# Dispersive determination of neutrino mass ordering

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2306.03463

# Unsolved issues in neutrino physics

- Today's talk will try to answer:
- Neutrino mass ordering

$$\Delta m^2_{21} \equiv m_2^2 - m_1^2 = (7.55^{+0.20}_{-0.16}) \times 10^{-5}\ \mathrm{eV}^2 \qquad \Delta m^2_{32} \equiv m_3^2 - m_2^2 = (2.424 \pm 0.03) \times 10^{-3}\ \mathrm{eV}^2$$

but normal ordering or inverted ordering?

- Why small mixing in quark sector, but large mixing in lepton sector?

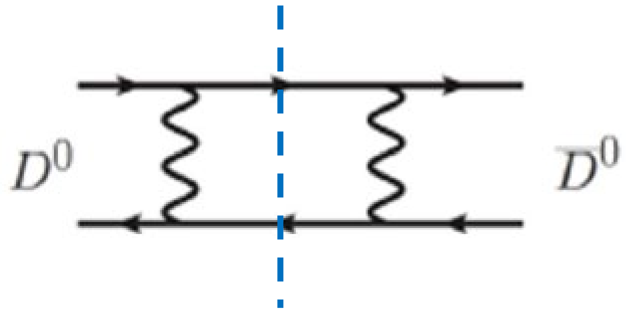
CKM: $\theta_{12} = 13.04° \pm 0.05°$, $\theta_{13} = 0.201° \pm 0.011°$, $\theta_{23} = 2.38° \pm 0.06°$

Pontecorvo–Maki–Nakagawa–Sakata: $\boldsymbol{\theta_{12} = 33.41°^{+0.75°}_{-0.72°}}$ $\quad \theta_{13} = 8.54°^{+0.11°}_{-0.12°}$

- Why lepton mixing has maximal angle $\theta_{23} \approx 45°$?
- Why neutrinos so light?

$$m_3 = \text{EW scale x } \frac{\text{charged lepton mass}}{\text{EW symmetry restoration scale}}$$

# Dispersion relation



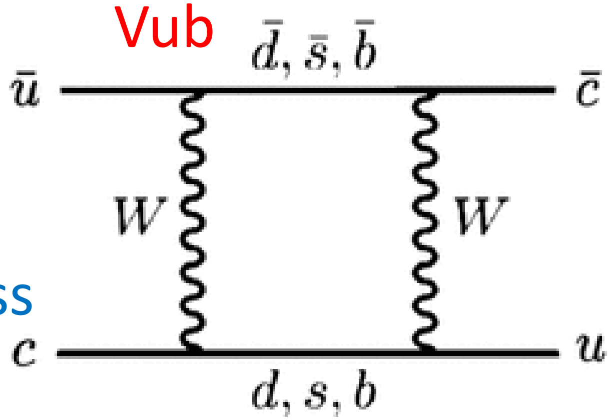

- Start with mixing of D meson with mass squared s

$$M_{12}(s) - \frac{i}{2}\Gamma_{12}(s) = \langle D^0|\mathcal{H}|\bar{D}^0\rangle$$

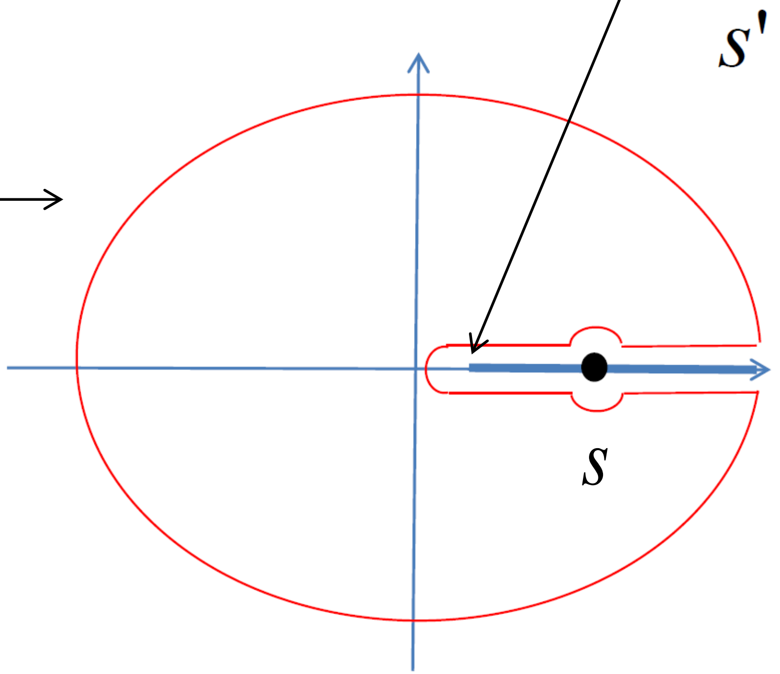
imaginary part

Vud
Vus
Vub

real part $$M_{12}(s) = \frac{P}{2\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{\Gamma_{12}(s')}{s-s'}$$




