Emergent particles of a dS universe:

Thermal interpretation of the stochastic formalism and beyond

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dS universe
= flat slicings of dS space (in this talk)

Abstract

- Thermal interpretation of the stochastic evolution of scalar field in dS universe in superhorizon (IR) regime ("stochastic formalism") is given
- Further physical significance of this interpretation is found through reinterpretation of the 1^{st} slow-roll condition and the Hubble expansion

Outline

- Introduction
  - Stochastic formalism of slow-rolling scalar field in inflation
- Giving thermal interpretation
  - General formalism / Heat bath model
- And beyond
  - 1^{st} slow-roll condition / Hubble expansion
- Discussion and Conclusion

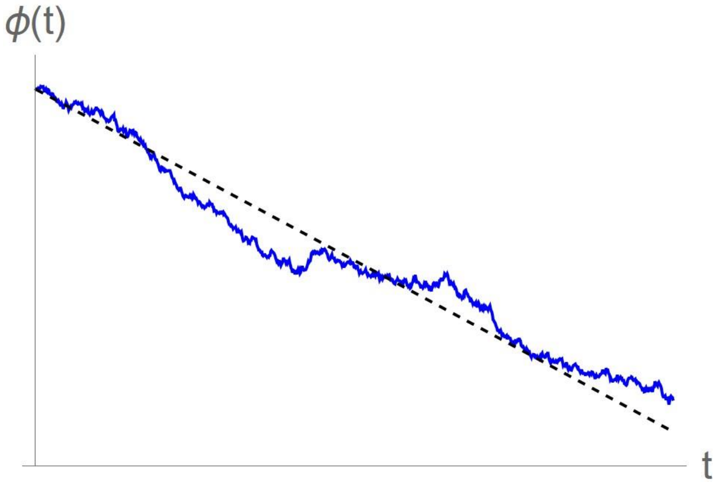
Starobinsky (1986)
Starobinsky, Yokoyama (1994)

Introduction

Polarski, Starobinsky (1996)
Lesgourgues, Polarski, Starobinsky (1997)
Kiefer, Polarski (1998)
Kiefer, Polarski (2009)

- The stochastic formalism
  - Effective theory of a slow-rolling scalar field in inflation
    - Wave number below a cutoff scale at $k = \epsilon a H$ ($\epsilon \ll 1$); "coarse-grained field"
  - Classicality of superhorizon modes
    - Measurement outcome of quantum state = Ensemble of classical random fields
  - Langevin equation ("classical" random evolution)

$$d\phi = -\frac{V'(\phi)}{3H}dt + \sqrt{\frac{H^3}{4\pi}}\, dW$$



Examples of papers mentioning intuitive arguments
Graham, Scherlis (2018)
Arvanitaki, Dimopoulos, Galanis, Racco, Simon, Thompson (2021)
Giudice, McCullough, You (2021)
Jung, Kim (2022)
...

Introduction

- Some similarities with dS thermodynamics:

  Appearance of $T_{dS} = H/2\pi$ in several points

  - Intuitive understanding of the formalism and result in particle physics literature
  - $\langle \Delta\phi^2 \rangle$ per Hubble time $\sim T_{dS}$
  - $d\rho/d\ln k$ at horizon crossing $\sim T_{dS}^4$
  - $\langle V(\phi) \rangle$ after reaching the equilibrium $\sim T_{dS}^4$

Introduction

Finelli, Marozzi, Starobinsky, Vacca, Venturi (2009)
Rigopoulos (2013) & (2016)

- But not actually a thermal effect
  - Not a result of the cosmological horizon associated with a local observer
    - Stochastic formalism: field is not described by a local observer.
      - Coarse-grained field in the "superhorizon limit"; not even at the horizon scale
      - Origin of the 'randomness' (~thermal) is different
  - Spin dependence, resultant spectrum, …
  - Appearance of T_{dS} in stochastic formalism should be understood as a result of the single-scale background; H

