four critical exponents $\beta, \tau, \gamma, \sigma$ of the Bethe lattice with $\underline{\boldsymbol{z = 4}}$ using **simulation**.

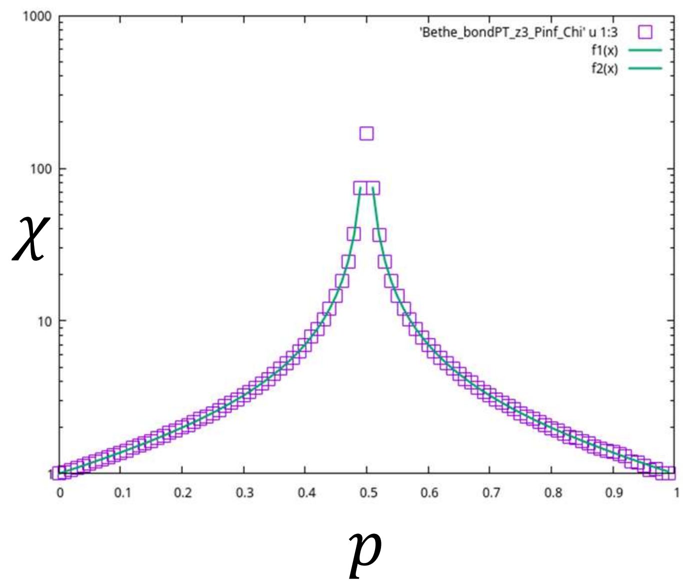
(2) Describe the simulation methods used in **(1)** briefly.



Problem 1. Bond percolation on the Bethe lattice

Reference) Results of $z = 3$





$$P_\infty = 1 - \left(\frac{1-p}{p}\right)^3 \text{ for } p \geq p_c = 0.5$$

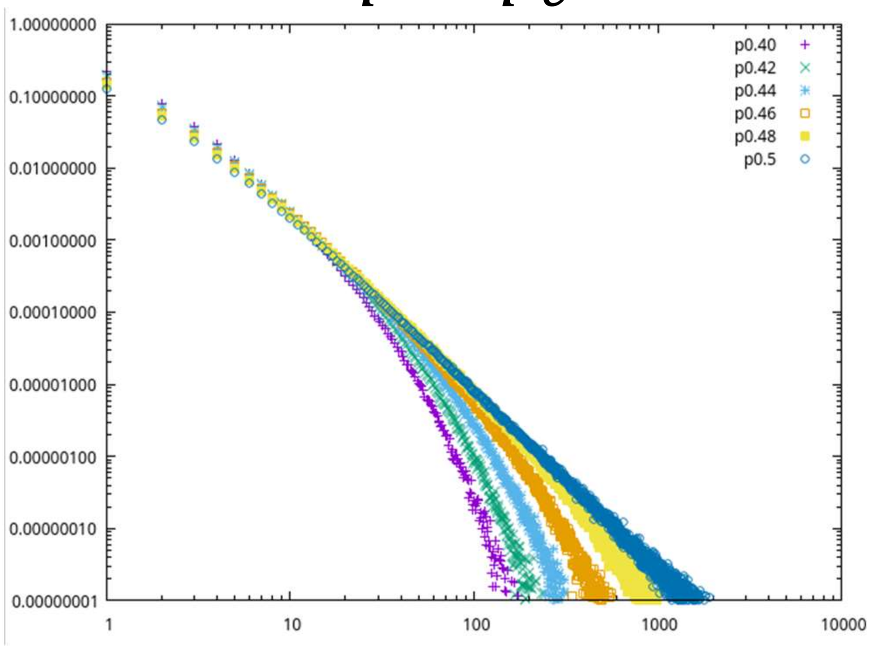
$$\chi = \begin{cases} \dfrac{1+p}{1-2p} & \text{for } p < p_c \\[2ex] \dfrac{2-p}{2p-1} & \text{for } p > p_c \end{cases}$$