invariant mass squared s




large circle contribution suppressed

branch cut caused by intermediate real states

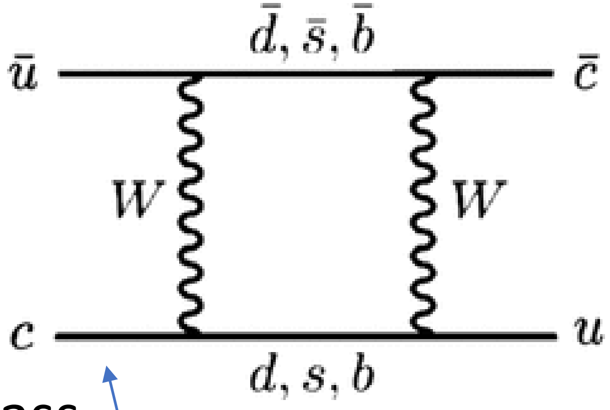
# Speculation

- Fundamental parameters in theory (like Standard Model) usually constrained by symmetries at Lagrangian level
- Physical observables are analytical
- Dispersion relations must be respected
- $\Gamma_{12}$ involves CKM matrix elements and fermion masses
- Additional dynamical constraints imposed by dispersion relations, given $M_{12}$ ?
- Turn out that dispersive constraints are so strong that Yukawa couplings in SM are in fact not free parameters

# Idea

- Neutral state mixing disappears at high energy, where electroweak symmetry is restored (working assumption)

Li, 2304.05921




EW symmetry broken
at low energy;
constrains fermion
masses and mixing angles

symmetry restored above restoration scale

invariant mass squared

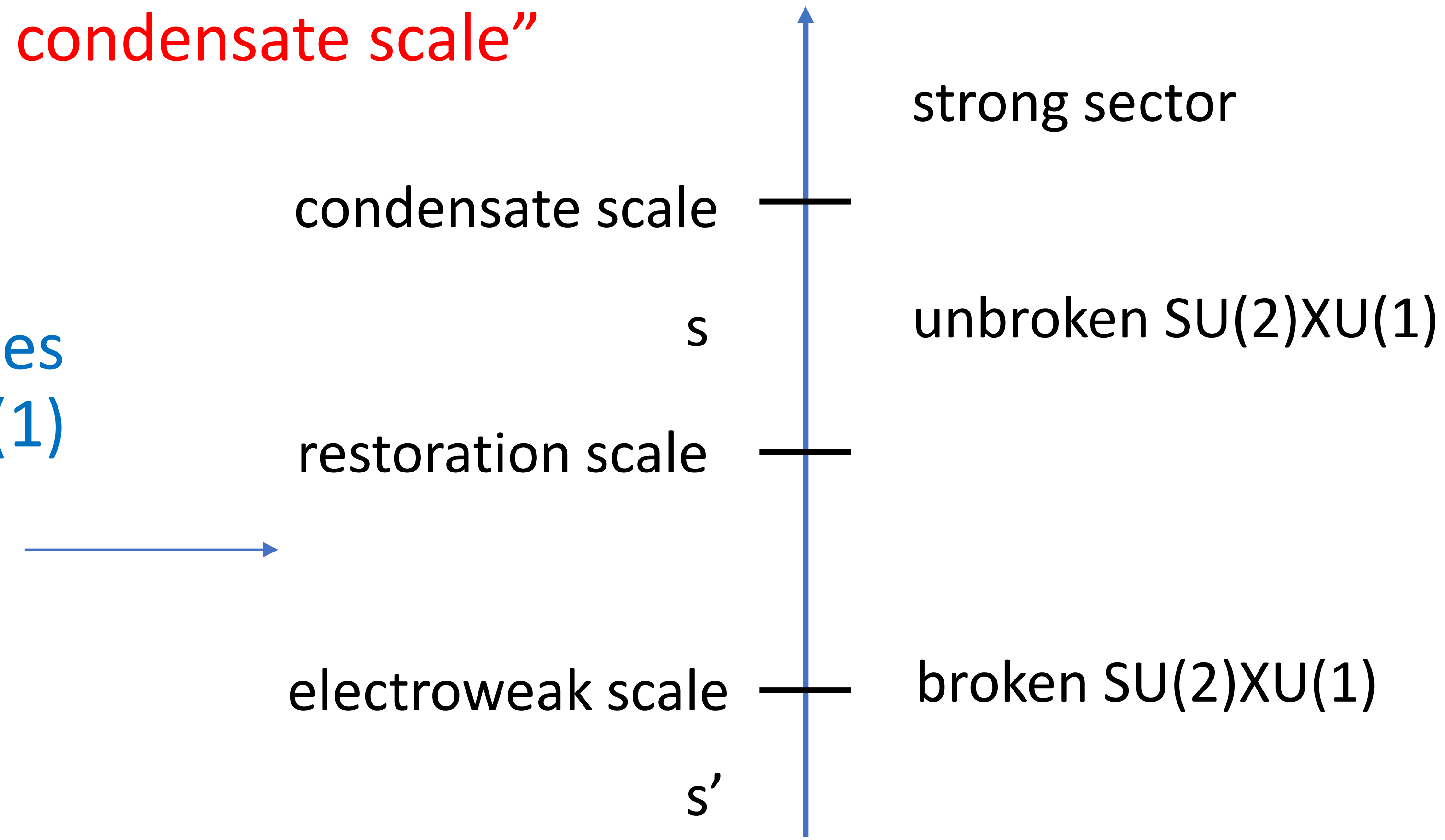
$$M_{12}(s) = \frac{1}{2\pi}\int ds' \frac{\Gamma_{12}(s')}{s-s'} \approx 0 \text{ at large } s$$

