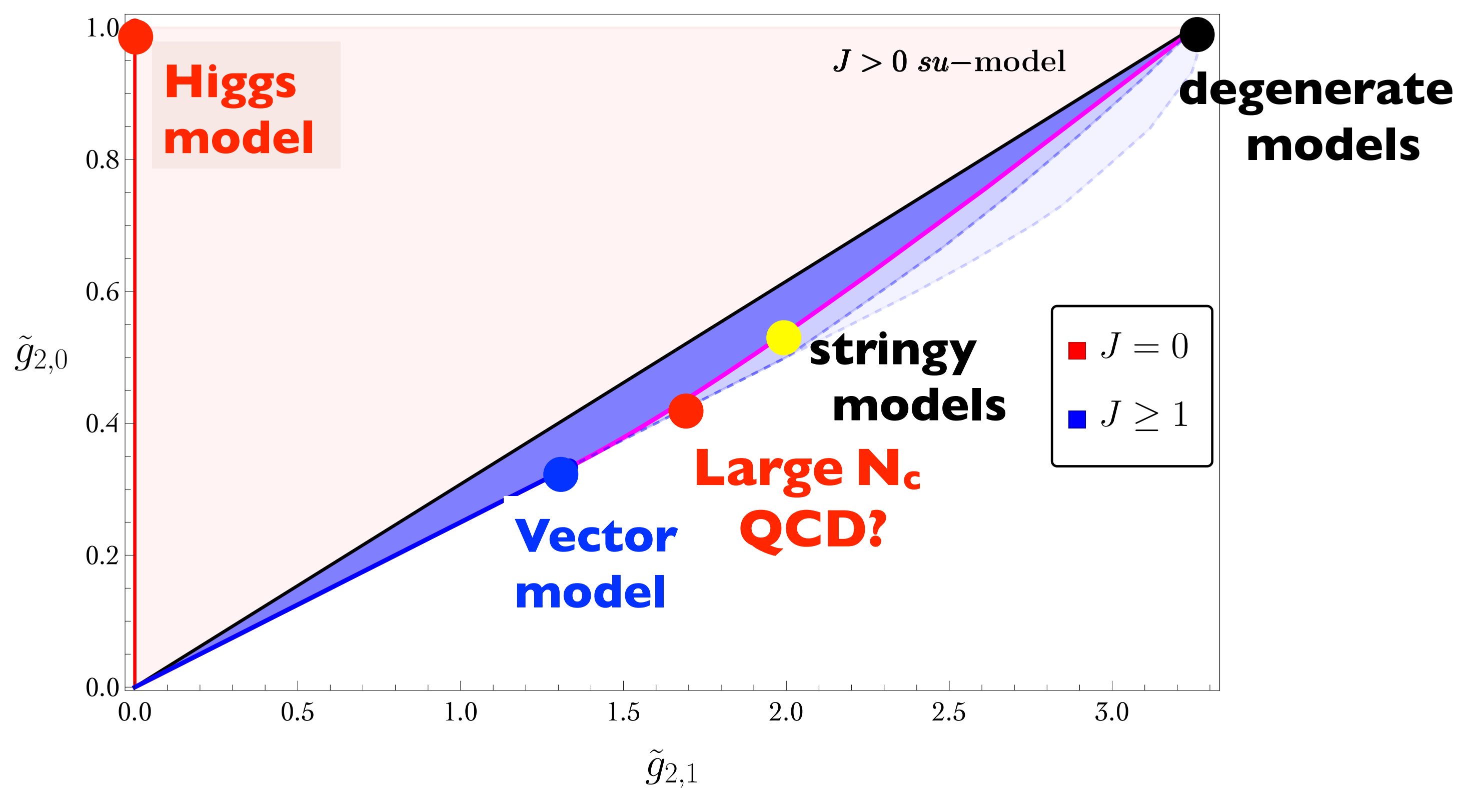
Bootstrapping Large-N_c QCD



Alex Pomarol, IFAE & UAB (Barcelona) & KIAS

based on 2211.12488 [hep-th] with C. Fernandez, F. Riva and F. Sciotti
2307.04729 [hep-th] with T. Ma and F. Sciotti

Motivation

- Understand better **Strongly-coupled theories** as plays an important role in nature, e.g. QCD

- They could also play an **important role BSM:**
  - Dark Matter
  - Hierarchy problem: Higgs composite

- To understand their physics, **simplifying techniques** are essential

Best examples:
- Taking $N_c \to \infty$ of $SU(N_c)$ (**large-N_c** limit)
- **Holography**: $CFT_4 \leftrightarrow AdS_5$

Large N_c limit

G. 't Hooft, Nucl. Phys. B 72, 461 (1974)

E. Witten, Nucl. Phys. B 160, 57 (1979)

quarks, gluons
SU($\mathbf{N_c}$)

$\xrightarrow[\mathbf{N_c} \to \infty]{}$

mesons ($q\bar{q}$ states), glueballs

weakly-coupled

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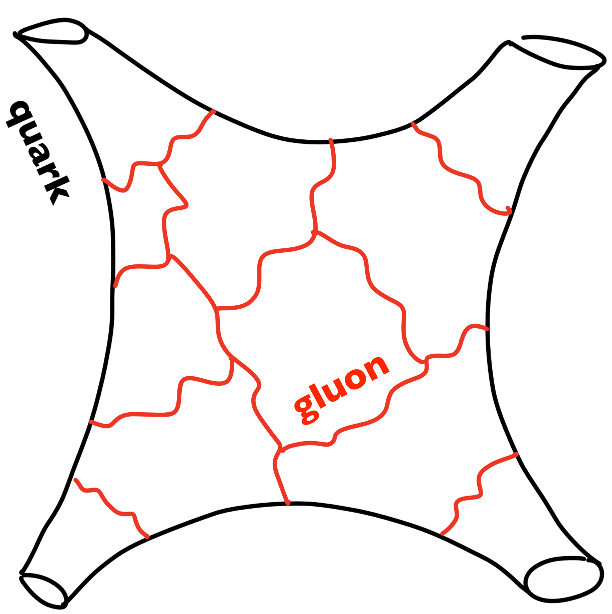
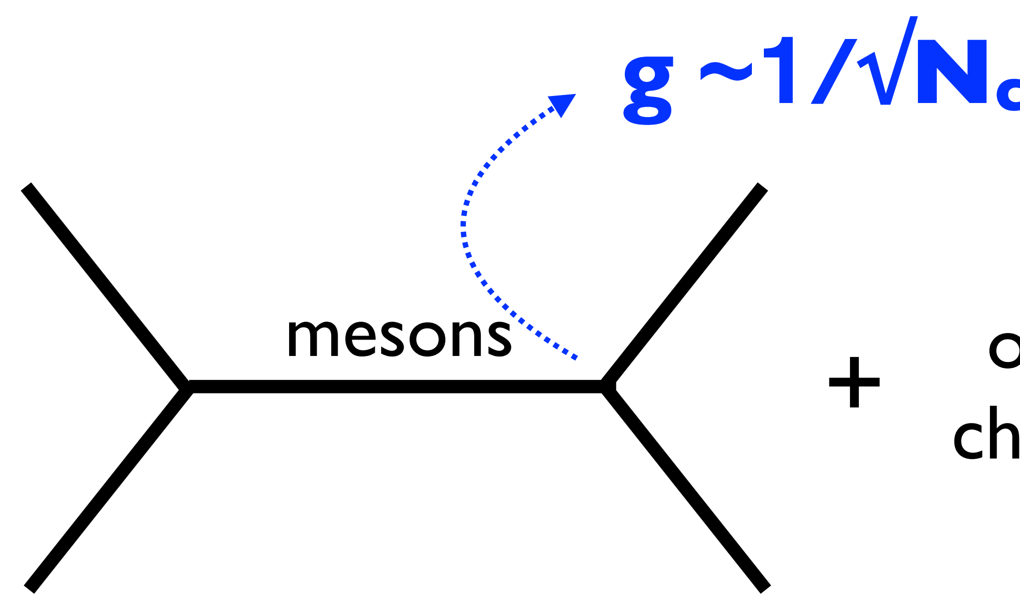


$\longrightarrow$



+ other channels

Large N_c limit

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Powerful simplification
but still difficult to get predictions

