The Supersymmetric Gegenbauer's Twin

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Work in progress

High1 Workshop on Particle, String and Cosmology
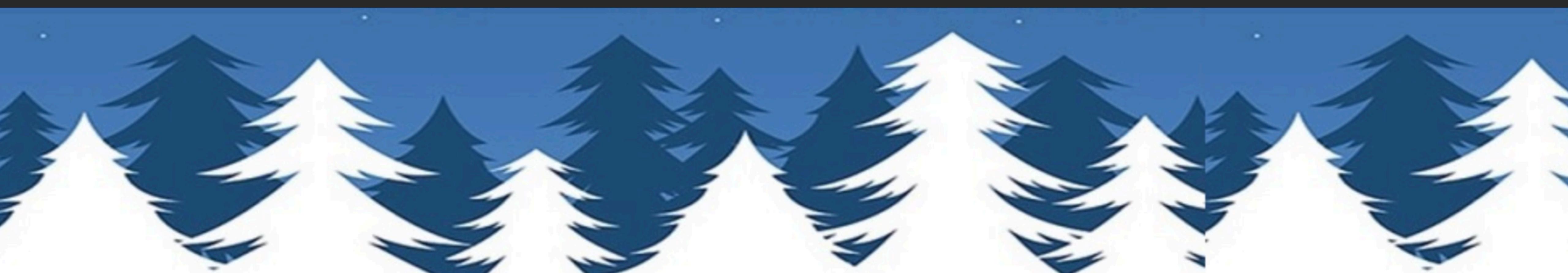
Gegenbauer Goldstones

$\pi \longrightarrow \langle q\bar{q} \rangle$

In **QCD** pions are light due to an approx $SU(2)_L \times SU(2)_R$

Quark masses are the only sym breaking parameters

Gegenbauer Goldstones

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$$H \longrightarrow \, ?$$

Can the higgs also be a pNGb?

Naturalness

Example: Breaking of a U(1) global symmetry

$$\mathcal{L} = \left| \partial_\mu \phi \right|^2 - \frac{\lambda}{4} \left(\left| \phi \right|^2 - \frac{f^2}{2} \right)^2$$