real part

imaginary part of box diagrams

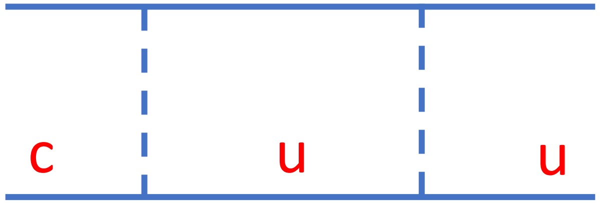
# Composite Higgs model

- Kaplan and Georgi, Phys. Lett. B136, 183 (1984):
- “The electroweak group is broken at a scale much smaller than the condensate scale”
- Hyperquark condensates misalign with SU(2)XU(1) vacuum owing to Yukawa couplings



# Proof of $M_{12}(s) \approx 0$

- Consider mixing of $Q_L\bar{q}_L$ , $\bar{Q}_L q_L$ neutral states
- Before breaking, all particles are massless, quarks in flavor eigenstates
- Mixing occurs via exchanges of charged or neutral scalars, whose strengths described by Yukawa matrices




neutral scalar exchanges

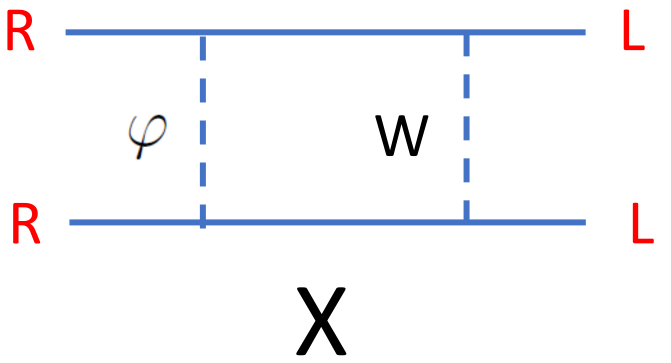
- After breaking, particles get masses, quarks turned to mass eigenstates
- Mixing occurs via W boson exchanges, whose strengths described by CKM matrix

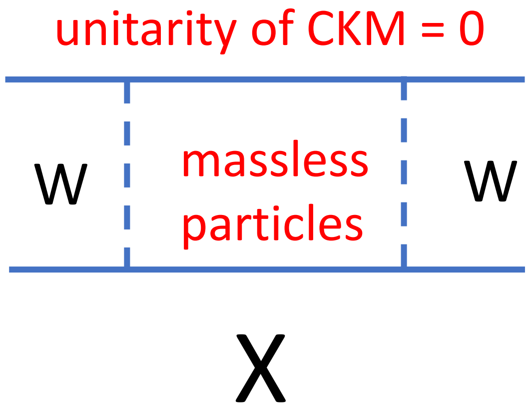
# Mixing in symmetric phase at leading order

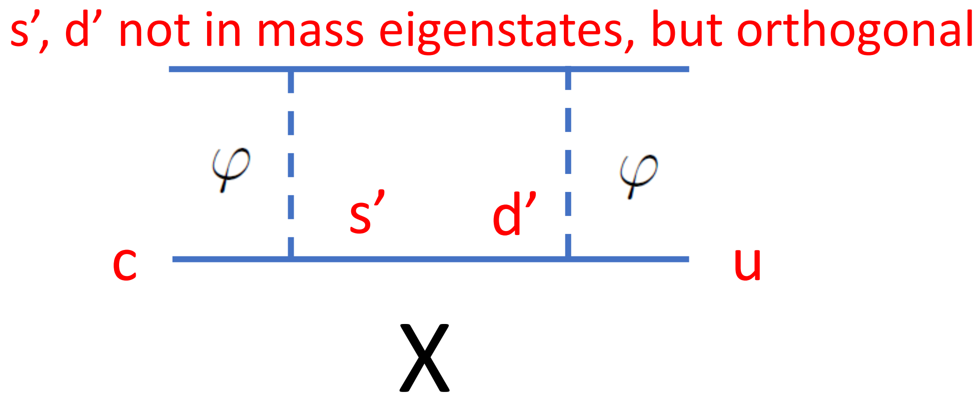
- Yukawa interaction $\overline{Q_L} Y_u u_R \varphi + \overline{Q_L} Y_d d_R \tilde{\varphi}$ $\qquad \varphi = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix},$

  left-handed doublet

- In symmetric phase, implement quark field transformation adopted in broken phase $u_L \to U_u u_L \quad u_R \longrightarrow V_u u_R \quad d_L \to U_d d_L \quad d_R \longrightarrow V_d d_R$
- Yukawa matrices diagonalized, but charged scalar currents exist
- down-type quarks, coupling to up-type quarks in mass eigenstates through charged scalar currents, are not in mass eigenstates



