

# Emergent particles of a dS universe:

Thermal interpretation of the stochastic formalism and beyond

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dS universe

= flat slicings of dS space (in this talk)

# Abstract

- Thermal interpretation of the stochastic evolution of scalar field in dS universe in superhorizon (IR) regime (“stochastic formalism”) is given
- Further physical significance of this interpretation is found through reinterpretation of the 1<sup>st</sup> slow-roll condition and the Hubble expansion

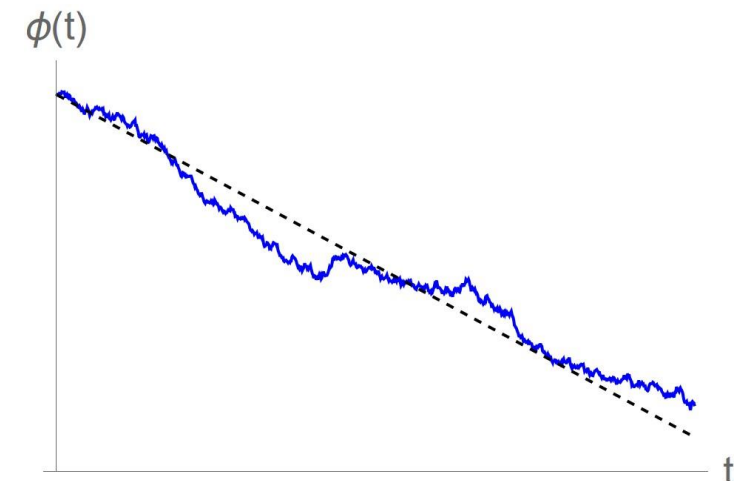
# Outline

- Introduction
  - Stochastic formalism of slow-rolling scalar field in inflation
- Giving thermal interpretation
  - General formalism / Heat bath model
- And beyond
  - 1<sup>st</sup> slow-roll condition / Hubble expansion
- Discussion and Conclusion

# Introduction

- The stochastic formalism
  - Effective theory of a slow-rolling scalar field in inflation
    - Wave number below a cutoff scale at  $k = \epsilon aH$  ( $\epsilon \ll 1$ ); "coarse-grained field"
  - Classicality of superhorizon modes
    - Measurement outcome of quantum state = Ensemble of classical random fields
  - Langevin equation ("classical" random evolution)

$$d\phi = -\frac{V'(\phi)}{3H} dt + \sqrt{\frac{H^3}{4\pi}} dW$$



# Introduction

- Some similarities with dS thermodynamics:

Appearance of  $T_{dS} = H/2\pi$  in several points

- Intuitive understanding of the formalism and result in particle physics literature
- $\langle \Delta\phi^2 \rangle$  per Hubble time  $\sim T_{dS}$
- $d\rho/d \ln k$  at horizon crossing  $\sim T_{dS}^4$
- $\langle V(\phi) \rangle$  after reaching the equilibrium  $\sim T_{dS}^4$

# Introduction

- But not actually a thermal effect
  - Not a result of the cosmological horizon associated with a local observer
    - Stochastic formalism: field is not described by a local observer.
      - Coarse-grained field in the "superhorizon limit"; not even at the horizon scale
      - Origin of the 'randomness' ( $\sim$ thermal) is different
  - Spin dependence, resultant spectrum, ...
  - Appearance of  $T_{dS}$  in stochastic formalism should be understood as a result of the single-scale background;  $H$

# Introduction

- Thermal interpretation by Rigopoulos (2013) & (2016)
  - Effective action of superhorizon modes has a term responsible for stochastic force
    - Schwinger-Keldysh formalism; effective action expressed in the Keldysh basis
  - Satisfies the fluctuation-dissipation relation with the Hubble friction at  $T_{dS}$
  - Interprets stochastic evolution as Brownian motion in a medium at  $T_{dS}$