Associated one pNGb

An explicit sym breaking in the UV

$$V = \epsilon \frac{\lambda}{f^{q-4}} \phi^q + \mathrm{h.c}$$

Generates the potential

$$V \propto \epsilon m_\rho^2 f^2 \cos \frac{q\Pi}{f}$$

Which is **stable** as a result of symmetry

Naturalness

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UV

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IR
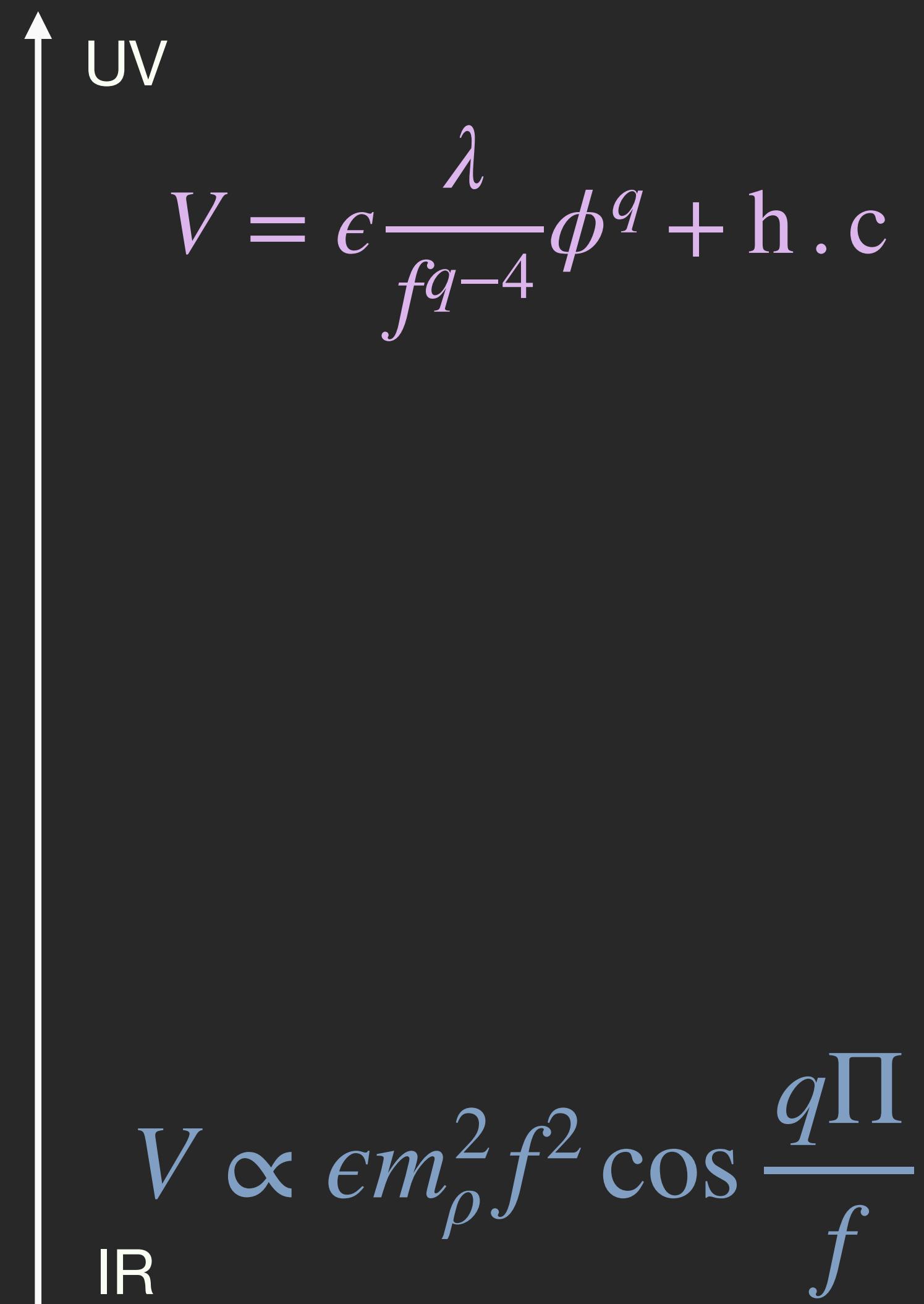
Abelian case