Introduction

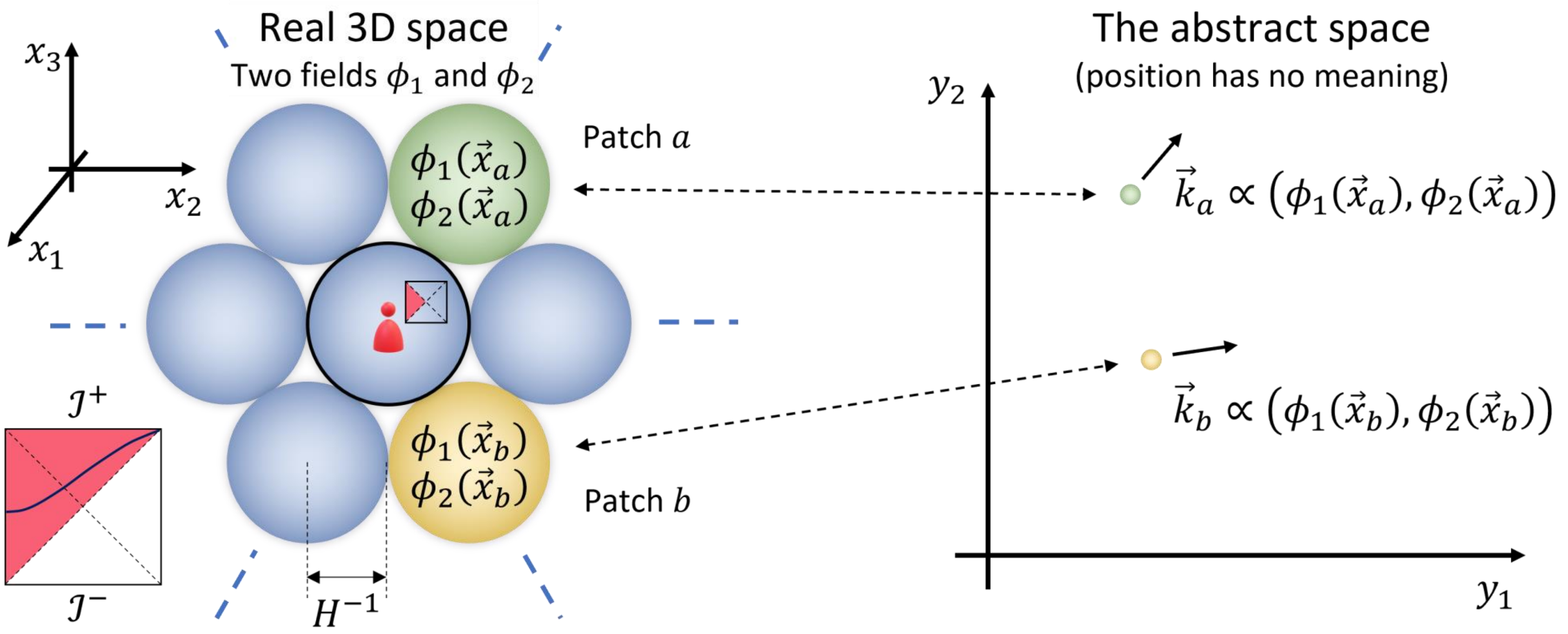
- Thermal interpretation by Rigopoulos (2013) & (2016)
  - Effective action of superhorizon modes has a term responsible for stochastic force
    - Schwinger-Keldysh formalism; effective action expressed in the Keldysh basis
  - Satisfies the fluctuation-dissipation relation with the Hubble friction at T_{dS}
  - Interprets stochastic evolution as Brownian motion in a medium at T_{dS}

Introduction

- Can we reach the same conclusion from physical considerations?
  - Proposing a general formalism & building a concrete heat bath model
  - We consistently make analogies to familiar physics, without introducing any unconventional relations
  - Arrive at the desired thermal interpretation and discover further physical significance
  - “Similarity between coarse-grained scalar fields in dS and conventional thermal physics”