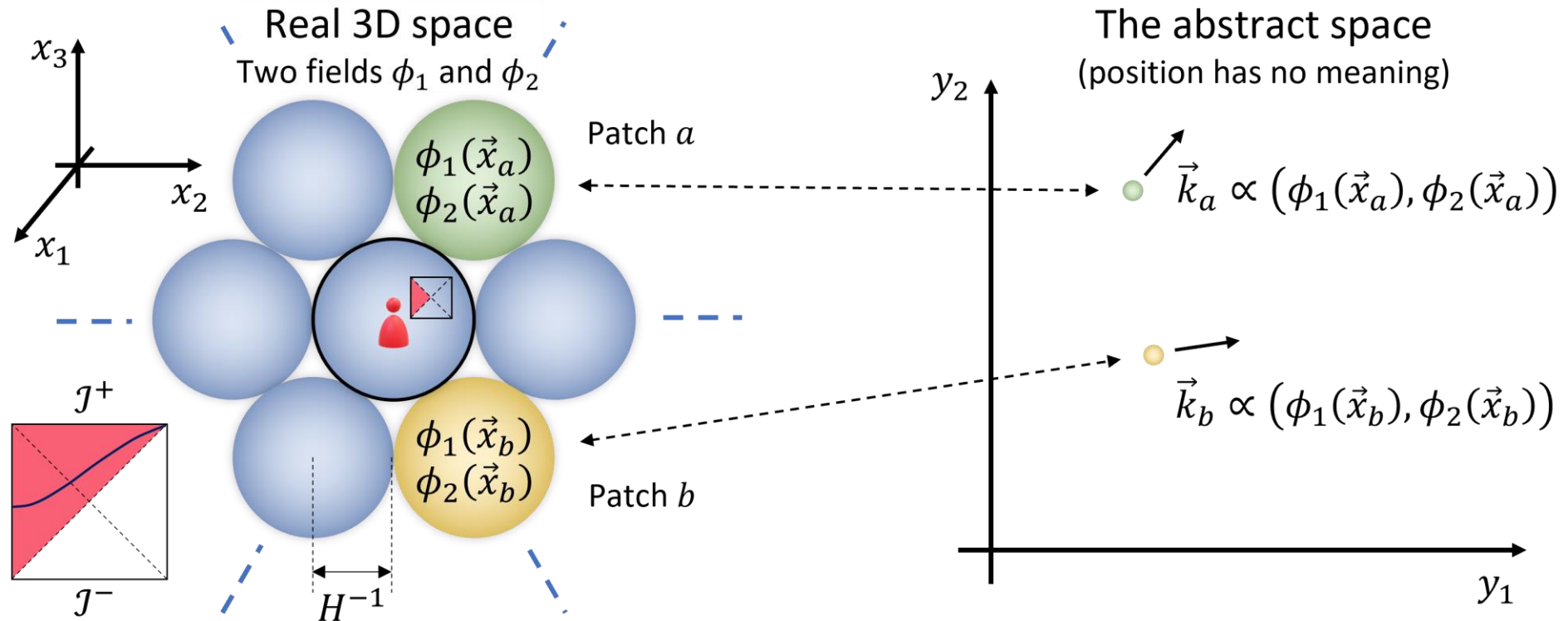
# Introduction

- Can we reach the same conclusion from physical considerations?
  - Proposing a general formalism & building a concrete heat bath model
  - We consistently make analogies to familiar physics, without introducing any unconventional relations
  - Arrive at the desired thermal interpretation and discover further physical significance
  - “Similarity between coarse-grained scalar fields in dS and conventional thermal physics”