# Box diagrams

- Heavy quark Q provides large s in box diagrams. Symmetry restores and intermediate particles become massless, $M_{12}(s) \approx 0$
- s' can be low, so $\Gamma_{12}(s')$ depends on CKM matrix elements associated with massive intermediate quarks in broken phase.
- Box-diagram contribution

Cheng 1982
Buras et al 1984

$$\Gamma_{12}(s) \;\propto\; -\frac{G_F^2}{16\pi}\sum_{i,j}\lambda_i\lambda_j\Gamma_{ij}(s)$$

$$\Gamma_{ij}(s) \;=\; \frac{1}{s^2}\frac{\sqrt{s^2-2s(m_i^2+m_j^2)+(m_i^2-m_j^2)^2}}{(m_W^2-m_i^2)(m_W^2-m_j^2)}$$
$$\times\left\{\left(m_W^4+\frac{m_i^2m_j^2}{4}\right)[2s^2-4s(m_i^2+m_j^2)+2(m_i^2-m_j^2)^2]+3m_W^2s(m_i^2+m_j^2)(m_i^2+m_j^2-s)\right\}$$

for D mixing $i,j = d,s,b \qquad \lambda_i \equiv V_{ci}^*V_{ui}$

# Constraints

- How to diminish dispersive integral $\int ds' \frac{\Gamma_{12}(s')}{s-s'}$ ?
- Asymptotic expansion

EW symmetry restoration scale

to have finite integral

$$\Gamma_{ij}(s') \approx \Gamma_{ij}^{(1)} s' + \Gamma_{ij}^{(0)} + \frac{\Gamma_{ij}^{(-1)}}{s'} + \cdots$$

$$\boxed{\sum_{i,j} \lambda_i \lambda_j \Gamma_{ij}^{(m)} \approx 0, \quad m = 1, 0, -1}$$

$$\Gamma_{ij}^{(1)} = \frac{4m_W^4 - 6m_W^2(m_i^2 + m_j^2) + 4m_i^2 m_j^2}{2(m_W^2 - m_i^2)(m_W^2 - m_j^2)}, \Rightarrow \Lambda^2/s$$

$$\Gamma_{ij}^{(0)} = -\frac{3(m_i^2 + m_j^2)\left[4m_W^4 - 4m_W^2(m_i^2 + m_j^2) + m_i^2 m_j^2\right]}{2(m_W^2 - m_i^2)(m_W^2 - m_j^2)} \Rightarrow (m_i^2 + m_j^2)\Lambda/s$$

$$\Gamma_{ij}^{(-1)} = \frac{3(m_i^4 + m_j^4)\left[4m_W^4 - 2m_W^2(m_i^2 + m_j^2) + m_i^2 m_j^2\right]}{2(m_W^2 - m_i^2)(m_W^2 - m_j^2)}. \Rightarrow (m_i^4 + m_j^4)\ln \Lambda/s$$

to diminish integral

$$\int ds' \frac{\Gamma_{12}(s')}{s-s'} \approx \frac{1}{s} \sum_{i,j} \lambda_i \lambda_j g_{ij} \qquad g_{ij} \equiv \int_{t_{ij}}^{\infty} ds' \left[\Gamma_{ij}(s') - \Gamma_{ij}^{(1)} s' - \Gamma_{ij}^{(0)} - \frac{\Gamma_{ij}^{(-1)}}{s'}\right]$$

$$\boxed{\sum_{i,j} \lambda_i \lambda_j g_{ij} \approx 0}$$

# Minimization

- Use unitarity to eliminate $\lambda_b$ and to rewrite constrains

$$r^2 R_{dd}^{(m)} + 2r R_{ds}^{(m)} + 1 \approx 0, , \quad m = 1, 0, -1, i$$

$$R_{dd}^{(m)} = \frac{\Gamma_{dd}^{(m)} - 2\Gamma_{db}^{(m)} + \Gamma_{bb}^{(m)}}{\Gamma_{ss}^{(m)} - 2\Gamma_{sb}^{(m)} + \Gamma_{bb}^{(m)}}, \quad R_{ds}^{(m)} = \frac{\Gamma_{ds}^{(m)} - \Gamma_{db}^{(m)} - \Gamma_{sb}^{(m)} + \Gamma_{bb}^{(m)}}{\Gamma_{ss}^{(m)} - 2\Gamma_{sb}^{(m)} + \Gamma_{bb}^{(m)}} \quad m = 1, 0, -1$$