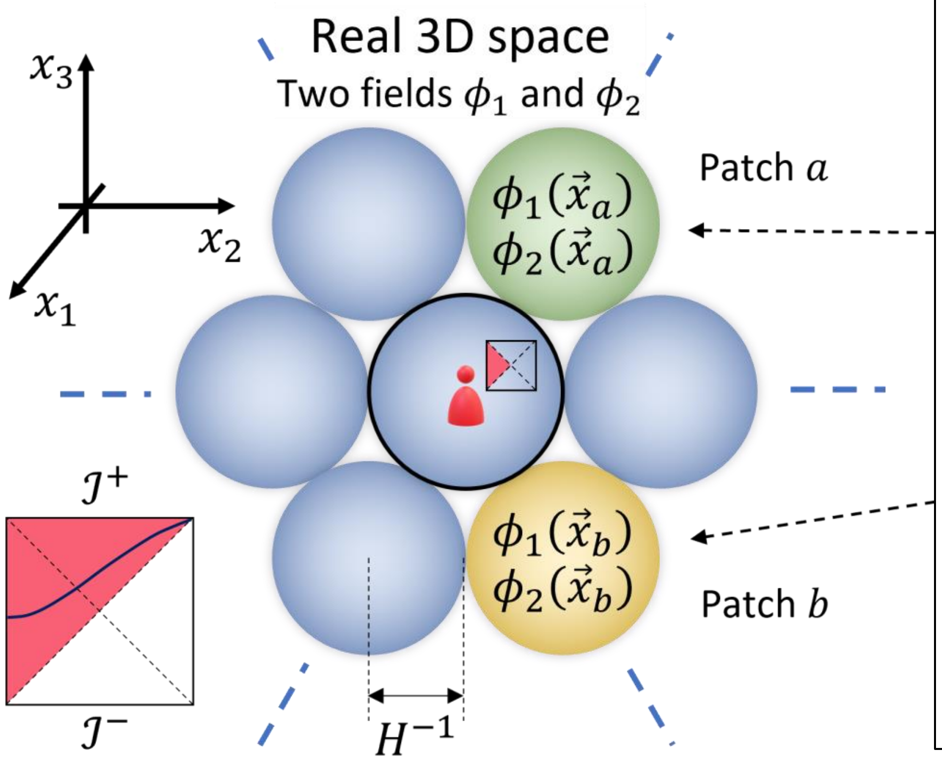
The general formalism

- We treat horizon-sized spatial regions as "particles" in a virtual space



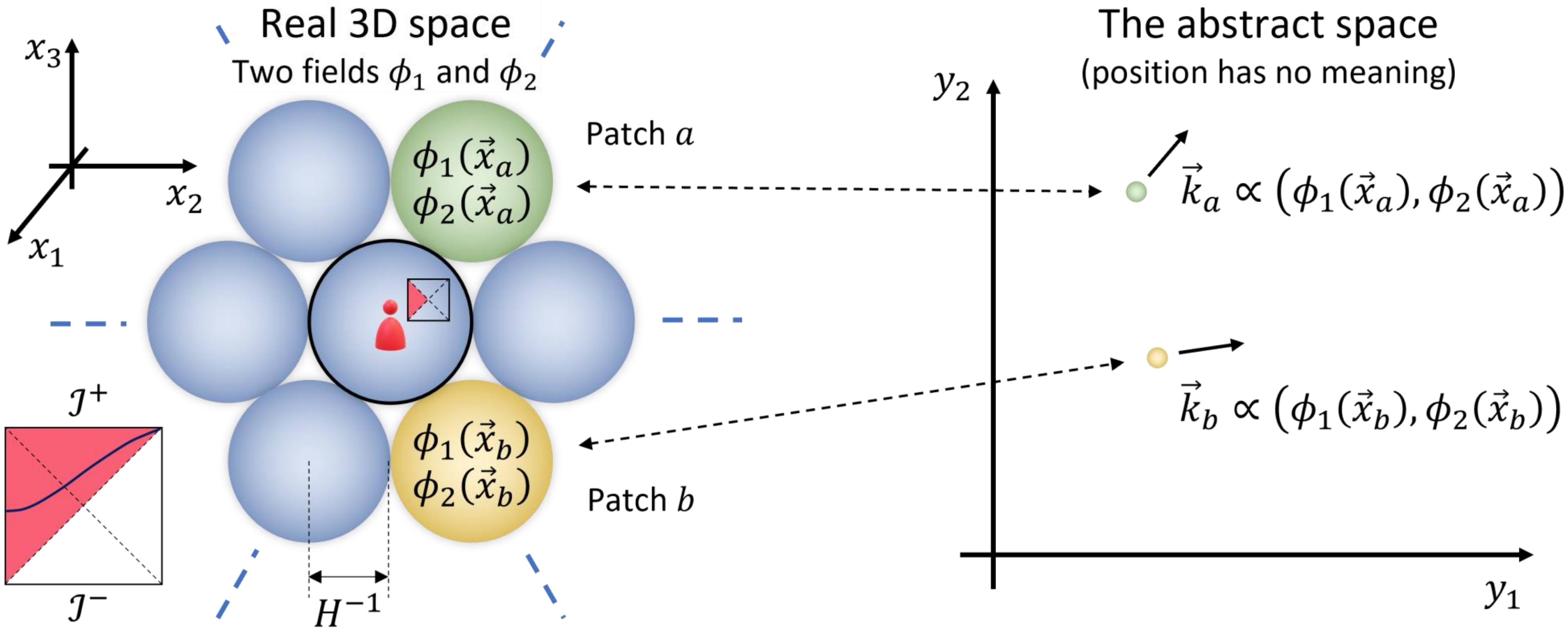
Causal patch
$\simeq$ Hubble volume

The general formalism



