Project for all

Find the factorial moment generating function

$$\mathcal{G}(z,t) := \sum_{n=-\infty}^{\infty} z^n P_n(t)$$

for the random walk, described by the following master equation

$$\frac{\partial P_n(t)}{\partial t} = rP_{n-1}(t) + lP_{n+1}(t) - (r+l)P_n(t),$$

in two ways. $[\text{Initial condition} : \mathcal{G}(z, t=0) = 1]$

1. Write down the differential equation for $\mathcal{G}$ and solve it.
2. Consider the probability that the walker is located at n after N steps, where N is the number of jumps up to time t.

For the second approach, it is convenient to use $\frac{1}{k!} = 0$ for negative integer k.

Project

Study the 2D Ising model with periodic boundary conditions at the critical point using the Metropolis algorithm. [Initial condition : $s_i = 1$ (all spin-up), $\beta = \frac{1}{8}$, $\nu = 1$, $d = 2$]

$$\beta H = -K \sum_{i,j=1}^{L} s_{i,j}(s_{i,j+1} + s_{i+1,j}), \quad K_c = \ln(1+\sqrt{2})/2.$$

- magnetization $m(t) := L^{-2} \sum_i s_i(t) \sim t^{-\beta/(\nu z)}$.
- energy per site $E(t) := -L^{-2} \sum_{i,j=1}^{L} s_{i,j}(s_{i,j+1} + s_{i+1,j}) + \ln 2/2 \sim t^{-(\nu d-1)/(\nu z)}$
- Compare two schemes : $\Delta t = -\log(1-Z)/L^2$ vs $\Delta t = 1/L^2$.
- analyze the effective exponents and CTS. Compare them to a power-law fitting.
- $T_{\max} = 10^4$ and $L = 2^{12}$ is recommended. But due to the time limit, small size ($L = 2^9$?) and short time (2×10^3?) would be fine. Compare Q for $b = 10^{0.7}$ and 10.