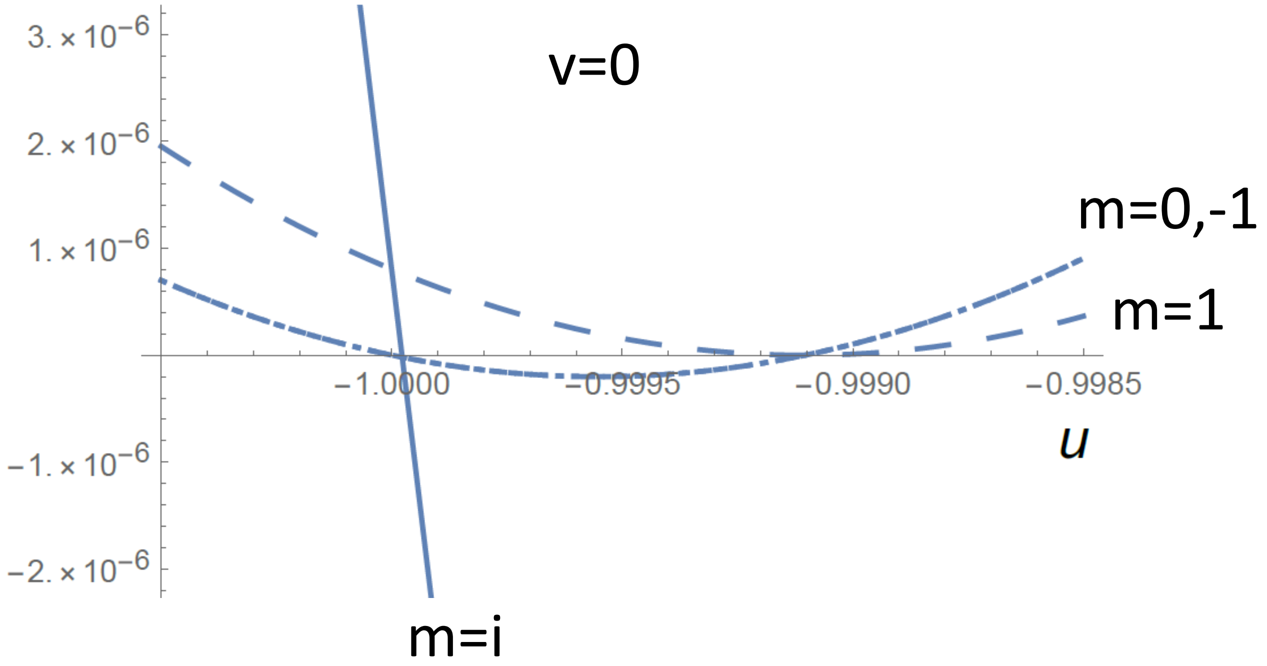
- Expression for $m = i$ similar, but with $g_{ij}$
- Ratio of CKM elements $r = \frac{\lambda_d}{\lambda_s} = \frac{V_{cd}^* V_{ud}}{V_{cs}^* V_{us}} \equiv u + iv,$

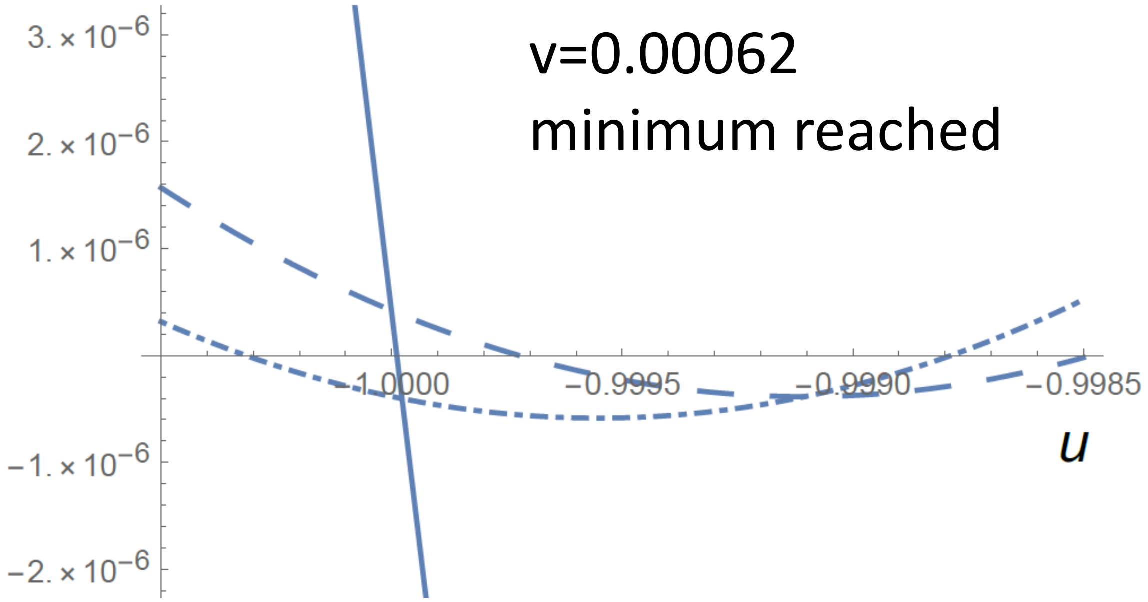
- Tune u and v to minimize the sum (real parts of constraints)

$$\sum_{m=1,-1,i} \left[ (u^2 - v^2) R_{dd}^{(m)} + 2u R_{ds}^{(m)} + 1 \right]^2$$