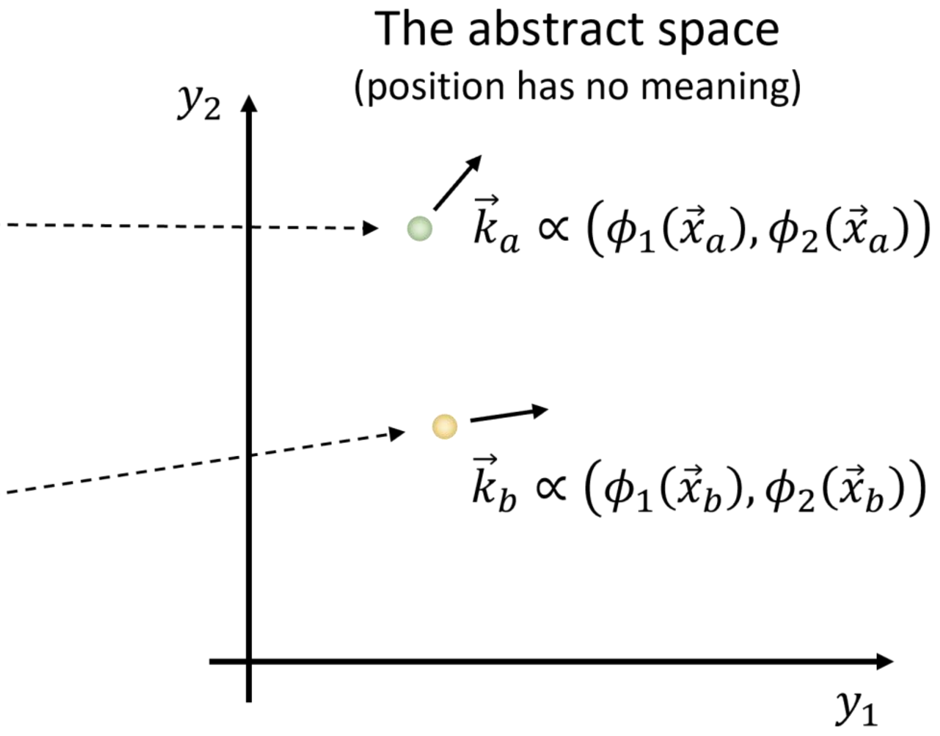
- The formalism depicts the space composed of many causal patches
  - "Horizon in an objective manner"
  - Note: We are developing a formalism, NOT a new theory!
- Each patch has its field value for the coarse-grained fields