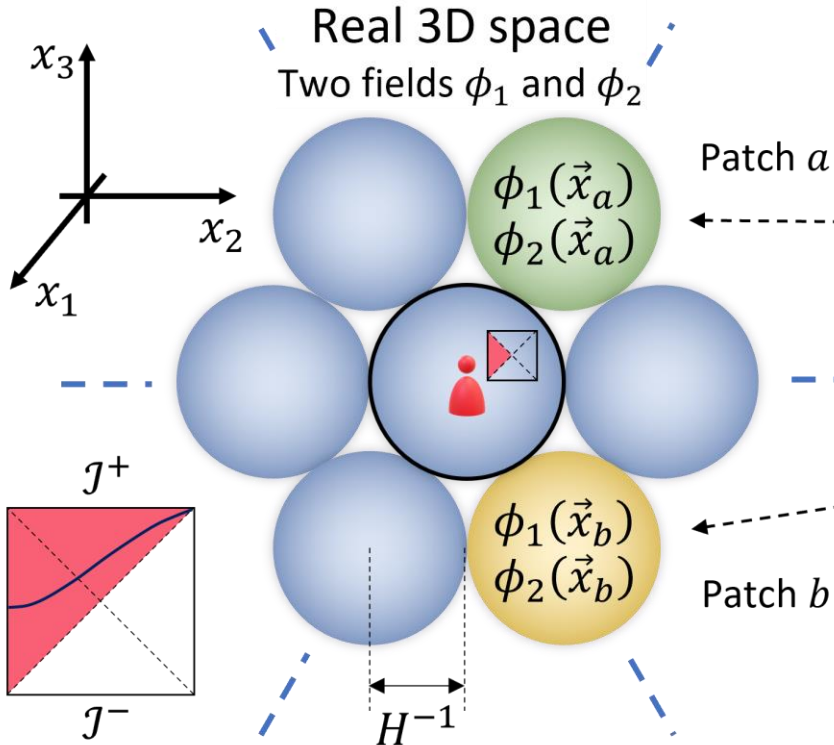
# The general formalism

- We treat horizon-sized spatial regions as “particles” in a virtual space



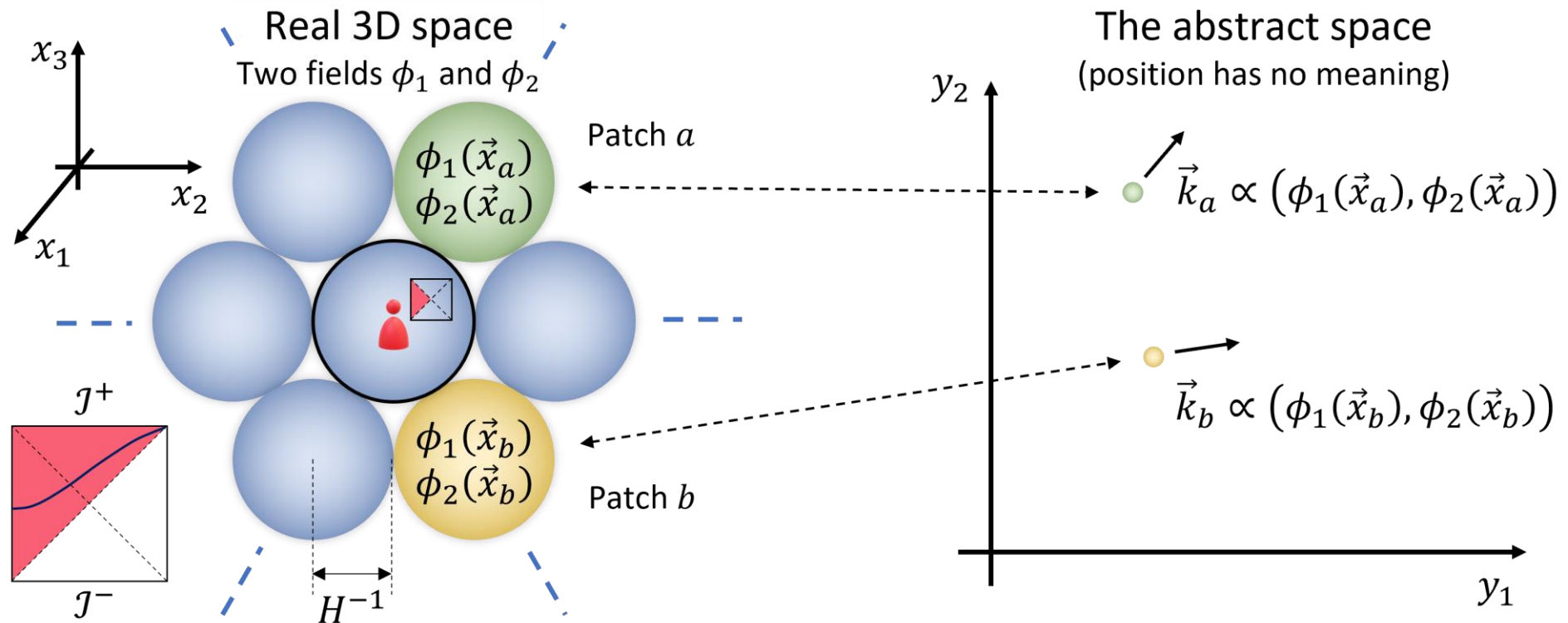
Causal patch  
 $\simeq$  Hubble volume

# The general formalism



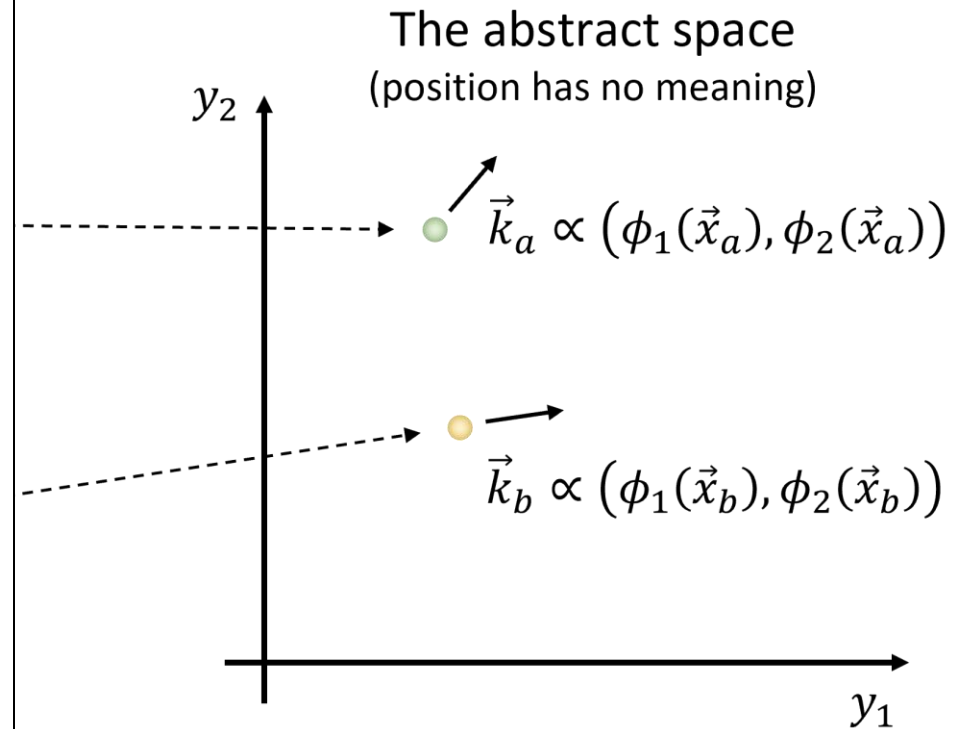
- The formalism depicts the space composed of many causal patches
  - “Horizon in an objective manner”
  - Note: We are developing **a formalism, NOT a new theory!**
- Each patch has its field value for the coarse-grained fields