$$V \propto \epsilon m_\rho^2 f^2 \cos \frac{q\Pi}{f}$$

$$\langle \Pi \rangle = f\pi n / q$$

$$m^2 = \epsilon q^2 m_\rho^2$$

$$\lambda = \epsilon q^4 m_\rho^2 / f^2$$
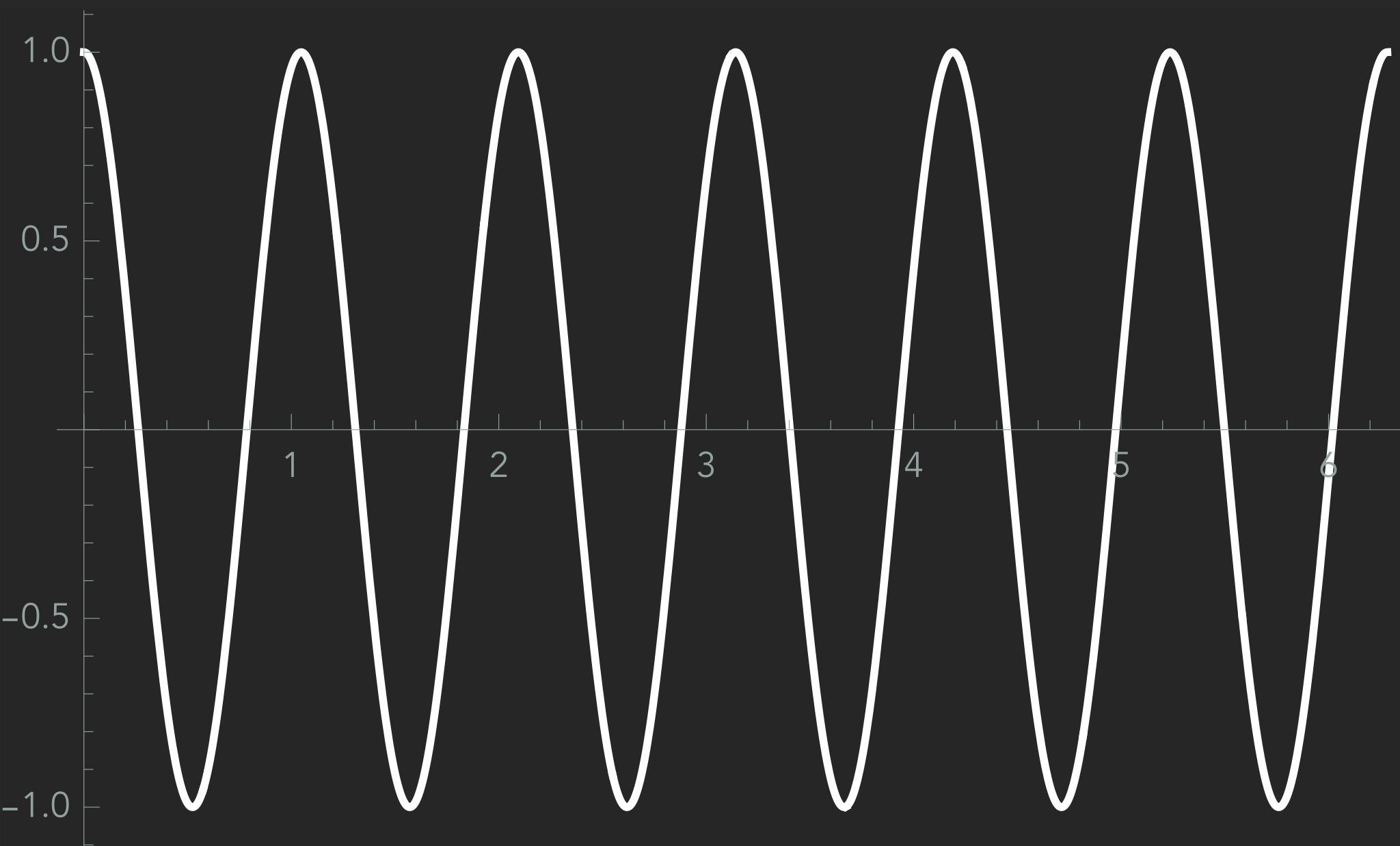
Natural small vev and mass

Non-abelian case

$$SO(N+1) \longrightarrow SO(N)$$

$$\mathscr{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{\lambda}{4}\left(\phi\cdot\phi - \frac{f^2}{2}\right)^2$$

N massless pNGbs

$$V = \frac{\lambda}{f^{n-4}}\epsilon_{a_1,a_2,\dots,a_n}\phi^{a_1}\phi^{a_2}\dots\phi^{a_n}$$

Spurion in a symmetric irrep (**Traceless**)