The general formalism



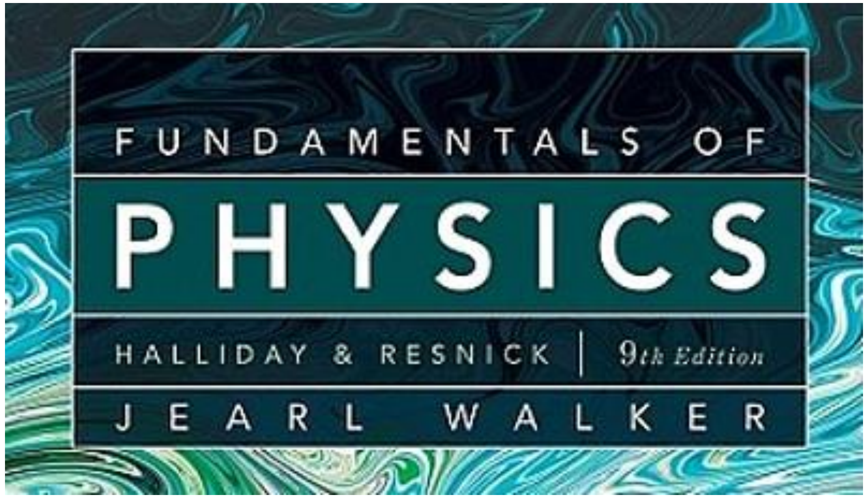
The general formalism

- Each patch is regarded as a particle in another space
  - “Emergent particle”
  - “The abstract space”

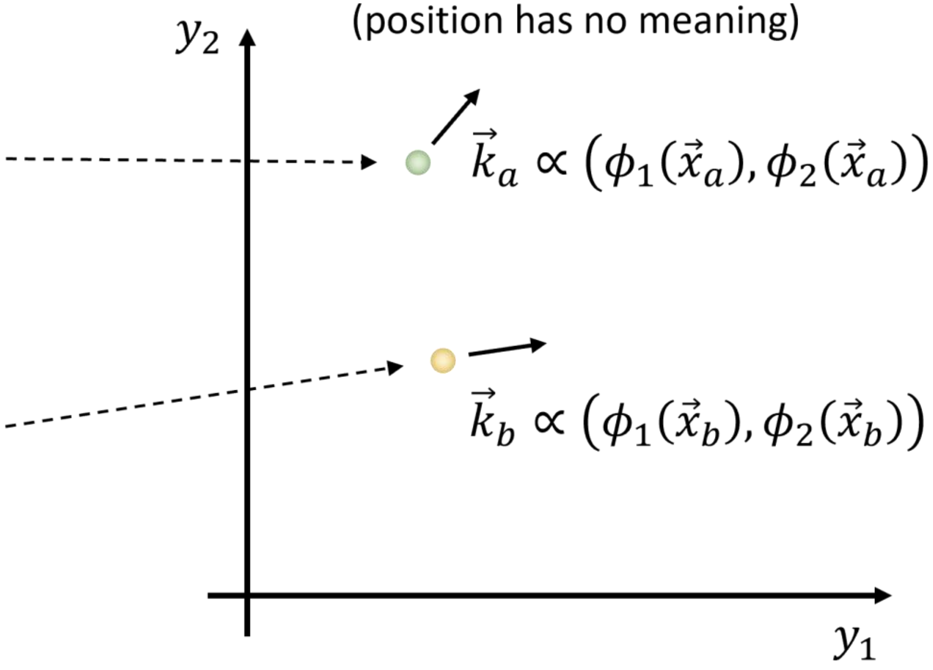


The general formalism

- Two principles assumed:
  1. Field value ∝ momentum
     - “Dual description”
  2. The usual classical physics in the abstract space
     - Set of relations between physical quantities in there



The abstract space
(position has no meaning)



The general formalism

- Translate several aspects of dS universe into the dynamics in the abstract space relying only on the assumed two principles
  - Stochastic field evolution + 1st slow-roll condition & Hubble expansion
- "Minimal non-minimal" setup
  - $V_0 = 3M_P^2 H^2$: Unspecified background energy density
  - ϕ: One real minimally coupled slow-rolling spectator scalar field (coarse-grained)
  - LOG, we assume that V_ϕ has a global minimum with nonzero mass.