Theory of <u>infinite</u> mesons of different spin J,
with unknown couplings and masses

f_0 (J=0), ρ (J=1), f_2 (J=2), ρ_3 (J=3), …

(as in real QCD)

$$\frac{\Gamma}{m} \sim \frac{1}{3}$$

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Constrain the theory just demanding "good" properties to its amplitudes: **Lorentz, Positivity, analyticity, crossing, ...**

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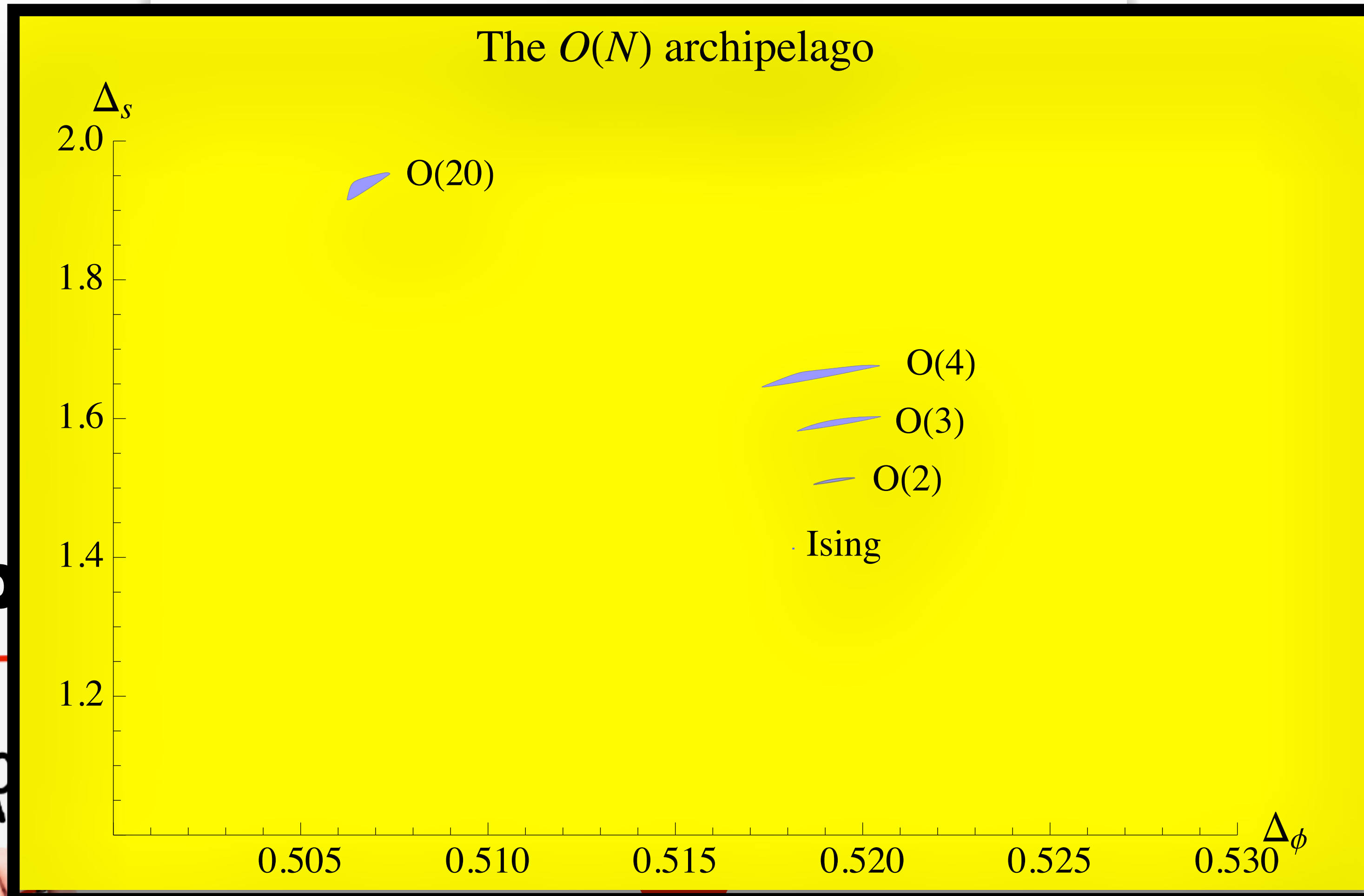

ooft, Nucl. Phys. B 72, 461 (1974)

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b ns

Constrain the theory just demanding "good" properties to its amplitudes: **Lorentz, Positivity, analyticity, crossing, ...**

), ρ_3 ($J=3$), ...

(as in real QCD)

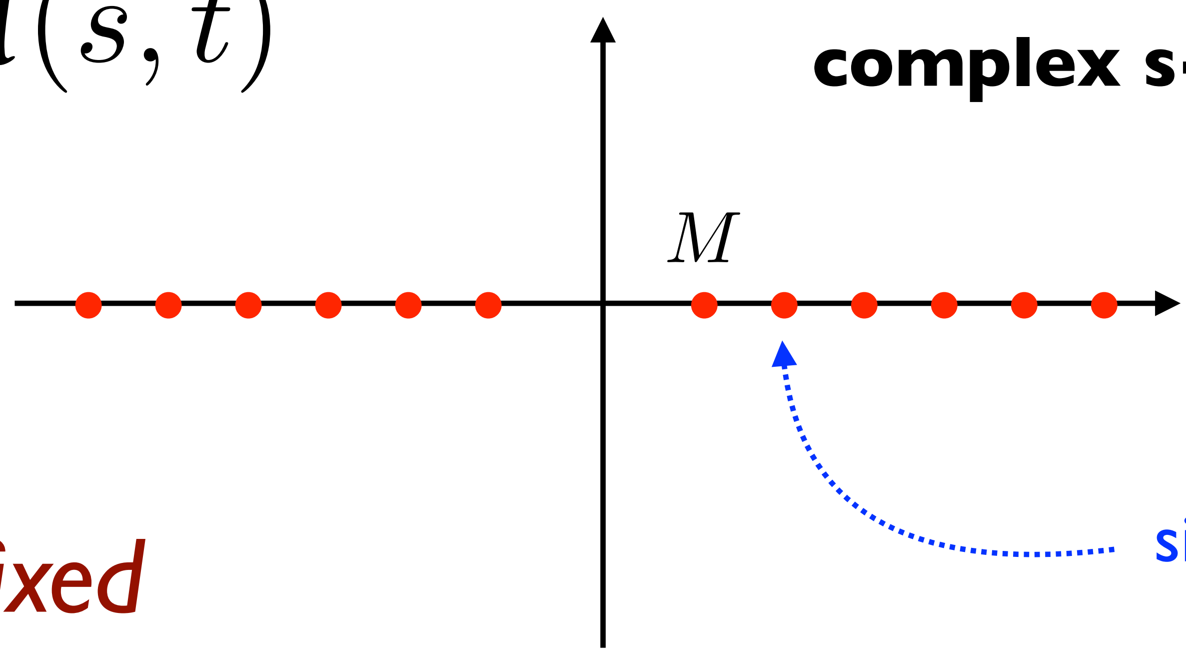
$$\frac{\Gamma}{m} \sim \frac{1}{3}$$

QCD large-N_c limit

Analytical structure of 2→2 amplitudes:

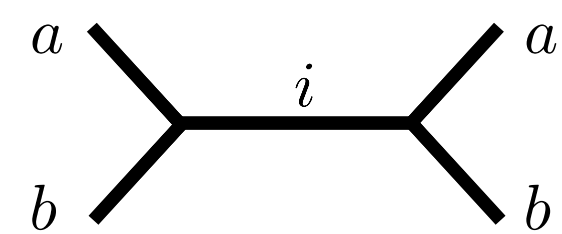
$$\mathcal{M}(s,t)$$

complex s-plane



u fixed

simple poles due to states in the s-channel:



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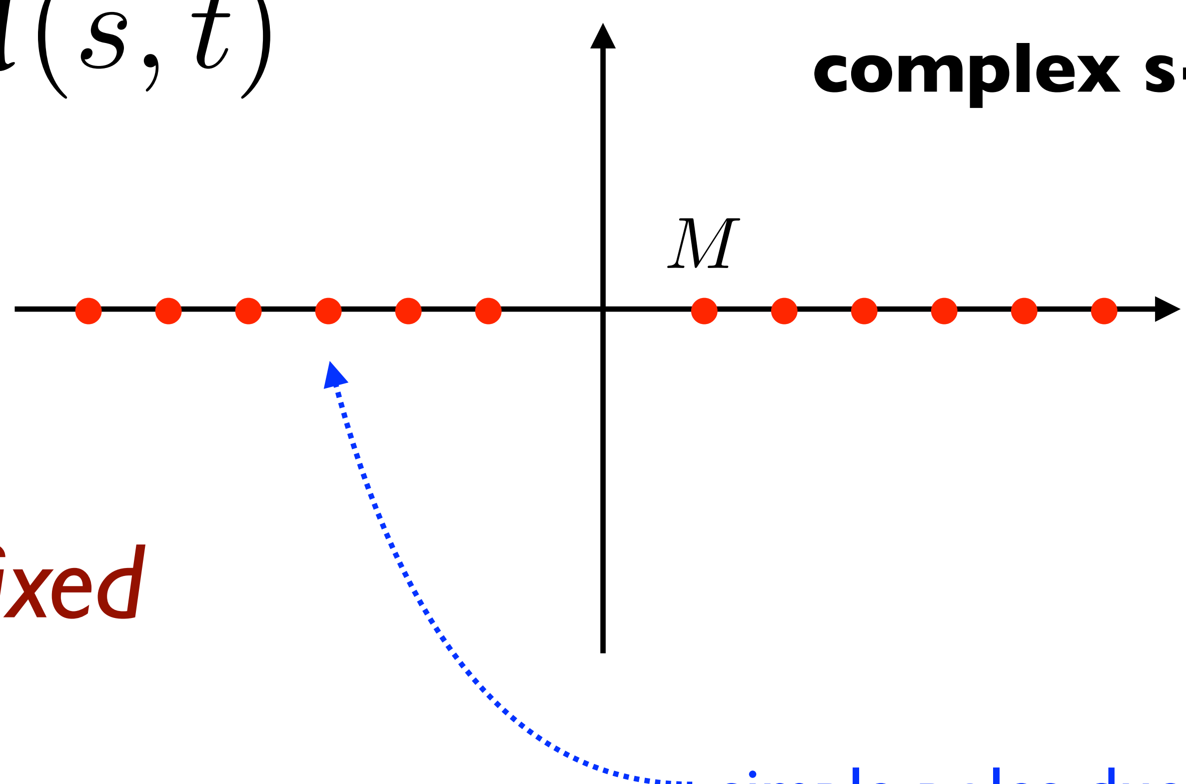
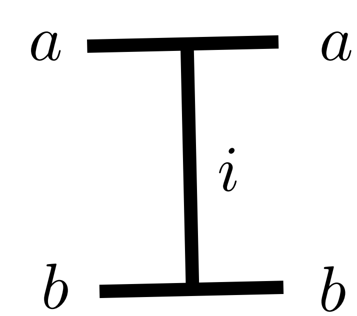


u fixed

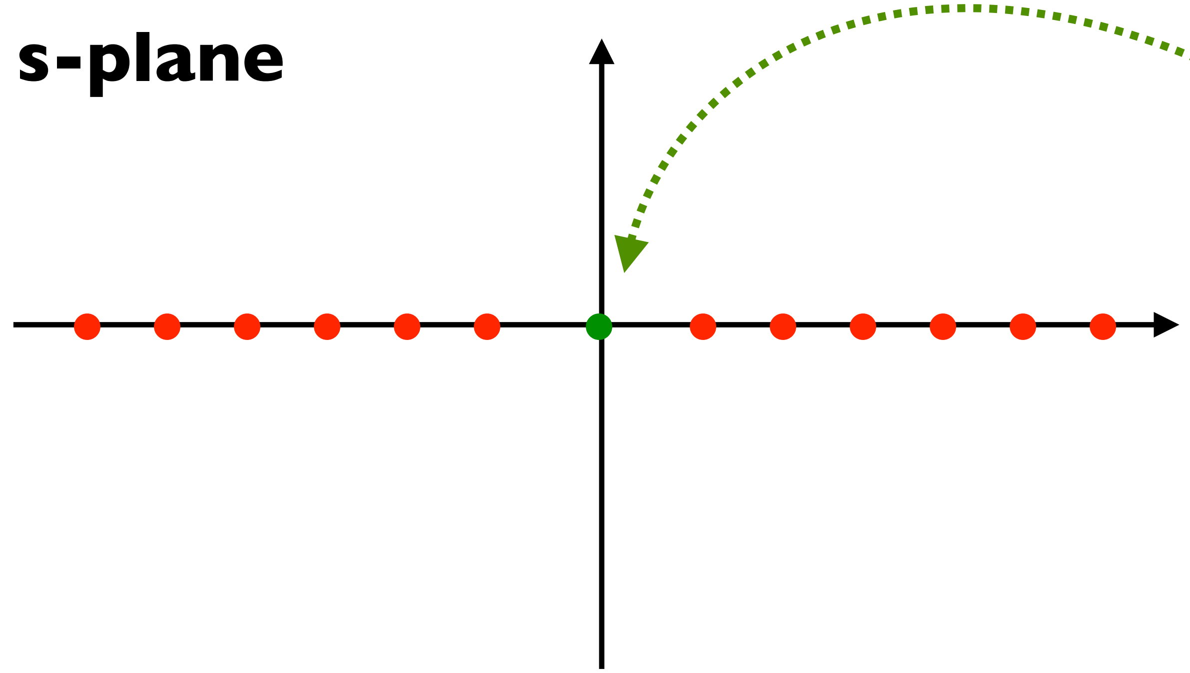
simple poles due to states in the t-channel:

$$s = -t - u + 4m^2$$



QCD large-N_c limit

This simple structure allows to get dispersion relations:



$$\frac{\mathcal{M}(s,t)}{s^{k+1}}$$

$k=1,2,3,\ldots$

u fixed

QCD large-N_c limit

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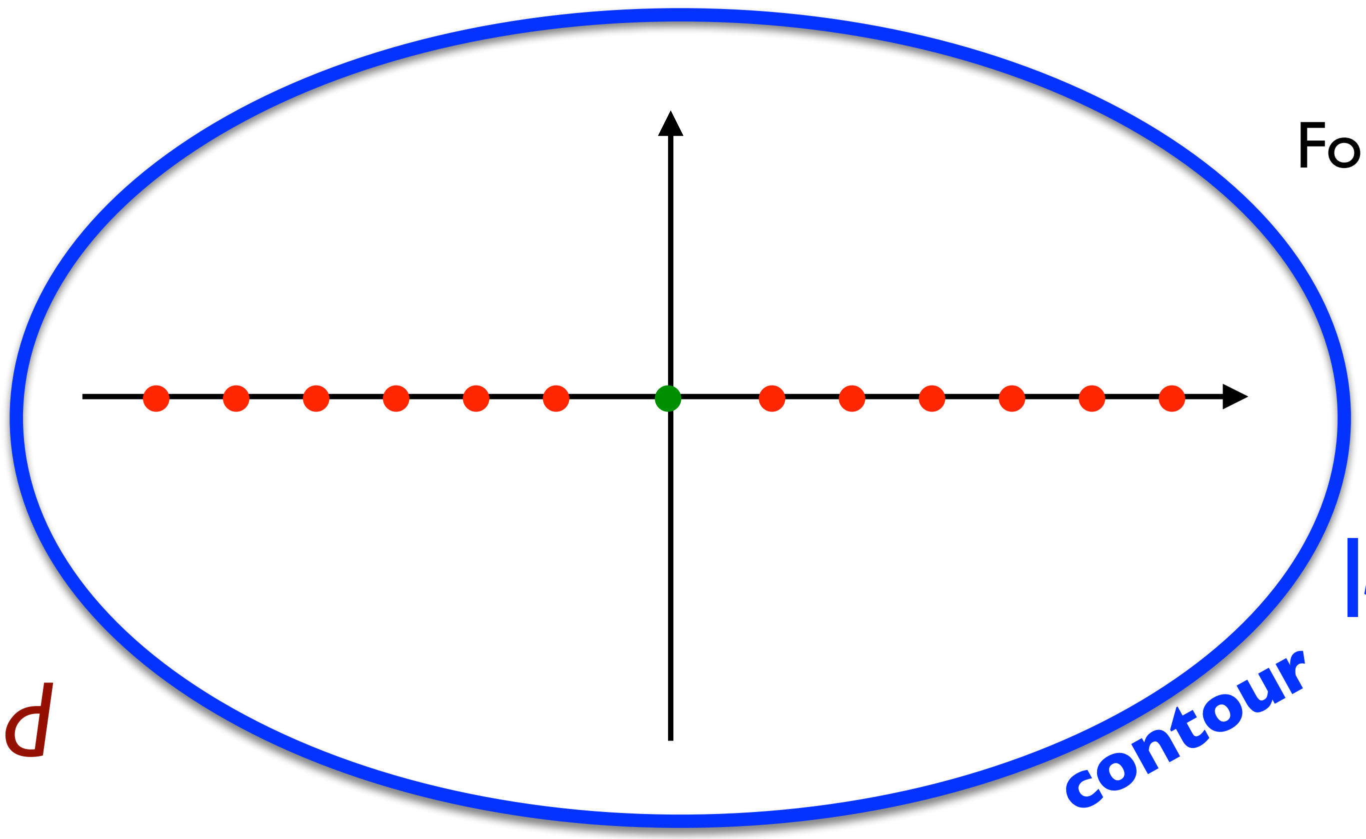