$$V = \epsilon m_\rho^2 f^2 G_n^{(N-1)/2}\left(\cos\Pi/f\right)$$

Radiatively stable

Non-abelian case

$$SO(N+1) \longrightarrow SO(N)$$

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Gegenbauer polynomial

Natural small vev

$$\frac{\langle\Pi\rangle}{f} \sim \frac{5.1}{n}$$
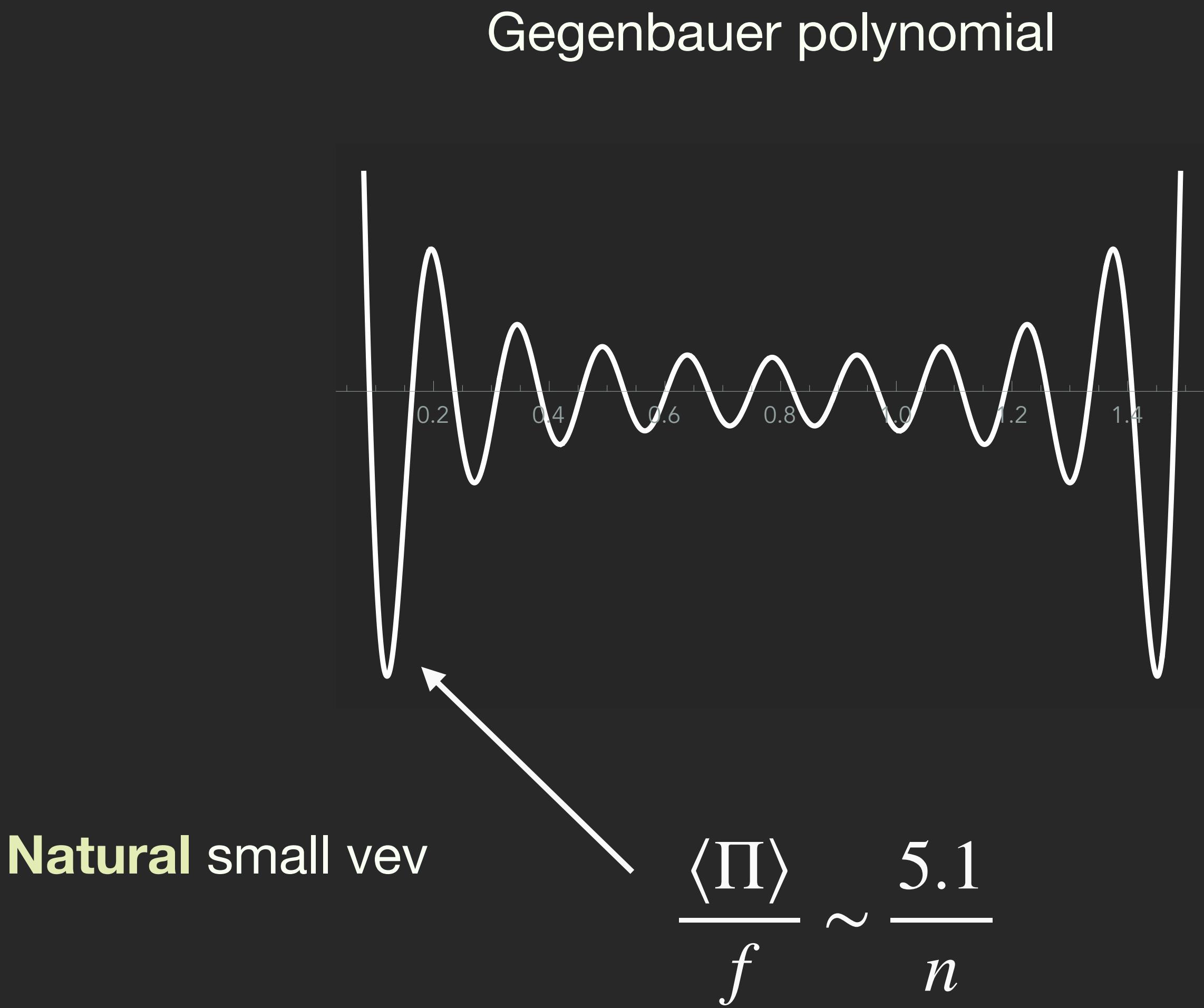
Gegenbauer as irreps

$$|t\phi - \tilde{\phi}|^{1-N} = \sum_{n=0}^{\infty} t^n K_n^{i_1 i_2 \ldots i_n} \left(\tilde{\phi} \right) \phi^{i_1} \phi^{i_2} \ldots \phi^{i_n}$$