    WLOG, $V_\phi = \frac{1}{2} m_\phi^2 \phi^2 + \cdots$; $\phi = 0$ to be the global minimum; $V_\phi(0) = 0$

The general formalism

- Identifying physical quantities in the abstract space
  - Uniquely identified after the assumed principles + heuristic arguments

Causal patch in dS	Emergent particle in the Abstract space	Equation
Field value ϕ	Momentum k	
Potential V_ϕ		
Background energy V_0		
Potential slope V'_ϕ		

The general formalism

- Identifying physical quantities in the abstract space
  - Uniquely identified after the assumed principles + heuristic arguments

Causal patch in dS	Emergent particle in the Abstract space	Equation
Field value ϕ	Momentum k	
Potential V_ϕ	Kinetic energy E_k	
Background energy V_0	Mass M	
Potential slope V'_ϕ		

Volume factor for conversion:
Volume of a patch = $4\pi/3H^3$

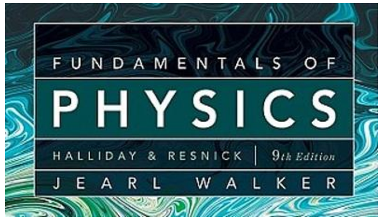
The general formalism

- Identifying physical quantities in the abstract space
  - Uniquely identified after the assumed principles + heuristic arguments

Causal patch in dS	Emergent particle in the Abstract space	Equation
Field value ϕ	Momentum k	
Potential V_ϕ	Kinetic energy E_k	$E_k = \frac{4\pi}{3H^3} V_\phi$
Background energy V_0	Mass M	$M = \frac{4\pi M_P^2}{H}$
Potential slope V'_ϕ		

Work-energy theorem:

$E_k = W = \int F\,dx = \int v(k)dk$



The general formalism

- Identifying physical quantities in the abstract space
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Causal patch in dS	Emergent particle in the Abstract space	Equation
Field value ϕ	Momentum k	$k = \frac{4\pi M_P m_\phi}{\sqrt{3}H^2}\phi$
Potential V_ϕ	Kinetic energy E_k	$E_k = \frac{4\pi}{3H^3}V_\phi$
Background energy V_0	Mass M	$M = \frac{4\pi M_P^2}{H}$
Potential slope V'_ϕ	Velocity v	$v = \frac{1}{\sqrt{3}M_P H m_\phi}V'_\phi$

The general formalism

- Langevin equation of ϕ → Brownian motion of emergent particle

$$d\phi = -\frac{V'(\phi)}{3H}dt + \sqrt{\frac{H^3}{4\pi}}\,dW \quad \longrightarrow \quad dk = -\frac{4\pi M_P^2 m_\phi^2}{3H^2}\,v\,dt + \sqrt{\frac{4M_P^2 m_\phi^2}{3H}}\,dW$$

  - Deterministic force $\propto -v$ (drag)
  - Continuous random impulses (Gaussian-distributed momentum kicks)
- Classical Brownian motion in a medium at a finite temperature
  - We build a concrete particle model of the heat bath

Unsuccessful trials:
Massive bath particles, elastic collisions, …

Heat bath model

- Abstract space filled with a heat bath of another type of particle
- Successful model: massless bath particles
  - c: "Speed of light (massless particles)"
  - T: Bath temperature
  - λ: Number density of bath particle
  - Bath particles are absorbed by emergent particles upon collision
    - ~ photons in our world

Bath particle

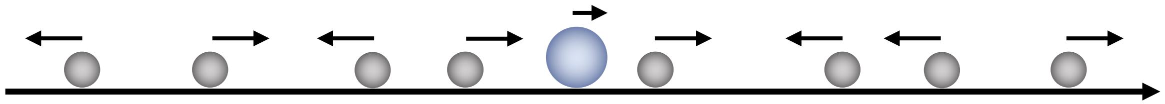
$$f(p) \propto e^{-\beta c|p|}$$

$$v = \pm c$$

Emergent particle

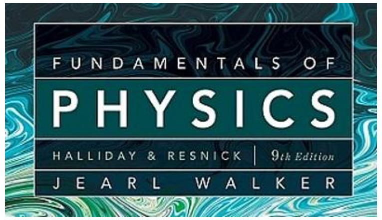
$$k = \frac{4\pi M_P m_\phi}{\sqrt{3}H^2}\phi$$

$$v = \frac{1}{\sqrt{3}M_P H m_\phi} V'_\phi$$



The abstract space (1D)

Heat bath model



- Pursuing the “kinetic theory of particles”
  - Randomly distributed and moving bath particles, colliding with emergent particles
  - I will show the results only (derivation in the backup slides)
- Momentum conservation at each collision
  - Deterministic force $\propto -v$
    - Average collision rate for each direction becomes asymmetric
  - Gaussian-distributed random kicks
    - Random nature of bath particles + central limit theorem applies for $\Delta t \gtrsim \frac{H^2}{M_P^2} \times \frac{1}{H}$