u fixed

For large enough k, we have:

$$\frac{\mathcal{M}(s,t)}{s^k} \underset{|s|\to\infty}{\longrightarrow} 0$$

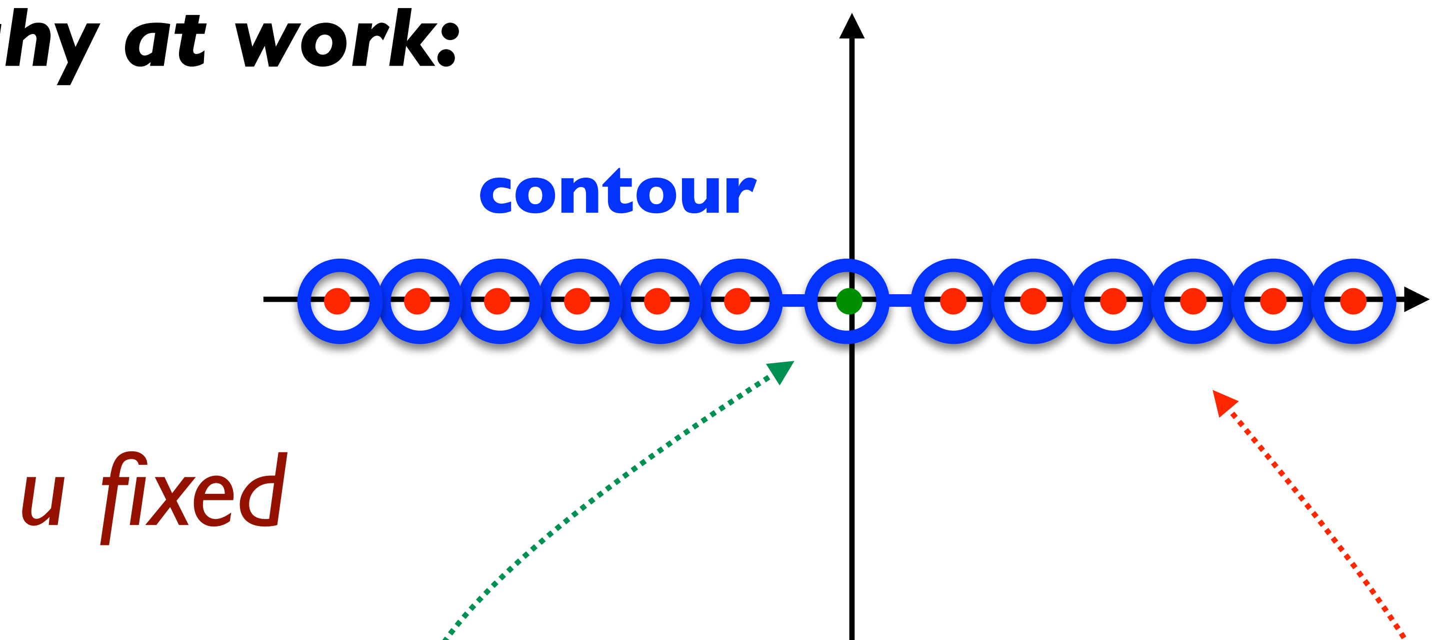
$|s| \to \infty$

$$\oint \frac{\mathcal{M}(s,t)}{s^{k+1}} = 0$$

QCD large-N_c limit

This simple structure allows to get dispersion relations:

Cauchy at work:

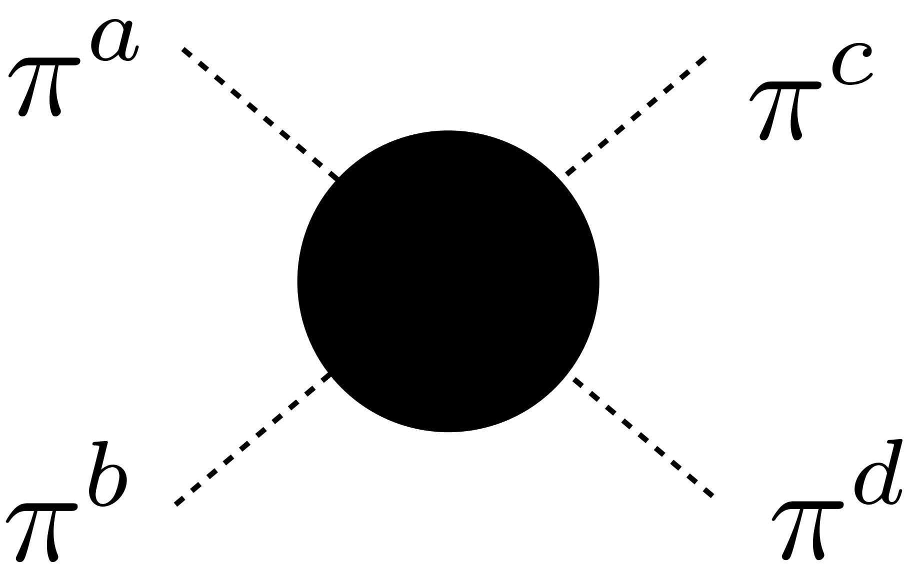


residue at the origin + sum of residues at the mass poles = 0

(*low-energy EFT parameters* ***related to*** *masses and couplings of mesons*)

Pion-Pion scattering

J. Albert and L. Rastelli, arXiv: 2203.11950



$\pi^a \in \mathbf{3}$ *massless*

Goldstones from
SU(2)⊗SU(2)→SU(2)

Isospin Invariance

(restricting to two quarks)

Extra condition from large-N_c QCD:

Mesons = $q\ \bar{q}$

Isospin = $I = 1/2 \otimes 1/2 = 0, 1$

☞ **no I =2 states**

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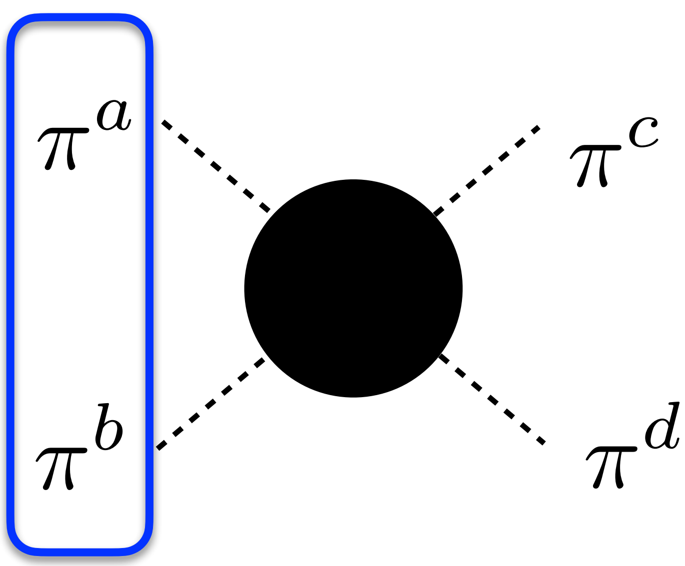
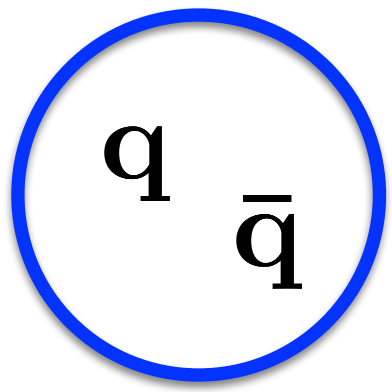