Similar to angular momentum decomposition

$$\phi = \frac{1}{\Pi} \sin \frac{\Pi}{f} \begin{pmatrix} \Pi_1 \\ \Pi_2 \\ \vdots \\ \Pi_N \\ \cot \frac{\Pi}{f} \end{pmatrix}$$

$$\tilde{\phi} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ f \end{pmatrix}$$

$$\Pi = \sqrt{\mathbf{\Pi} \cdot \mathbf{\Pi}}$$

Gegenbauer as irreps

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Taylor expansion on t

$$K_n^{i_1 i_2 \ldots i_n} \left(\phi \right) = \frac{1}{n!} \frac{\partial^n \phi^{1-N}}{\partial \phi^{i_1} \partial \phi^{i_2} \ldots \partial \phi_{i_n}}$$

$$|t\phi - \tilde{\phi}|^{1-N} = \sqrt{1 - 2t \cos \Pi / f + t^2}$$

$$= \sum_{n=0}^{\infty} t^n G_n^{(N-1)/2} \left(\cos \Pi / f \right)$$

Generating function

$$\left(1 - 2tx + t^2 \right)^{-\lambda} = \sum_{n=0}^{\infty} t^N G_n^{\lambda}(x)$$

Gegenbauer as irreps

$$|t\phi - \tilde{\phi}|^{1-N} = \sum_{n=0}^{\infty} t^n K_n^{i_1 i_2 \ldots i_n} \left(\tilde{\phi} \right) \phi^{i_1} \phi^{i_2} \ldots \phi^{i_n}$$

Taylor expansion on t

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$$|t\phi - \tilde{\phi}|^{1-N} = \sqrt{1 - 2t \cos \Pi/f + t^2}$$

$$= \sum_{n=0}^{\infty} t^n G_n^{(N-1)/2} \left(\cos \Pi/f \right)$$

$$\left(1 - 2tx + t^2 \right)^{-\lambda} = \sum_{n=0}^{\infty} t^N G_n^\lambda (x)$$

Tuning

i) Higgs vev

$$V_T \approx \frac{3y_t^2}{16\pi^2} f^2 M_T^2 \left(a_1 \cos \frac{\Pi}{f} + a_2 \cos \frac{2\Pi}{f} \right)$$

$$\langle \Pi \rangle = 0$$

$$\langle \Pi \rangle \approx n\pi f \qquad n \in \mathbb{Z}$$

Phenomenologically not viable

$$V_G = \epsilon f^2 M^2 G_n^{3/2} \left(\cos \frac{\Pi}{f} \right)$$

Large n allows for natural $v^2 \ll f^2$

ii) Higgs mass

$$V_T \approx \frac{3y_t^2}{16\pi^2} f^2 M_T^2 \left(a_1 \cos \frac{\Pi}{f} + a_2 \cos \frac{2\Pi}{f} \right)$$

$$m_h^2 \sim \frac{3y_t^2}{16\pi^2} M_T^2$$

$M_T \gtrsim 900$ **GeV**, adds additional fine tuning $\longrightarrow$ **Gegenbauer's twin**