# The general formalism



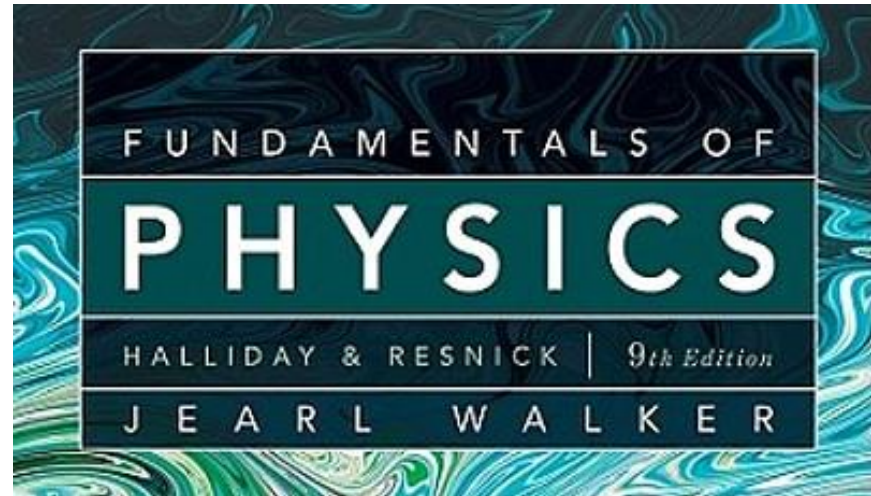
# The general formalism

- Each patch is regarded as a particle in another space
  - “Emergent particle”
  - “The abstract space”

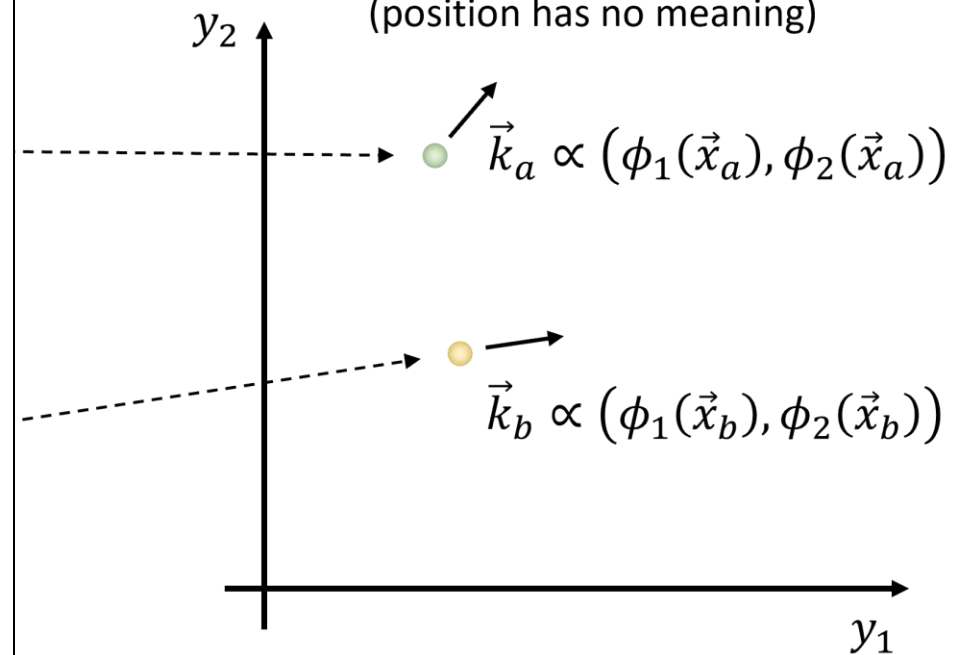


# The general formalism

- Two principles assumed:
  1. Field value  $\propto$  momentum
    - “Dual description”
  2. The usual classical physics in the abstract space
    - Set of relations between physical quantities in there



The abstract space  
(position has no meaning)



# The general formalism

- Translate several aspects of dS universe into the dynamics in the abstract space relying only on the assumed two principles
  - Stochastic field evolution + 1<sup>st</sup> slow-roll condition & Hubble expansion
- “Minimal non-minimal” setup
  - $V_0 = 3M_P^2 H^2$ : Unspecified background energy density
  - $\phi$ : One real minimally coupled slow-rolling spectator scalar field (coarse-grained)
  - LOG, we assume that  $V_\phi$  has a global minimum with nonzero mass.  
WLOG,  $V_\phi = \frac{1}{2} m_\phi^2 \phi^2 + \dots$ ;  $\phi = 0$  to be the global minimum;  $V_\phi(0) = 0$

# The general formalism

- Identifying physical quantities in the abstract space
  - Uniquely identified after the assumed principles + heuristic arguments