Heat bath model

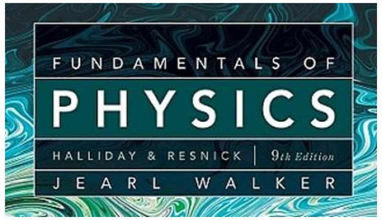
- In equation,

$$\Delta k = -\left[2\lambda \int_0^\infty p f(p) dp\right] v\, \Delta t + \left[2\lambda c \int_0^\infty p^2 f(p) dp\right]^{1/2} \Delta W$$

- Desired thermal motion. Adjusting the model parameters can fit the coefficients.
  - We may claim a thermal interpretation (1st objective)? But…

- Three model parameters, two equations from coefficients
  - Underconstrained?

Heat bath model



- Energy conservation?
  - We assumed bath particles are absorbed (perfectly inelastic); kinetic energy loss
  - But energy conservation is expected once we assumed the usual classical physics in the abstract space. Where should it go?
- Energy postulation
  - In the abstract space, the Hubble expansion is exponential creation of massive particles; requires continuous energy gain from somewhere.

Heat bath model

- We *postulate* that the lost kinetic energy amounts the energy required for the Hubble expansion.
- In equation,

Postulation entered

$$-\frac{\langle \Delta E \rangle}{\Delta t} \simeq 2\lambda c^2 \int_0^\infty p f(p) dp = 12\pi M_P^2$$

per emergent particle.

  - Total kinetic energy loss $\propto$ number of emergent particle → desired exponential growth
- This gives the third equation for the model parameters

Heat bath model

- Parameters of the successful model
  - $c = \frac{3H}{m_\phi}$
  - $T = \frac{H}{2\pi} = T_{dS}$
  - $\lambda = \frac{8\pi^2 M_P^2 m_\phi}{H^2}$

$$\longrightarrow \quad dk = -\frac{4\pi M_P^2 m_\phi^2}{3H^2}\, v\, dt + \sqrt{\frac{4M_P^2 m_\phi^2}{3H}}\, dW$$

$$\downarrow$$

$$d\phi = -\frac{V'(\phi)}{3H} dt + \sqrt{\frac{H^3}{4\pi}}\, dW$$

Giving thermal interpretation

Stochastic field evolution ⟷ Brownian motion in the abstract space

- Superhorizon fluctuation modes
- Quantum field evolution

- Heat bath of massless particles
- Classical thermal motion

↑

Emergent particle formalism

1. Dual description of scalar field
2. Usual classical physics in the abstract space

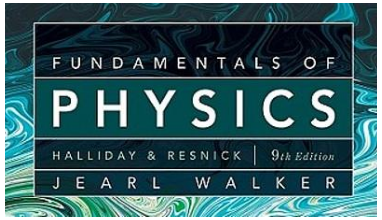
$$k = \frac{4\pi M_P m_\phi}{\sqrt{3}H^2}\phi, \qquad E_k = \frac{4\pi}{3H^3}V_\phi, \qquad M = \frac{4\pi M_P^2}{H}, \qquad v = \frac{1}{\sqrt{3}M_P H m_\phi}V'_\phi$$

Giving thermal interpretation

- Arrived at the same conclusion of Rigopoulos (2013) & (2016)
  - Brownian motion in a medium at T_{dS}
- But our approach is simply started from sudden assumptions
  - We can always claim a thermal interpretation if we “declare” any random-walking variable as a momentum
    - “Assume that stock market price is a momentum”; “Thermal interpretation of stock market price”; “Temperature of monetary heat bath”
    - NONSENSE

And beyond (1st slow-roll condition)

- The physical significance is given by the reappearance of other seemingly unrelated quantities and phenomena in consistent ways
- $c = \frac{3H}{m_\phi}$: the "speed of light" in the abstract space
  - Introduced as the speed of massless bath particles
  - Would be the speed limit for massive particles once we assumed the usual classical physics in the abstract space; determines the "relativistic regime"
  - What would be the value of c when reverted to the usual field variables?