Twin Higgs
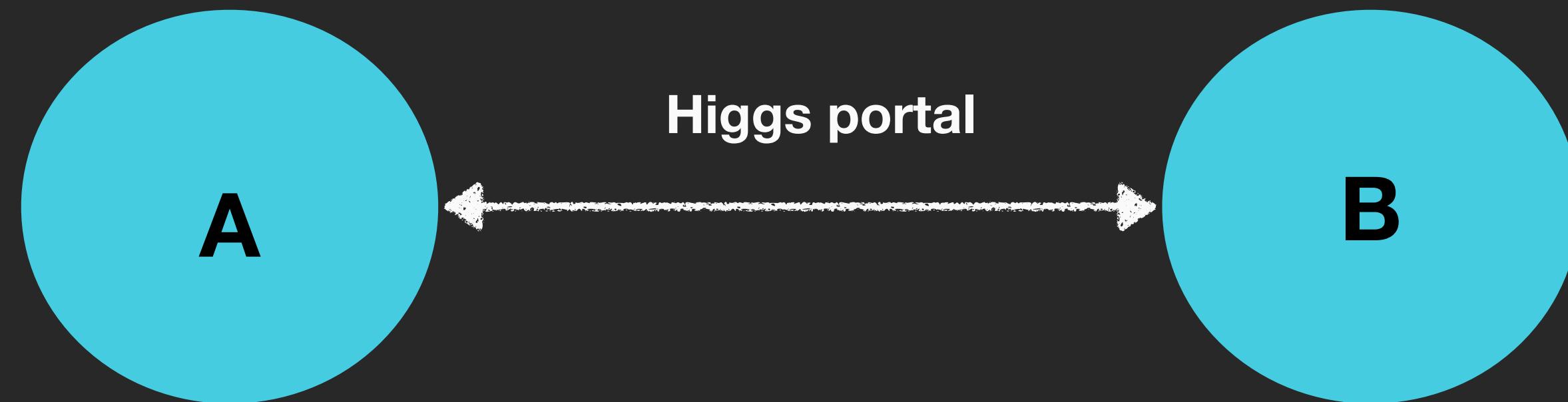
A ← Higgs portal → B

Global $SO(8)$

$$V = \lambda \left(\left| H_A \right|^2 + \left| H_B \right|^2 \right)^2 - \Lambda^2 \left(\left| H_A \right|^2 + \left| H_B \right|^2 \right)$$

Masses respect the accidental SO(8) symmetry

**Twin higgs is itself a PnGb model** $SO(8) \rightarrow SO(7)$

7 Goldstones

4 into the Higgs boson

3 into the gauge bosons of sector B

Twin Higgs

A ⟷ Higgs portal ⟷ **B**

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9
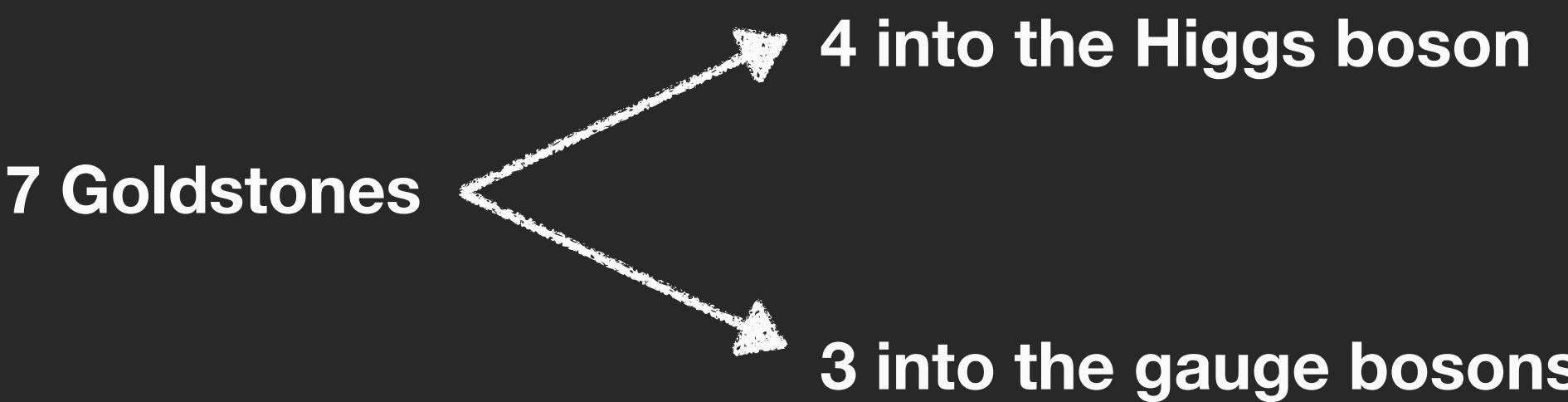
Gegenbauer's Twin

$$V_t \approx \frac{3y_t^4}{64\pi^2} f^4 \left[\sin^4 \frac{h}{f} \log \frac{a}{\sin^2 \frac{h}{f}} + \cos^4 \frac{h}{f} \log \frac{a}{\cos^2 \frac{h}{f}} \right]$$