Causal patch in $dS$	Emergent particle in the Abstract space	Equation
Field value $\phi$	Momentum $k$	
Potential $V_\phi$		
Background energy $V_0$		
Potential slope $V'_\phi$		

# The general formalism

- Identifying physical quantities in the abstract space
  - Uniquely identified after the assumed principles + heuristic arguments

Causal patch in $dS$	Emergent particle in the Abstract space	Equation
Field value $\phi$	Momentum $k$	
Potential $V_\phi$	Kinetic energy $E_k$	
Background energy $V_0$	Mass $M$	
Potential slope $V'_\phi$		



Volume factor for conversion:  
 Volume of a patch =  $4\pi/3H^3$

# The general formalism

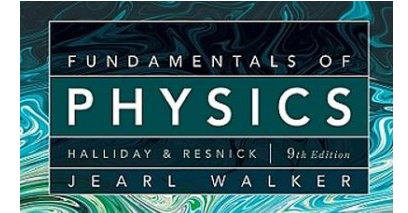
- Identifying physical quantities in the abstract space
  - Uniquely identified after the assumed principles + heuristic arguments

Causal patch in $dS$	Emergent particle in the Abstract space	Equation
Field value $\phi$	Momentum $k$	
Potential $V_\phi$	Kinetic energy $E_k$	$E_k = \frac{4\pi}{3H^3} V_\phi$
Background energy $V_0$	Mass $M$	$M = \frac{4\pi M_P^2}{H}$
Potential slope $V'_\phi$		

# The general formalism

Work-energy theorem:

$$E_k = W = \int F dx = \int v(k) dk$$



- Identifying physical quantities in the abstract space
  - Uniquely identified after the assumed principles + heuristic arguments

Causal patch in $dS$	Emergent particle in the Abstract space	Equation
Field value $\phi$	Momentum $k$	$k = \frac{4\pi M_P m_\phi}{\sqrt{3}H^2} \phi$
Potential $V_\phi$	Kinetic energy $E_k$	$E_k = \frac{4\pi}{3H^3} V_\phi$
Background energy $V_0$	Mass $M$	$M = \frac{4\pi M_P^2}{H}$
Potential slope $V'_\phi$	Velocity $v$	$v = \frac{1}{\sqrt{3}M_P H m_\phi} V'_\phi$

# The general formalism

- Langevin equation of  $\phi \rightarrow$  Brownian motion of emergent particle

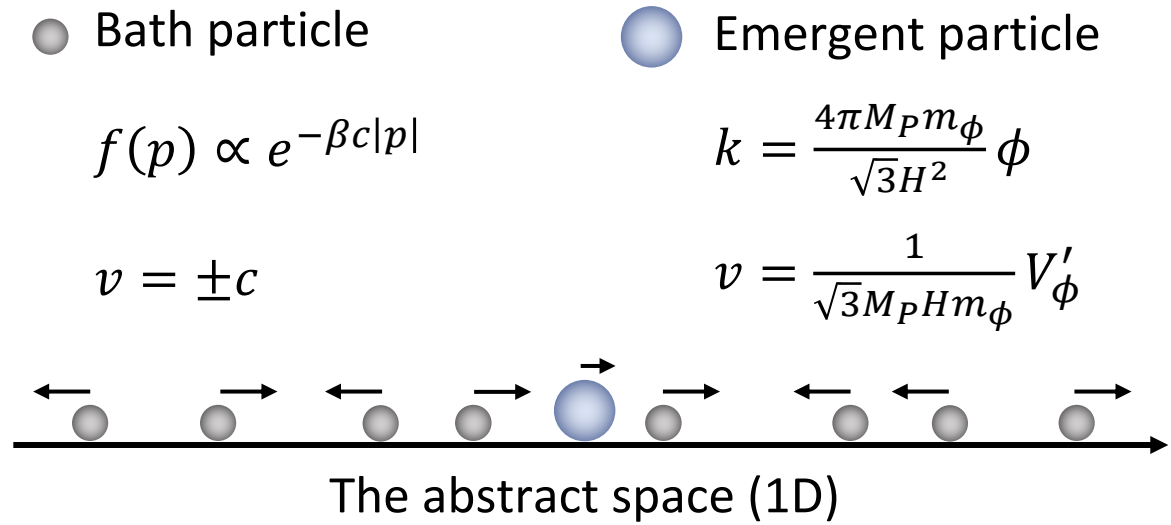
$$d\phi = -\frac{V'(\phi)}{3H} dt + \sqrt{\frac{H^3}{4\pi}} dW \quad \longrightarrow \quad dk = -\frac{4\pi M_P^2 m_\phi^2}{3H^2} v dt + \sqrt{\frac{4M_P^2 m_\phi^2}{3H}} dW$$

- Deterministic force  $\propto -v$  (drag)
- Continuous random impulses (Gaussian-distributed momentum kicks)
- Classical Brownian motion in a medium at a finite temperature
  - We build a concrete particle model of the heat bath

# Heat bath model

Unsuccessful trials:  
Massive bath particles, elastic collisions, ...