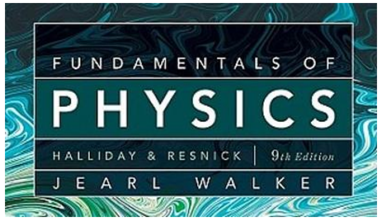
And beyond (1st slow-roll condition)

- $v = \frac{1}{\sqrt{3}M_P H m_\phi} V'_\phi$. What is the potential slope when $v = c$ is reached?

$$V'_\phi\Big|_{v=c} = 3\sqrt{3}M_P H^2$$

- 1st (potential) slow-roll parameter $\epsilon_V = \left(V'_\phi/3\sqrt{2}M_P H^2\right)^2$
  - Surprisingly, $V'_\phi\Big|_{v=c}$ is where $\epsilon_V \simeq 1$ (only $\sqrt{2/3} \approx 0.82$ difference)

And beyond (1st slow-roll condition)



- Unexpected but consistent agreement
  - Abstract space: c is the speed of light
    - Once the classical physics is assumed, c would be the speed limit for massive particles & physics in $v \ll c$ regime breaks down when $v \simeq c$
  - Our space: $\epsilon_V \simeq 1$ is the potential slope that (quasi-) dS background breaks down
  - These two are connected by $v = \frac{1}{\sqrt{3}M_P H m_\phi} V'_\phi$ from the general formalism

∴ 1st slow-roll condition is reinterpreted as the speed of light in the abstract space

And beyond (Hubble expansion)

- $c = \frac{3H}{m_\phi}$ relies on the energy postulation (backup slides)
  - $v \simeq c \Leftrightarrow \epsilon_V \simeq 1$ is a result of the energy conservation in the abstract space

$\therefore$ Hubble expansion is reinterpreted as transfer of conserved energy in the abstract space

  - Thermal interpretation is extended also to the Hubble expansion
  - Particle creation should be realized with the "quantum" emergent particles

Emergent particle formalism

$\downarrow$ Langevin eq.

Heat bath model

$\downarrow$ Momentum conservation in collisions

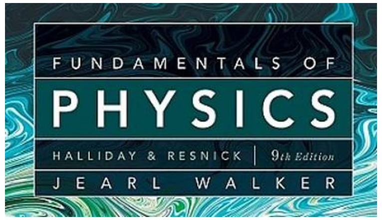
Thermal interpretation of stochastic formalism

1. Dual description of scalar field
2. Usual classical mechanics in the abstract space

$$k = \frac{4\pi M_P m_\phi}{\sqrt{3}H^2}\phi, \qquad E_k = \frac{4\pi}{3H^3}V_\phi$$

$$M = \frac{4\pi M_P^2}{H}, \qquad v = \frac{1}{\sqrt{3}M_P H m_\phi}V'_\phi$$

Interesting, but can be done by simple “declaration”



Emergent particle formalism

Langevin eq.

Heat bath model

Momentum conservation in collisions

Thermal interpretation of stochastic formalism

Speed of light → Speed limit & "Relativistic regime"

$v = \frac{1}{\sqrt{3}M_P H m_\phi} V'_\phi$

Model parameter

$v \simeq c$

$\Leftrightarrow$

$\epsilon_V \simeq 1$

Energy conservation

Completes the model

Hubble expansion as transfer of conserved energy

Determines $c = 3H/m_\phi$

All being self-consistent with the 2nd assumed principle; similarity to conventional thermal physics

Discussion

- Possible future works (disclaimer: all are speculations)
  - Theoretical side
    - Any deeper connection to dS thermodynamics?
      - Reappearance of T_{dS} and S_{dS} only for horizon sized patches for emergent particles
      - But the stochastic formalism assumes an artificial sharp cutoff between UV and IR…
  - Practical side
    - Can our approach help calculating the inflationary quantities after incorporating higher order effects (deviation from exact dS)?

Summary

- Stochastic formalism for slow-rolling fields in inflation has similarities with thermodynamics but not a thermal effect.
- We arrived at the thermal interpretation through the emergent particle formalism and the Heat bath model.
- Consistent reinterpretation of the 1^{st} slow-roll condition and the Hubble expansion are also achieved, suggesting the physical significance.

Thank you for the attention!

THK; 2310.15216 [gr-qc]