I =2

$\mathcal{M}_s^{I=2}$ cannot have poles in s

Extra condition from large-N_c QCD:

Mesons = 

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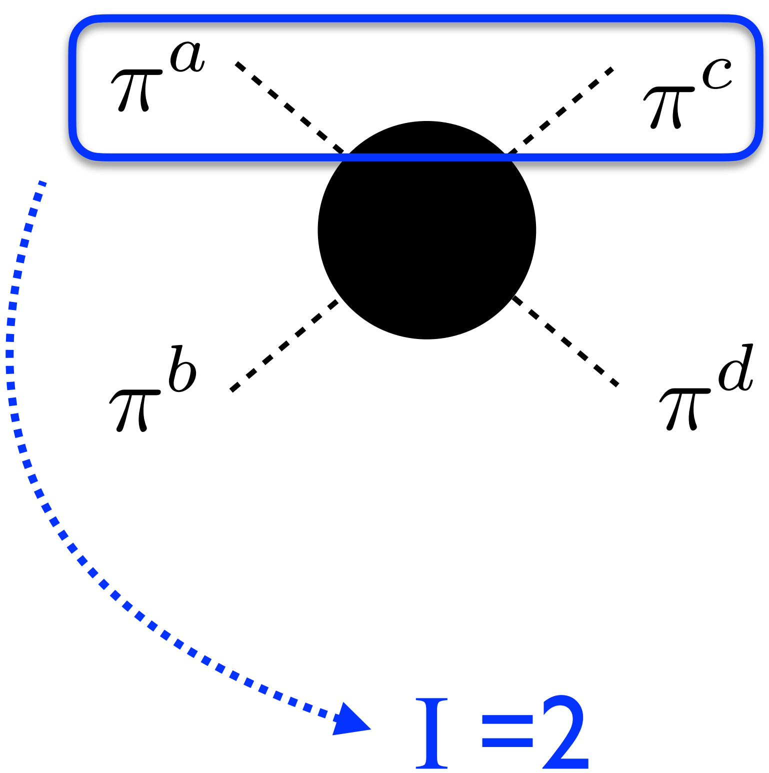


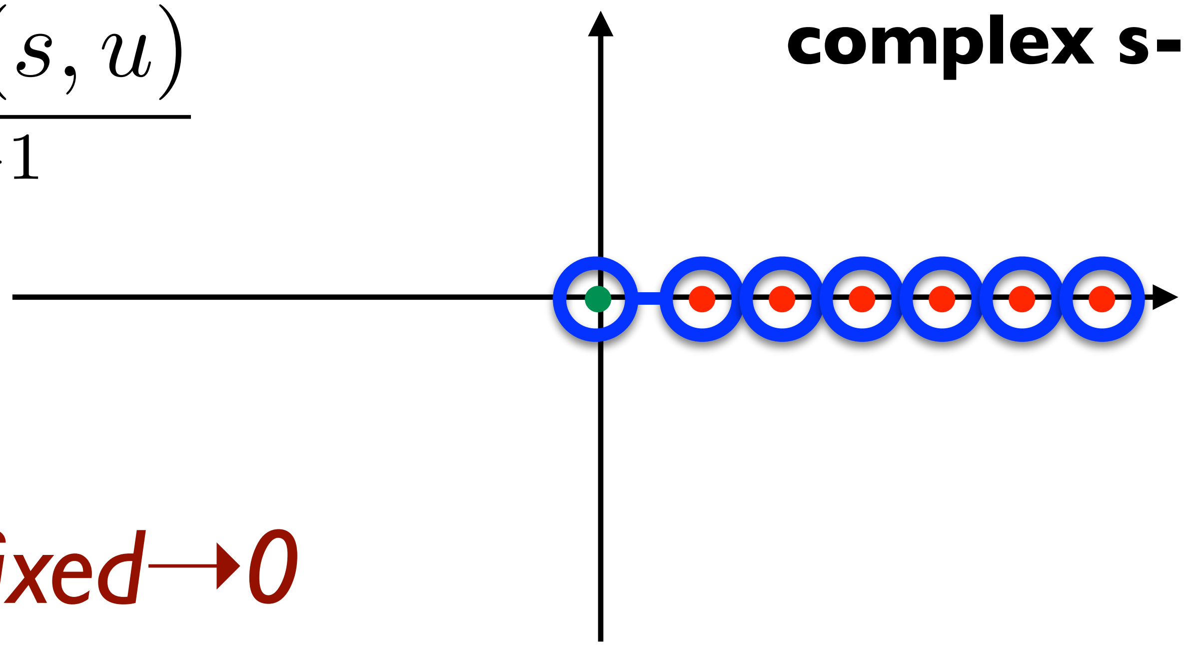
$\mathcal{M}_t^{I=2}$ cannot have poles in t

Working with $\mathcal{M}_t^{I=2}(s,u)$

(that cannot have poles in the t-channel)

crossing s⬌u invariant

$$\frac{\mathcal{M}_t^{I=2}(s,u)}{s^{k+1}}$$



u fixed→0

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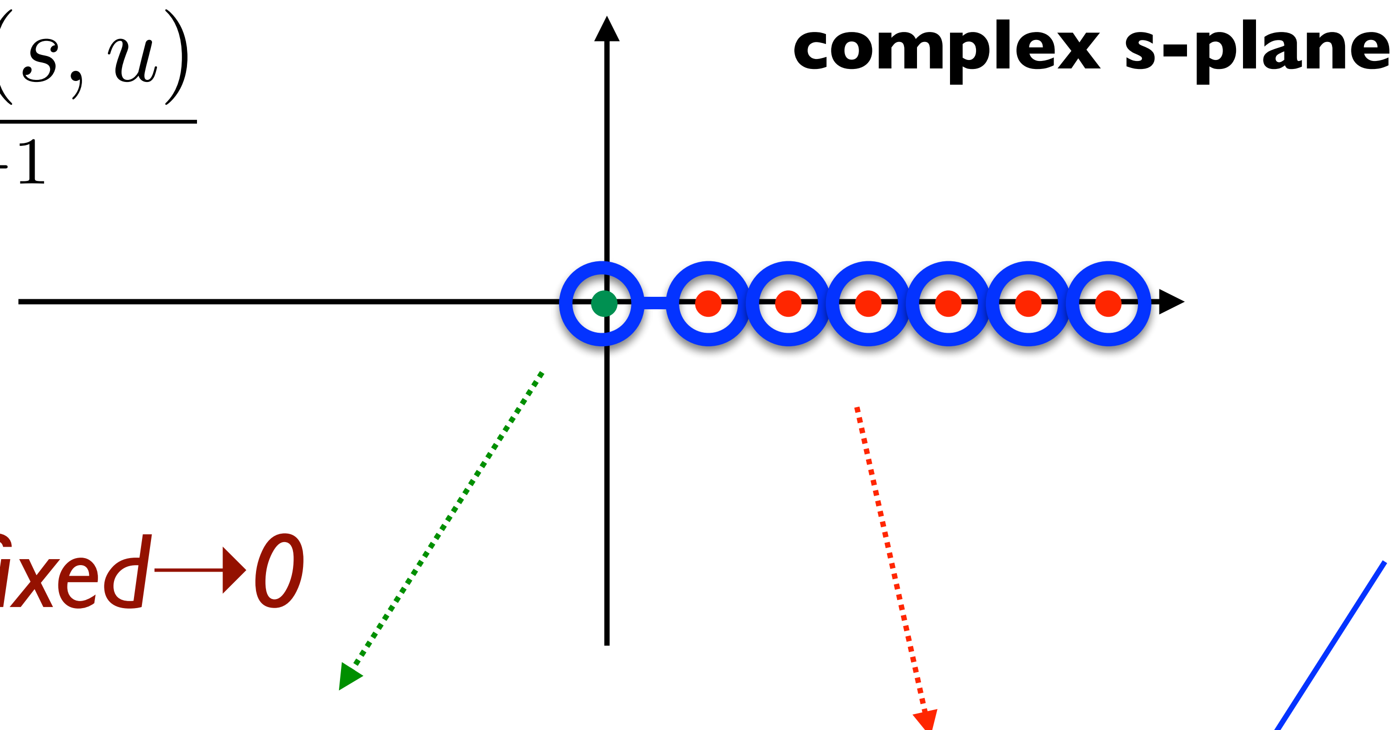

u fixed→0

partial-wave expansion

$$\mathrm{Res}\,\frac{\mathcal{M}_t^{I=2}(s,u)}{s^{k+1}} = \sum_i \frac{|g_{\pi\pi\,i}|^2}{m_i^{2k}} P_{J_i}\left(1+\frac{2u}{m_i^2}\right)$$