- Abstract space filled with a heat bath of another type of particle
- Successful model: massless bath particles
  - $c$ : "Speed of light (massless particles)"
  - $T$ : Bath temperature
  - $\lambda$ : Number density of bath particle
  - Bath particles are absorbed by emergent particles upon collision
    - $\sim$  photons in our world



# Heat bath model

- Pursuing the “kinetic theory of particles”
  - Randomly distributed and moving bath particles, colliding with emergent particles
  - I will show the results only (derivation in the backup slides)
- Momentum conservation at each collision
  - Deterministic force  $\propto -v$ 
    - Average collision rate for each direction becomes asymmetric
  - Gaussian-distributed random kicks
    - Random nature of bath particles + central limit theorem applies for  $\Delta t \gtrsim \frac{H^2}{M_P^2} \times \frac{1}{H}$

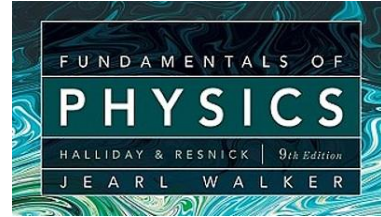
# Heat bath model

- In equation,

$$\Delta k = - \left[ 2\lambda \int_0^\infty p f(p) dp \right] v \Delta t + \left[ 2\lambda c \int_0^\infty p^2 f(p) dp \right]^{1/2} \Delta W$$

- Desired thermal motion. Adjusting the model parameters can fit the coefficients.
  - We may claim a thermal interpretation (1<sup>st</sup> objective)? But...
- Three model parameters, two equations from coefficients
  - Underconstrained?

# Heat bath model



- Energy conservation?
  - We assumed bath particles are absorbed (perfectly inelastic); kinetic energy loss
  - But energy conservation is expected once we assumed the usual classical physics in the abstract space. Where should it go?
- Energy postulation
  - In the abstract space, the Hubble expansion is exponential creation of massive particles; requires continuous energy gain from somewhere.

# Heat bath model

- We *postulate* that the lost kinetic energy amounts the energy required for the Hubble expansion.

- In equation,

$$-\frac{\langle \Delta E \rangle}{\Delta t} \simeq 2\lambda c^2 \int_0^\infty p f(p) dp = 12\pi M_p^2$$

Postulation entered



per emergent particle.

- Total kinetic energy loss  $\propto$  number of emergent particle  $\rightarrow$  desired exponential growth
- This gives the third equation for the model parameters



# Heat bath model

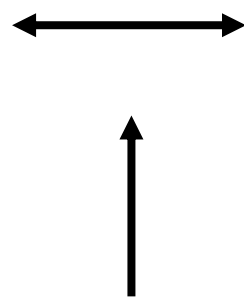
- Parameters of the successful model

$$\left. \begin{aligned} \bullet \quad c &= \frac{3H}{m_\phi} \\ \bullet \quad T &= \frac{H}{2\pi} = T_{dS} \\ \bullet \quad \lambda &= \frac{8\pi^2 M_P^2 m_\phi}{H^2} \end{aligned} \right\} \longrightarrow dk = -\frac{4\pi M_P^2 m_\phi^2}{3H^2} v dt + \sqrt{\frac{4M_P^2 m_\phi^2}{3H}} dW$$
$$\downarrow$$
$$d\phi = -\frac{V'(\phi)}{3H} dt + \sqrt{\frac{H^3}{4\pi}} dW$$

# Giving thermal interpretation

Stochastic field evolution

- Superhorizon fluctuation modes
- Quantum field evolution



Brownian motion in the abstract space

- Heat bath of massless particles
- Classical thermal motion

Emergent particle formalism

1. Dual description of scalar field
2. Usual classical physics in the abstract space

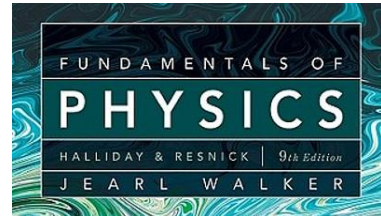
$$k = \frac{4\pi M_P m_\phi}{\sqrt{3}H^2} \phi, \quad E_k = \frac{4\pi}{3H^3} V_\phi, \quad M = \frac{4\pi M_P^2}{H}, \quad v = \frac{1}{\sqrt{3}M_P H m_\phi} V'_\phi$$

# Giving thermal interpretation

- Arrived at the same conclusion of Rigopoulos (2013) & (2016)
  - Brownian motion in a medium at  $T_{dS}$
- But our approach is simply started from sudden assumptions
  - We can always claim a thermal interpretation if we “declare” any random-walking variable as a momentum
    - “Assume that stock market price is a momentum”; “Thermal interpretation of stock market price”; “Temperature of monetary heat bath”
    - NONSENSE

# And beyond (1<sup>st</sup> slow-roll condition)

- The physical significance is given by the reappearance of other seemingly unrelated quantities and phenomena in consistent ways
- $c = \frac{3H}{m_\phi}$ : the “speed of light” in the abstract space
  - Introduced as the speed of massless bath particles
  - Would be the speed limit for massive particles once we assumed the usual classical physics in the abstract space; determines the “relativistic regime”
  - What would be the value of  $c$  when reverted to the usual field variables?