Including the explicit symmetry breaking

$$V_G^n = \epsilon f^4 G_n^{3/2} \left(\cos \frac{2h}{f} \right)$$
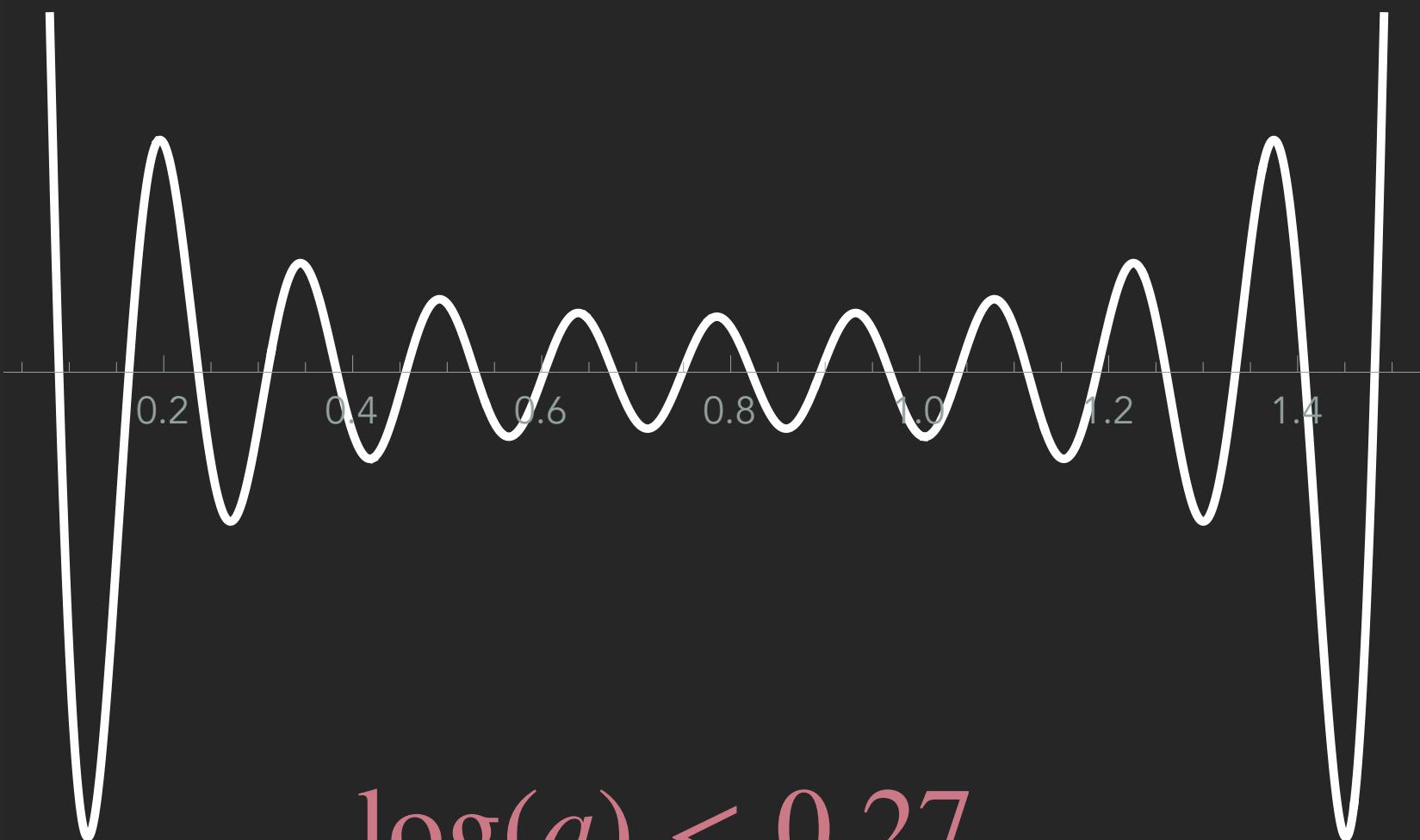
$$\log(a) < 0.27$$