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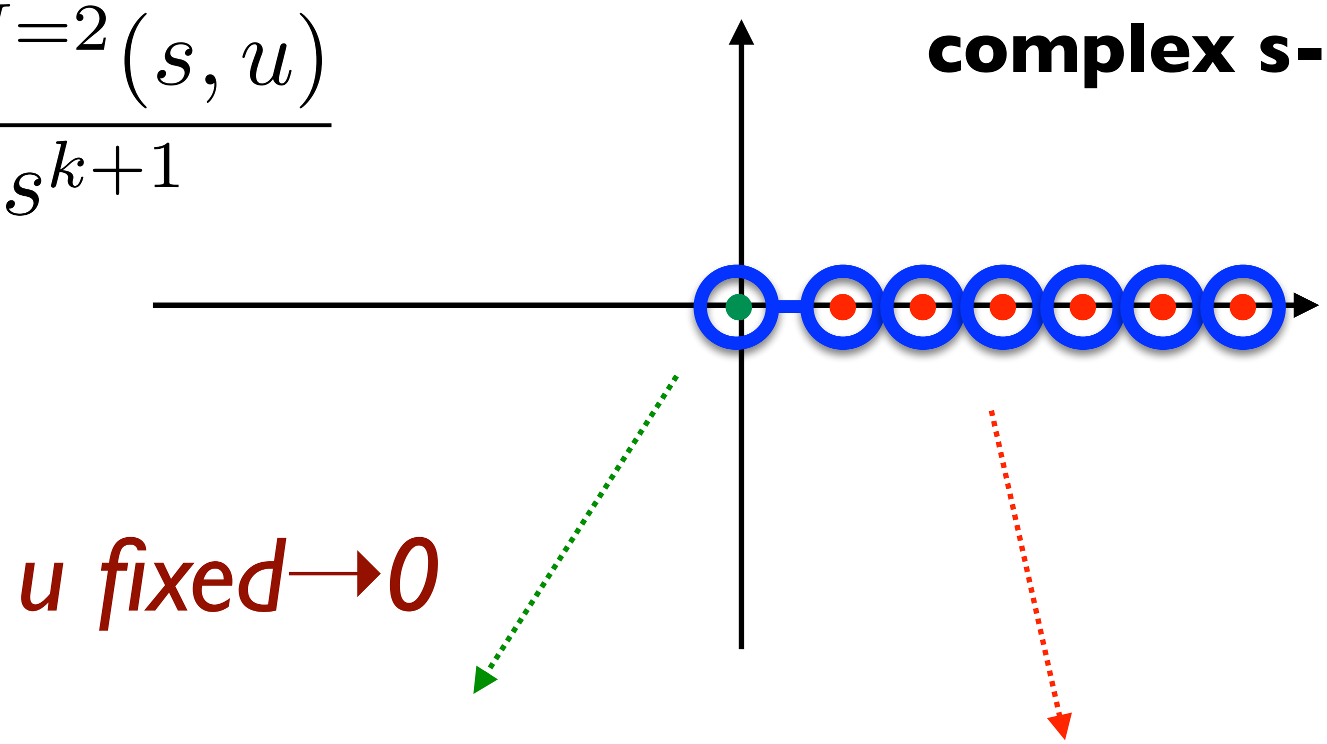

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Wilson coefficients

$$\mathcal{M}_t^{I=2}(s,u) \xrightarrow{s,u\to 0} \boxed{g_{1,0}}(s+u) + \boxed{g_{2,0}}(s^2+u^2) + \boxed{g_{2,1}}su + \cdots$$

Legendre pol. and derivatives (all positive!)

small u expansion:

$$k=1: \qquad g_{1,0}+g_{2,1}u+g_{3,1}u^2+\ldots = \sum_i |g_{\pi\pi\,i}|^2 \left(\frac{P_{J_i}(1)}{m_i^2} + 2\frac{P'_{J_i}(1)}{m_i^4}u + 2\frac{P''_{J_i}(1)}{m_i^6}u^2 + \ldots \right),$$

$$k=2: \qquad g_{2,0}+g_{3,1}u+g_{4,2}u^2+\ldots = \sum_i |g_{\pi\pi\,i}|^2 \left(\frac{P_{J_i}(1)}{m_i^4} + 2\frac{P'_{J_i}(1)}{m_i^6}u + 2\frac{P''_{J_i}(1)}{m_i^8}u^2 + \ldots \right),$$

$$k=3: \qquad g_{3,0}+g_{4,1}u+g_{5,2}u^2+\ldots = \sum_i |g_{\pi\pi\,i}|^2 \left(\frac{P_{J_i}(1)}{m_i^6} + 2\frac{P'_{J_i}(1)}{m_i^8}u + 2\frac{P''_{J_i}(1)}{m_i^{10}}u^2 + \ldots \right),$$

$$\vdots$$

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$$\vdots$$

$$\left\{\begin{aligned} g_{n,0} &= \sum_i \frac{g_{i\pi\pi}^2}{m_i^{2n}} \\ g_{n+1,1} &= \sum_i \frac{g_{i\pi\pi}^2\, J_i(J_i+1)}{m_i^{2(n+1)}} \end{aligned}\right.$$

all states contribute positively!

➥ the larger the J, the smaller $g_{i\pi\pi}/m_i$

small u expansion:

$$k=1: \qquad g_{1,0} + g_{2,1} u + \boxed{g_{3,1}} u^2 + \dots = \sum_i |g_{\pi\pi\, i}|^2 \left(\frac{P_{J_i}(1)}{m_i^2} + 2\frac{P'_{J_i}(1)}{m_i^4} u + 2\frac{P''_{J_i}(1)}{m_i^6} u^2 + \dots \right) ,$$

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$$\vdots$$

due to crossing, overconstrained system!

☞ **infinite constraints in the spectrum and couplings**

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due to crossing, overconstrained system!

☞ **infinite constraints in the spectrum and couplings**

e.g. $$\sum_i \frac{|g_{\pi\pi\,i}|^2}{m_i^6} J_i(J_i+1)(J_i-2)(J_i+3)=0$$

small u expansion:

$$
\begin{aligned}
k=1: \quad & g_{1,0}+g_{2,1}u+\boxed{g_{3,1}}u^2+\ldots = \sum_i |g_{\pi\pi\,i}|^2\left(\frac{P_{J_i}(1)}{m_i^2}+2\frac{P'_{J_i}(1)}{m_i^4}u+2\frac{P''_{J_i}(1)}{m_i^6}u^2+\ldots\right),\\
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& \vdots
\end{aligned}
$$

due to crossing, overconstrained system!