# And beyond (1<sup>st</sup> slow-roll condition)

- $v = \frac{1}{\sqrt{3}M_P H m_\phi} V'_\phi$ . What is the potential slope when  $v = c$  is reached?

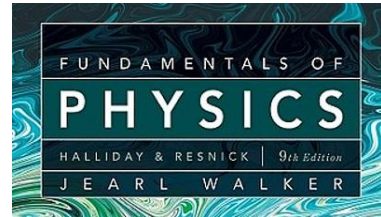
$$V'_\phi \Big|_{v=c} = 3\sqrt{3}M_P H^2$$

- 1<sup>st</sup> (potential) slow-roll parameter  $\epsilon_V = \left( V'_\phi / 3\sqrt{2}M_P H^2 \right)^2$ 
  - Surprisingly,  $V'_\phi \Big|_{v=c}$  is where  $\epsilon_V \simeq 1$  (only  $\sqrt{2/3} \approx 0.82$  difference)

# And beyond (1<sup>st</sup> slow-roll condition)

- Unexpected but consistent agreement
  - Abstract space:  $c$  is the speed of light
    - Once the classical physics is assumed,  $c$  would be the speed limit for massive particles & physics in  $v \ll c$  regime breaks down when  $v \simeq c$
  - Our space:  $\epsilon_V \simeq 1$  is the potential slope that (quasi-) dS background breaks down
  - These two are connected by  $v = \frac{1}{\sqrt{3}M_P H m_\phi} V'_\phi$  from the general formalism

$\therefore$  1<sup>st</sup> slow-roll condition is reinterpreted as the speed of light in the abstract space



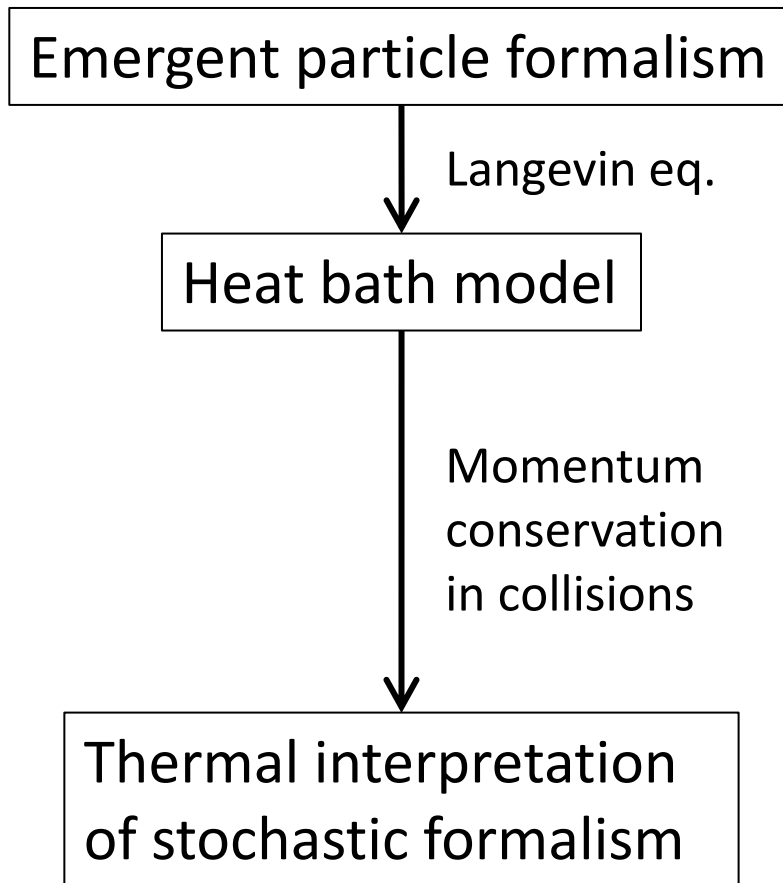
# And beyond (Hubble expansion)

- $c = \frac{3H}{m_\phi}$  relies on the energy postulation (backup slides)

- $v \simeq c \Leftrightarrow \epsilon_V \simeq 1$  is a result of the energy conservation in the abstract space

∴ Hubble expansion is reinterpreted as transfer of conserved energy in the abstract space

- Thermal interpretation is extended also to the Hubble expansion
- Particle creation should be realized with the “quantum” emergent particles