# Results

$m_d = 0.005\ \mathrm{GeV} \quad m_s = 0.12\ \mathrm{GeV} \quad m_b = 4.0\ \mathrm{GeV} \quad m_W = 80.377\ \mathrm{GeV}$




PDG

$$r = \frac{V_{cd}^* V_{ud}}{V_{cs}^* V_{us}} = -1.0 + (6.2^{+1.2}_{-1.0}) \times 10^{-4} i \qquad u = -1.00029 \pm 0.00002, \quad v = 0.00064 \pm 0.00002$$

variation of ms by 0.01 GeV

they agree well

# Pontecorvo–Maki–Nakagawa–Sakata matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{bmatrix}.$$

Chau-Keung parametrization

# PDG

| | Ref. [188] w/o SK-ATM | | Ref. [188] w SK-ATM | | Ref. [189] w SK-ATM | | Ref. [190] w SK-ATM | |
|---|---|---|---|---|---|---|---|---|
| NO | Best Fit Ordering | | Best Fit Ordering | | Best Fit Ordering | | Best Fit Ordering | |
| Param | bfp $\pm 1\sigma$ | $3\sigma$ range | bfp $\pm 1\sigma$ | $3\sigma$ range | bfp $\pm 1\sigma$ | $3\sigma$ range | bfp $\pm 1\sigma$ | $3\sigma$ range |
| $\frac{\sin^2\theta_{12}}{10^{-1}}$ | $3.10^{+0.13}_{-0.12}$ | $2.75 \to 3.50$ | $3.10^{+0.13}_{-0.12}$ | $2.75 \to 3.50$ | $3.04^{+0.14}_{-0.13}$ | $2.65 \to 3.46$ | $3.20^{+0.20}_{-0.16}$ | $2.73 \to 3.79$ |
| $\theta_{12}/^\circ$ | $33.82^{+0.78}_{-0.76}$ | $31.61 \to 36.27$ | $33.82^{+0.78}_{-0.76}$ | $31.61 \to 36.27$ | $33.46^{+0.87}_{-0.88}$ | $30.98 \to 36.03$ | $34.5^{+1.2}_{-1.0}$ | $31.5 \to 38.0$ |
| $\frac{\sin^2\theta_{23}}{10^{-1}}$ | $5.58^{+0.20}_{-0.33}$ | $4.27 \to 6.09$ | $5.63^{+0.18}_{-0.24}$ | $4.33 \to 6.09$ | $5.51^{+0.19}_{-0.80}$ | $4.30 \to 6.02$ | $5.47^{+0.20}_{-0.30}$ | $4.45 \to 5.99$ |
| $\theta_{23}/^\circ$ | $48.3^{+1.2}_{-1.9}$ | $40.8 \to 51.3$ | $48.6^{+1.0}_{-1.4}$ | $41.1 \to 51.3$ | $47.9^{+1.1}_{-4.0}$ | $41.0 \to 50.9$ | $47.7^{+1.2}_{-1.7}$ | $41.8 \to 50.7$ |
| $\frac{\sin^2\theta_{13}}{10^{-2}}$ | $2.241^{+0.066}_{-0.065}$ | $2.046 \to 2.440$ | $2.237^{+0.066}_{-0.065}$ | $2.044 \to 2.435$ | $2.14^{+0.09}_{-0.07}$ | $1.90 \to 2.39$ | $2.160^{+0.083}_{-0.069}$ | $1.96 \to 2.41$ |
| $\theta_{13}/^\circ$ | $8.61^{+0.13}_{-0.13}$ | $8.22 \to 8.99$ | $8.60^{+0.13}_{-0.13}$ | $8.22 \to 8.98$ | $8.41^{+0.18}_{-0.14}$ | $7.9 \to 8.9$ | $8.45^{+0.16}_{-0.14}$ | $8.0 \to 8.9$ |
| $\delta_{CP}/^\circ$ | $222^{+38}_{-28}$ | $141 \to 370$ | $221^{+39}_{-28}$ | $144 \to 357$ | $238^{+41}_{-33}$ | $149 \to 358$ | $218^{+38}_{-27}$ | $157 \to 349$ |
| $\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$ | $7.39^{+0.21}_{-0.20}$ | $6.79 \to 8.01$ | $7.39^{+0.21}_{-0.20}$ | $6.79 \to 8.01$ | $7.34^{+0.17}_{-0.14}$ | $6.92 \to 7.91$ | $7.55^{+0.20}_{-0.16}$ | $7.05 \to 8.24$ |
| $\frac{\Delta m^2_{32}}{10^{-3}\ \mathrm{eV}^2}$ | $2.449^{+0.032}_{-0.030}$ | $2.358 \to 2.544$ | $2.454^{+0.029}_{-0.031}$ | $2.362 \to 2.544$ | $2.419^{+0.035}_{-0.032}$ | $2.319 \to 2.521$ | $2.424 \pm 0.03$ | $2.334 \to 2.524$ |