☞ **infinite constraints in the spectrum and couplings**

$$\text{e.g.}\quad \sum_i \frac{|g_{\pi\pi\,i}|^2}{m_i^6} J_i(J_i+1)(J_i-2)(J_i+3)=0$$

☞ **also from dispersion relations at fixed t**

Implications of Positivity bounds

Lets assume at $|s|\to\infty$ & either t or u fixed:

$$\frac{\mathcal{M}_t^{I=2}(s,u)}{s} \to 0$$

$k_{min}=1$

expected from Regge theory

Infinite set of Sum Rules:

$$\sum_i \frac{|g_{\pi\pi\, i}|^2}{m_i^6} J_i(J_i+1)(J_i-2)(J_i+3) = 0$$

$$\sum_i \frac{|g_{\pi\pi i}|^2}{m_i^{10}} J_i(J_i-1)(J_i+1)(J_i+2)(J_i^2+J_i-15) = 0$$

$$\sum_i \frac{|g_{\pi\pi i}|^2}{m_i^{14}} J_i(J_i-2)(J_i-1)(J_i+1)(J_i+2)(J_i+3)(J_i^2+J_i-28) = 0$$

$$\vdots$$

Infinite set of Sum Rules:

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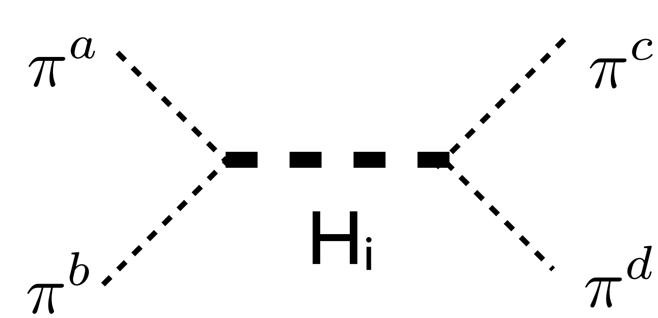
$$\vdots$$

$J_i = 0$ states satisfy all constraints

possible UV completion:

Theory of Scalars (*Higgs mechanism*)



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$$\boxed{\frac{|g_{\pi\pi\, 1}|^2}{m_{J=1}^6} = 9\frac{|g_{\pi\pi\, 3}|^2}{m_{J=3}^6} + 35\frac{|g_{\pi\pi\, 4}|^2}{m_{J=4}^6} + \cdots}$$

spin-1 must be in the spectrum with the largest coupling

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spin-1 must be in the spectrum with the largest coupling

☞ **Vector Meson Dominance** (VMD),

assumed in the past to explain QCD experimental data

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$$\vdots$$

spin-2 must be in the spectrum

Infinite set of Sum Rules:

$$\sum_i \frac{|g_{\pi\pi\,i}|^2}{m_i^6} J_i(J_i+1)(J_i-2)(J_i+3) = 0$$

$$\sum_i \frac{|g_{\pi\pi i}|^2}{m_i^{10}} J_i(J_i-1)(J_i+1)(J_i+2)(J_i^2+J_i-15) = 0$$

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$$\vdots$$

spin-3 must be in the spectrum

Infinite set of Sum Rules:

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$$\vdots$$

$$\Downarrow$$

non-scalar UV completions require **all spin states** with couplings to pions decreasing with J

From the constraints, we find numerically (~50 constraint, J_{max}~1000):

Upper bound on couplings

(normalized to m_i^2/F_π^2)

J	$\lvert g_{\pi\pi i}\rvert^2$
1	0.78
2	0.18
3	0.03

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Upper bound on couplings

(normalized to m_i^2/F_π^2)

J	$\lvert g_{\pi\pi i}\rvert^2$	Exp. QCD
1	0.78	0.5
2	0.18	0.18
3	0.03	

Constraints on Wilson coefficients

$$\mathcal{L} = \frac{F_\pi^2}{4}\mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U\right) + L_1 \mathrm{Tr}^2\left(\partial_\mu U^\dagger \partial^\mu U\right) + L_2 \mathrm{Tr}\left(\partial_\mu U^\dagger \partial_\nu U\right)\mathrm{Tr}\left(\partial^\mu U^\dagger \partial^\nu U\right) + L_3 \mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U \partial_\nu U^\dagger \partial^\nu U\right)$$

$$\mathbf{e}^{\mathbf{i}\,\sigma^{\mathbf{a}}\pi^{\mathbf{a}}/\mathbf{F}_\pi}$$

Constraints on Wilson coefficients

$$\mathcal{L} = \frac{F_\pi^2}{4}\mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U\right) + L_1 \mathrm{Tr}^2\left(\partial_\mu U^\dagger \partial^\mu U\right) + L_2 \mathrm{Tr}\left(\partial_\mu U^\dagger \partial_\nu U\right)\mathrm{Tr}\left(\partial^\mu U^\dagger \partial^\nu U\right) + L_3 \mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U \partial_\nu U^\dagger \partial^\nu U\right)$$

O(s²): $\tilde{g}_{2,0} = 4(2L_1 + 3L_2 + L_3)\frac{M^2}{F_\pi^2}$, $\tilde{g}_{2,1} = 16L_2\frac{M^2}{F_\pi^2}$

mass of the 1st meson

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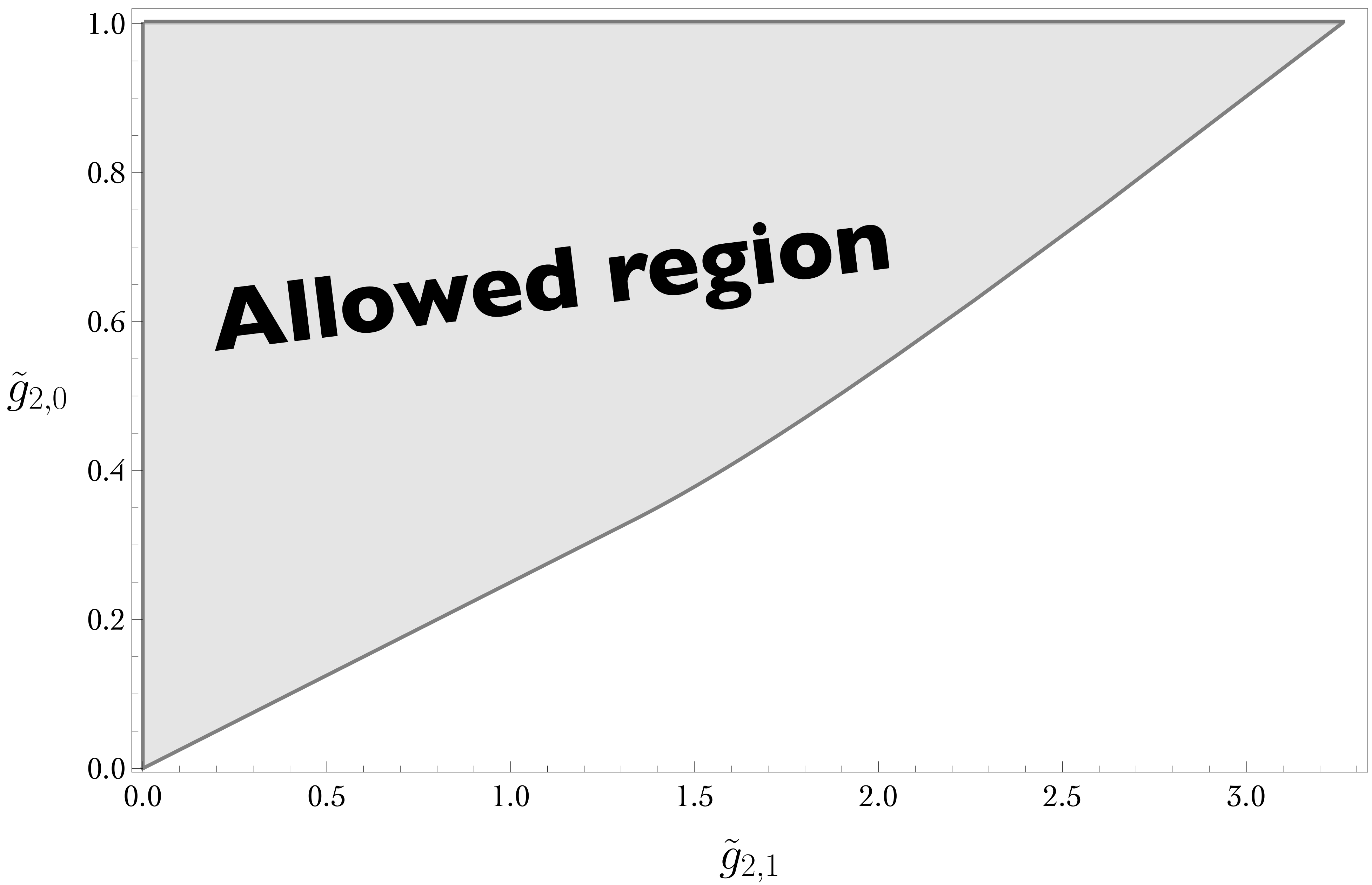