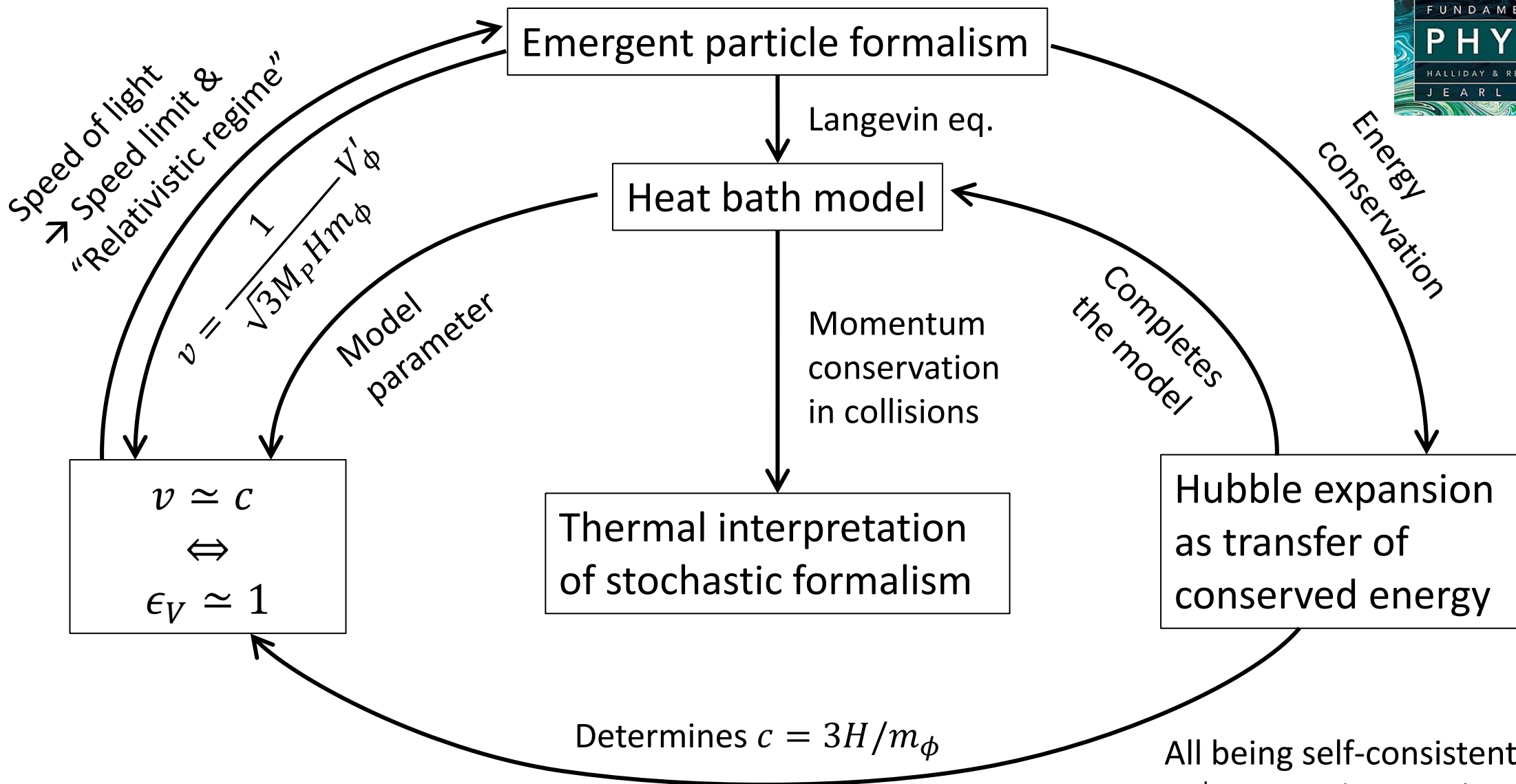
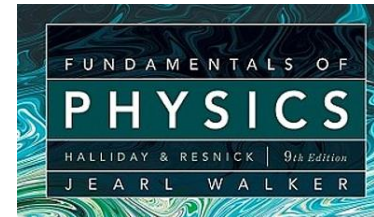
1. Dual description of scalar field
2. Usual classical mechanics in the abstract space

$$k = \frac{4\pi M_P m_\phi}{\sqrt{3}H^2} \phi, \quad E_k = \frac{4\pi}{3H^3} V_\phi$$

$$M = \frac{4\pi M_P^2}{H}, \quad v = \frac{1}{\sqrt{3}M_P H m_\phi} V'_\phi$$

Interesting, but can be done by simple “declaration”





All being self-consistent with the 2<sup>nd</sup> assumed principle; similarity to conventional thermal physics

# Discussion

- Possible future works (disclaimer: all are speculations)
  - Theoretical side
    - Any deeper connection to dS thermodynamics?
      - Reappearance of  $T_{dS}$  and  $S_{dS}$  only for horizon sized patches for emergent particles
      - But the stochastic formalism assumes an artificial sharp cutoff between UV and IR...
  - Practical side
    - Can our approach help calculating the inflationary quantities after incorporating higher order effects (deviation from exact dS)?

# Summary

- Stochastic formalism for slow-rolling fields in inflation has similarities with thermodynamics but not a thermal effect.
- We arrived at the thermal interpretation through the emergent particle formalism and the Heat bath model.
- Consistent reinterpretation of the 1<sup>st</sup> slow-roll condition and the Hubble expansion are also achieved, suggesting the physical significance.

# Thank you for the attention!

THK; 2310.15216 [gr-qc]