| IO | $\Delta\chi^2 = 6.2$ | | $\Delta\chi^2 = 10.4$ | | $\Delta\chi^2 = 9.5$ | | $\Delta\chi^2 = 11.7$ | |
| $\frac{\sin^2\theta_{12}}{10^{-1}}$ | $3.10^{+0.13}_{-0.12}$ | $2.75 \to 3.50$ | $3.10^{+0.13}_{-0.12}$ | $2.75 \to 3.50$ | $3.03^{+0.14}_{-0.13}$ | $2.64 \to 3.45$ | $3.20^{+0.20}_{-0.16}$ | $2.73 \to 3.79$ |
| $\theta_{12}/^\circ$ | $33.82^{+0.78}_{-0.76}$ | $31.61 \to 36.27$ | $33.82^{+0.78}_{-0.75}$ | $31.62 \to 36.27$ | $33.40^{+0.87}_{-0.81}$ | $30.92 \to 35.97$ | $34.5^{+1.2}_{-1.0}$ | $31.5 \to 38.0$ |
| $\frac{\sin^2\theta_{23}}{10^{-1}}$ | $5.63^{+0.19}_{-0.26}$ | $4.30 \to 6.12$ | $5.65^{+0.17}_{-0.22}$ | $4.36 \to 6.10$ | $5.57^{+0.17}_{-0.24}$ | $4.44 \to 6.03$ | $5.51^{+0.18}_{-0.30}$ | $4.53 \to 5.98$ |
| $\theta_{23}/^\circ$ | $48.6^{+1.1}_{-1.5}$ | $41.0 \to 51.5$ | $48.8^{+1.0}_{-1.2}$ | $41.4 \to 51.3$ | $48.2^{+1.0}_{-1.4}$ | $41.8 \to 50.9$ | $47.9^{+1.0}_{-1.7}$ | $42.3 \to 50.7$ |
| $\frac{\sin^2\theta_{13}}{10^{-2}}$ | $2.261^{+0.067}_{-0.064}$ | $2.066 \to 2.461$ | $2.259^{+0.065}_{-0.065}$ | $2.064 \to 2.457$ | $2.18^{+0.08}_{-0.07}$ | $1.95 \to 2.43$ | $2.220^{+0.074}_{-0.076}$ | $1.99 \to 2.44$ |
| $\theta_{13}/^\circ$ | $8.65^{+0.13}_{-0.12}$ | $8.26 \to 9.02$ | $8.64^{+0.12}_{-0.13}$ | $8.26 \to 9.02$ | $8.49^{+0.15}_{-0.14}$ | $8.0 \to 9.0$ | $8.53^{+0.14}_{-0.15}$ | $8.1 \to 9.0$ |
| $\delta_{CP}/^\circ$ | $285^{+24}_{-26}$ | $205 \to 354$ | $282^{+23}_{-25}$ | $205 \to 348$ | $247^{+26}_{-27}$ | $193 \to 346$ | $281^{+23}_{-27}$ | $202 \to 349$ |
| $\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$ | $7.39^{+0.21}_{-0.20}$ | $6.79 \to 8.01$ | $7.39^{+0.21}_{-0.20}$ | $6.79 \to 8.01$ | $7.34^{+0.17}_{-0.14}$ | $6.92 \to 7.91$ | $7.55^{+0.20}_{-0.16}$ | $7.05 \to 8.24$ |
| $\frac{\Delta m^2_{32}}{10^{-3}\ \mathrm{eV}^2}$ | $-2.509^{+0.032}_{-0.032}$ | $-2.603 \to -2.416$ | $-2.510^{+0.030}_{-0.031}$ | $-2.601 \to -2.419$ | $-2.478^{+0.035}_{-0.033}$ | $-2.577 \to -2.375$ | $-2.50\pm^{+0.04}_{-0.03}$ | $-2.59 \to -2.39$ |

# Lepton mixing

- Apply the same formalism to lepton $\mu^- e^+$-$\mu^+ e^-$ mixing through similar box diagrams with intermediate neutrino channels

PMNS

- Correspondence $m_{d,s,b} \leftrightarrow m_{1,2,3}$ $\quad V^*_{cd}V_{ud}/(V^*_{cs}V_{us}) \leftrightarrow r = U^*_{\mu 1}U_{e1}/(U^*_{\mu 2}U_{e2})$
- Normal hierarchy (NH) $\quad m_1^2 = 10^{-6}\ \mathrm{eV}^2$

de Salas et al, 2018

$$\Delta m^2_{21} \equiv m_2^2 - m_1^2 = (7.55^{+0.20}_{-0.16}) \times 10^{-5}\ \mathrm{eV}^2 \qquad \Delta m^2_{32} \equiv m_3^2 - m_2^2 = (2.424 \pm 0.03) \times 10^{-3}\ \mathrm{eV}^2$$