Recap

Tuning

Gegenbauer's Goldstones $\longrightarrow$ Natural EW scale

Gegenbauer's Twin $\longrightarrow$ Natural Higgs mass

$\log(a) < 0.27$

iii) Couplings

Radiative corrections in the Gegenbauer's Twin

$$V_t \approx \frac{3y_t^4}{64\pi^2} f^4 \left[\sin^4 \frac{h}{f} \log \frac{a}{\sin^2 \frac{h}{f}} + \cos^4 \frac{h}{f} \log \frac{a}{\cos^2 \frac{h}{f}} \right]$$

a is a UV parameter $\qquad a \approx \frac{g_\star^2}{y_t^2} \qquad\qquad M_\star \approx g_\star f$

For a pNGb model, $g_\star \sim 4\pi$, thus, the UV is **strongly coupled**

For the Gegenbauer Higgs, $a \lesssim 2$, the UV is perturbative $\longrightarrow$ **SUSY Gegenbauer's twin**

SUSY Gegenbauer's Twin

$$W = \mu \Phi_u \Phi_d + \lambda S \Phi_u \Phi_d + MS^2$$

$$V = \left(m_{Hu}^2 + \mu^2\right)|\phi_u|^2 + \left(m_{Hd}^2 + \mu^2\right)|\phi_d|^2 - b\left(\phi_u \phi_d + h.c.\right) + \lambda^2|\phi_u \phi_d|^2$$

$\mathbb{Z}_2$ **between MSSM and its mirror** $\longrightarrow$ **approximate global U(4) symmetry**

The Gegenbauer explicitly breaks SUSY

$$V_G^{(n)} = \epsilon f^2 M^2 F(\beta) G_n^{3/2}\left(\cos\left(\frac{2h}{f}\right)\right)$$

$$\tan\beta \equiv \frac{\mu^2 + m_{Hd}^2}{\mu^2 + m_{Hu}^2}$$

SUSY Gegenbauer's Twin

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Sources of symmetry breaking

$$V_G^{(n)} = \epsilon f^2 M^2 F(\beta) G_n^{3/2}\left(\cos\left(\frac{2h}{f}\right)\right) \quad \text{Explicit}$$

$$V_{g',g} = \frac{g^2 + g''^2}{8}\left[\left(\left|\phi_u^A\right|^2 - \left|\phi_d^A\right|^2\right)^2 + \left(\left|\phi_u^B\right|^2 - \left|\phi_d^B\right|^2\right)^2\right] \quad \text{D-terms}$$

$$V_t = \frac{3y_t^4}{16\pi^2}\left(\left|\phi_u^A\right|^4 \log\frac{m_{\tilde{t}}^2}{m_t^{A2}} + \left|\phi_u^B\right|^4 \log\frac{m_{\tilde{t}}^2}{m_t^{B2}}\right) \quad \text{Top sector}$$

SUSY contribution to the pNGb potential

$$V_{\cancel{U(4)}} = \frac{3(y_t f \sin\beta)^4}{64\pi^2} \left[\sin^4\left(\frac{h}{f}\right) \log\frac{a}{\sin^2\left(\frac{h}{f}\right)} + \cos^4\left(\frac{h}{f}\right) \log\frac{a}{\cos^2\left(\frac{h}{f}\right)} \right]$$

$$a = \frac{2m_{\tilde{t}}^2}{(y_t f \sin\beta)^2} e^{\frac{2\pi^2 \cos^2(2\beta)(g'^2 + g^2)}{3(y_t \cos\beta)^4}}$$

$m_{\tilde{t}} \sim \mathcal{O}(f)$: we have a model that is a hybrid between a **SUSY** model and a **pNGb** model

Summary

✅ A pNGb potential can be decomposed in a sum of Gegenbauer polynomials

✅ The twin Gegenbauer points to a perturbative coupled UV

✅ We provided a SUSY UV completion, valid up to very small scales

✅ Since SUSY is perturbative, IR parameters can be calculated

✅ The SUSY-Gegenbauer less tuned that a SUSY model by an order of magnitude

On-going work: LHC-HL. Precision measurements vs direct searches