Problem 1. Bond percolation on the Bethe lattice

Reference) Results of $z = 3$

$p < p_c$

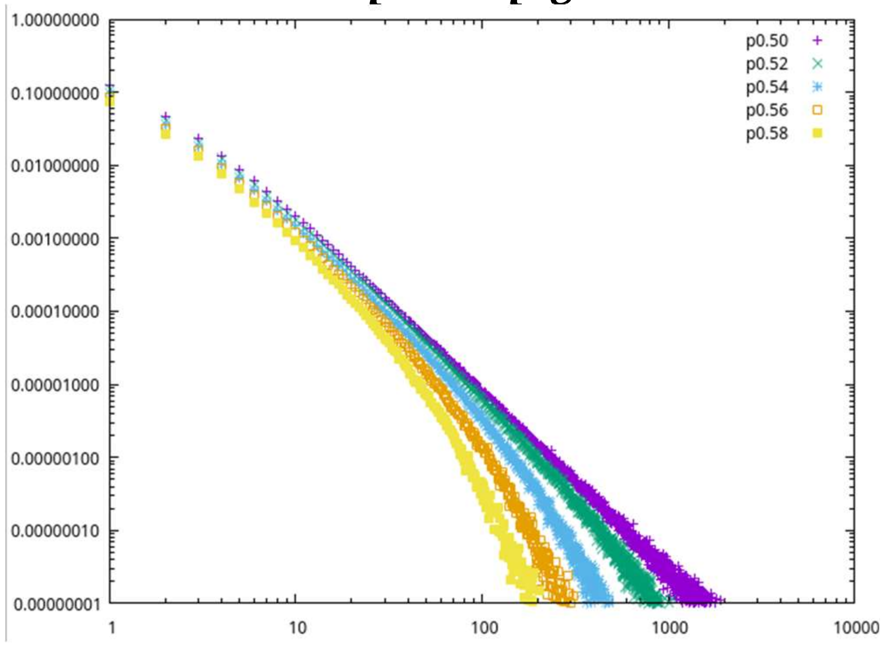
n_s



s

$p > p_c$

n_s



s

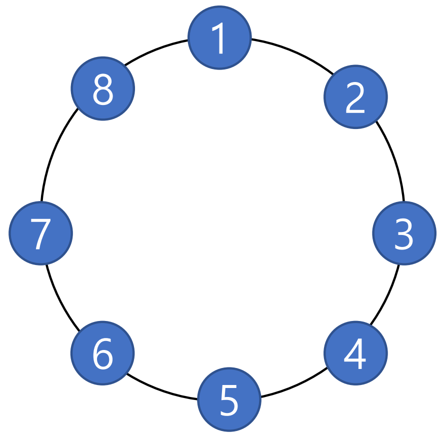
Problem 2. q-twisted states of identical Kuramoto oscillators on a ring of length N

Identical Kuramoto oscillators $\phi_i \in [0, 2\pi)$ $(i = 1, \dots, N)$ on a ring of length N follows

$$\dot{\phi}_i = [\sin(\phi_{i+1} - \phi_i) + \sin(\phi_{i-1} - \phi_i)]$$

with p.b.c. $\phi_{N+1} \equiv \phi_1$ and $\phi_0 \equiv \phi_N$.

$N = 8$

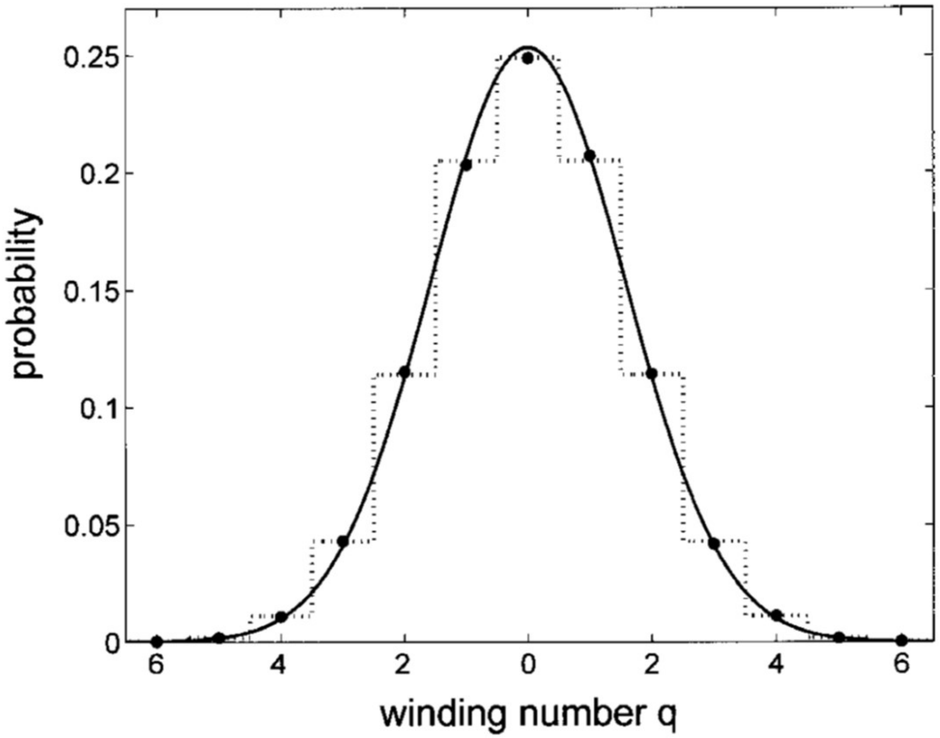


In this system, there exist q-twisted states in which

$$\phi_i = \left[\frac{2\pi q}{N} i\right] \bmod 2\pi.$$

(1) Show that the condition for stable q-twisted states and
(2) obtain basin stability of stable q-twisted states using $N = 80$.

Problem 2. q-twisted states of identical Kuramoto oscillators on a ring of length N



D. A. Wiley *et al.* Chaos **16**, 015103 (2006).

S. Lee *et al.* Phys. Rev. E **98**, 062221 (2018).