- Predict

LO analysis so far

global fit

$$r = \frac{U^*_{\mu 1}U_{e1}}{U^*_{\mu 2}U_{e2}} \approx \underline{-1.0} - 0.02i \qquad r = -\underline{(0.738^{+0.050}_{-0.048})} - (0.179^{+0.136}_{-0.125})i$$

- Inverted hierarchy (IH) $\quad r \approx -1.0 - O(10^{-5})i \qquad r = -(1.03^{+0.05}_{-0.16}) - (0.356^{+0.015}_{-0.048})i$
- NH and observed PMNS matrix satisfy constraint at order of magnitude

# Why neutrinos so light?

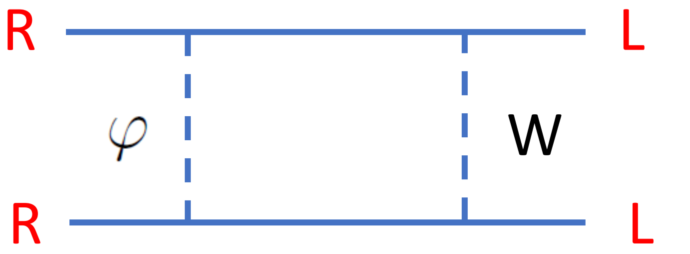
So far, connections between mixing angles and mass ratios

Haven’t addressed absolute neutrino masses

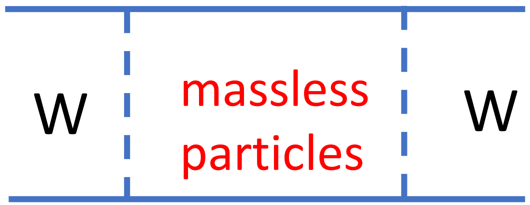
alternative to see-saw mechanism

# NLO in symmetric phase

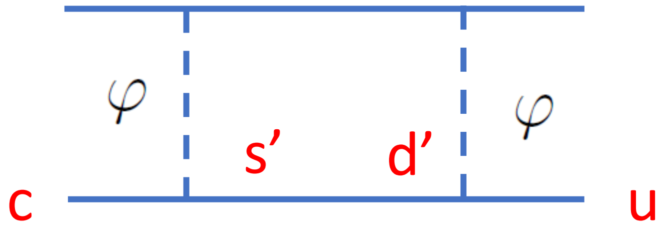
- Mixing disappears above EW restoration scale only at LO, in fact



chirality could be changed

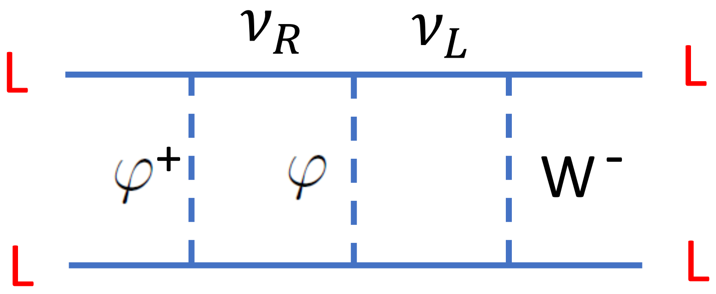


unitarity persists



orthogonality persists

- At NLO,



real part, dominant diagram

does not vanish, and determines smallness of neutrino masses

# Estimate m3

- Consider 2-3 generation mixing to reach maximal neutrino mass

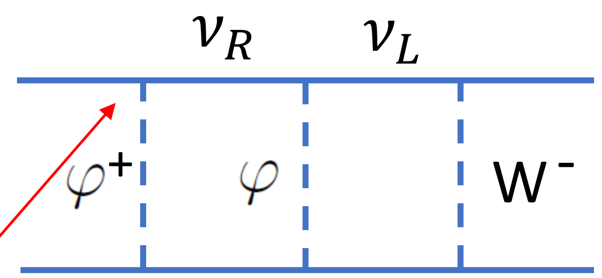
loop factor

$$\frac{G_F^2}{8\pi^2}\frac{m_3^4}{m_W^4}\frac{\Lambda^4}{2} \approx 2\left(\frac{1}{16\pi^2}\right)^2\left(\frac{g}{2\sqrt{2}}\right)^2\frac{m_\tau m_\mu m_3^2}{v^4}$$

dispersive integral

M12

number of diagrams




$$m_3^2 = \frac{1}{8\pi^2}\frac{m_W^4}{v^2}\frac{m_\tau m_\mu m_W^2}{\Lambda^4} \sim \boxed{1.5\times 10^{-3}\mathrm{eV}^2}$$

50 TeV, between b' mass 2.7 TeV and t' mass 200 TeV

Li, 2309.15602

measured value $\Delta m_{32}^2 \equiv m_3^2 - m_2^2 = (2.424 \pm 0.03)\times 10^{-3}\ \mathrm{eV}^2$

# Analyticity dictates scalar sector and explains SM flavor structure?