"Polyhedronal" bounds

EFTs are "**EFT-hedron**"

Constraints on Wilson coefficients

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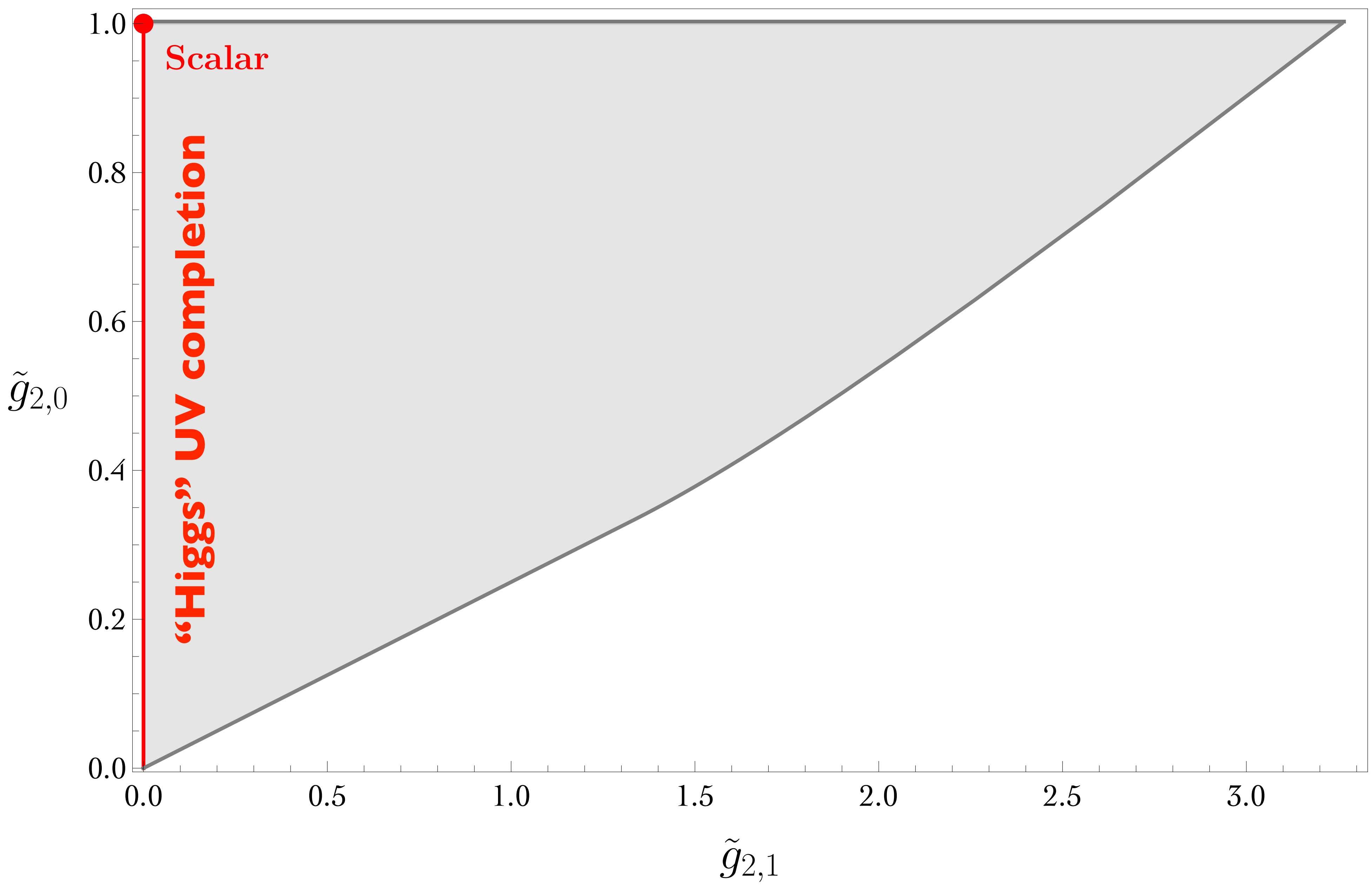

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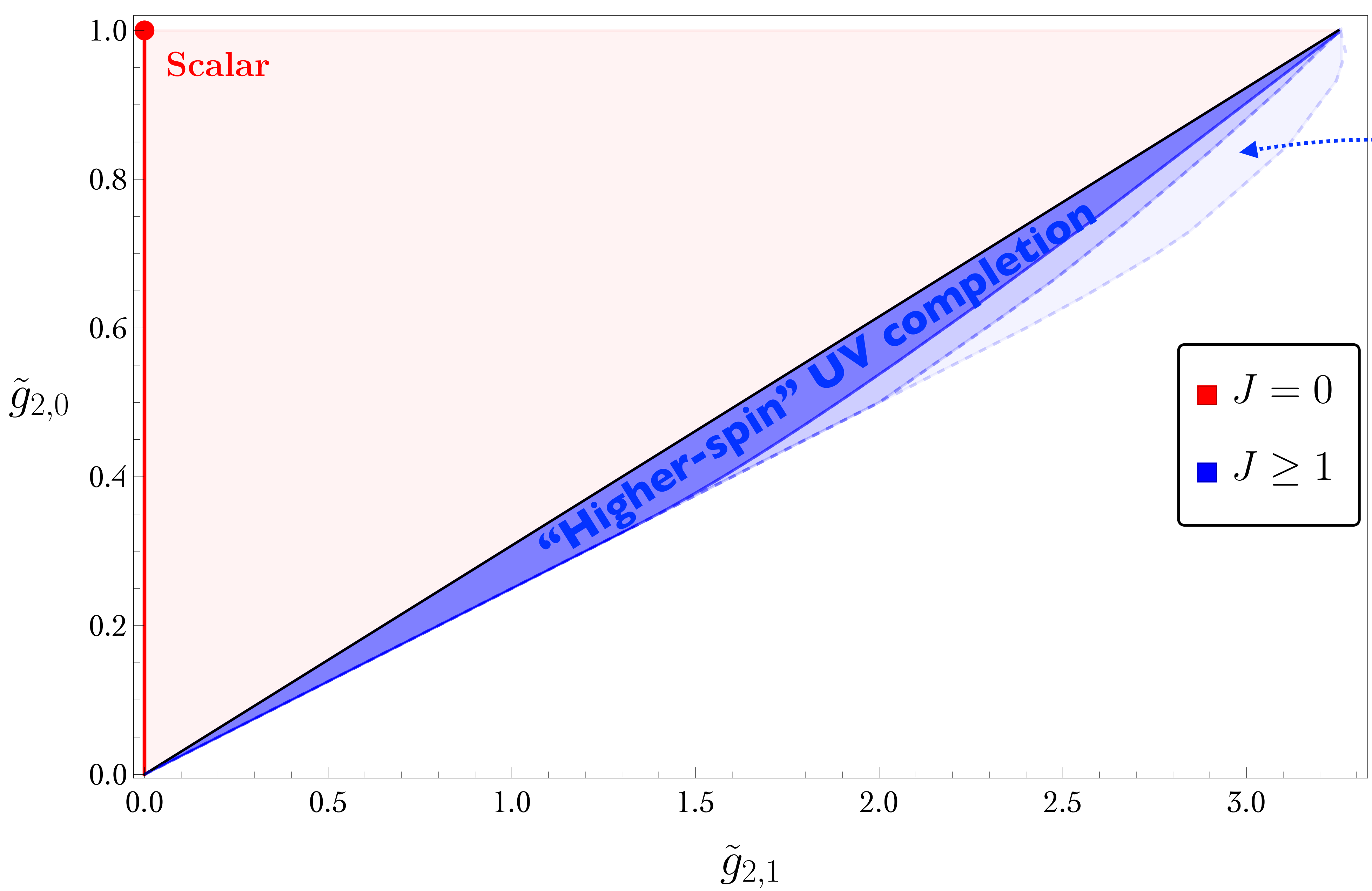

obtained numerically

Constraints on Wilson coefficients

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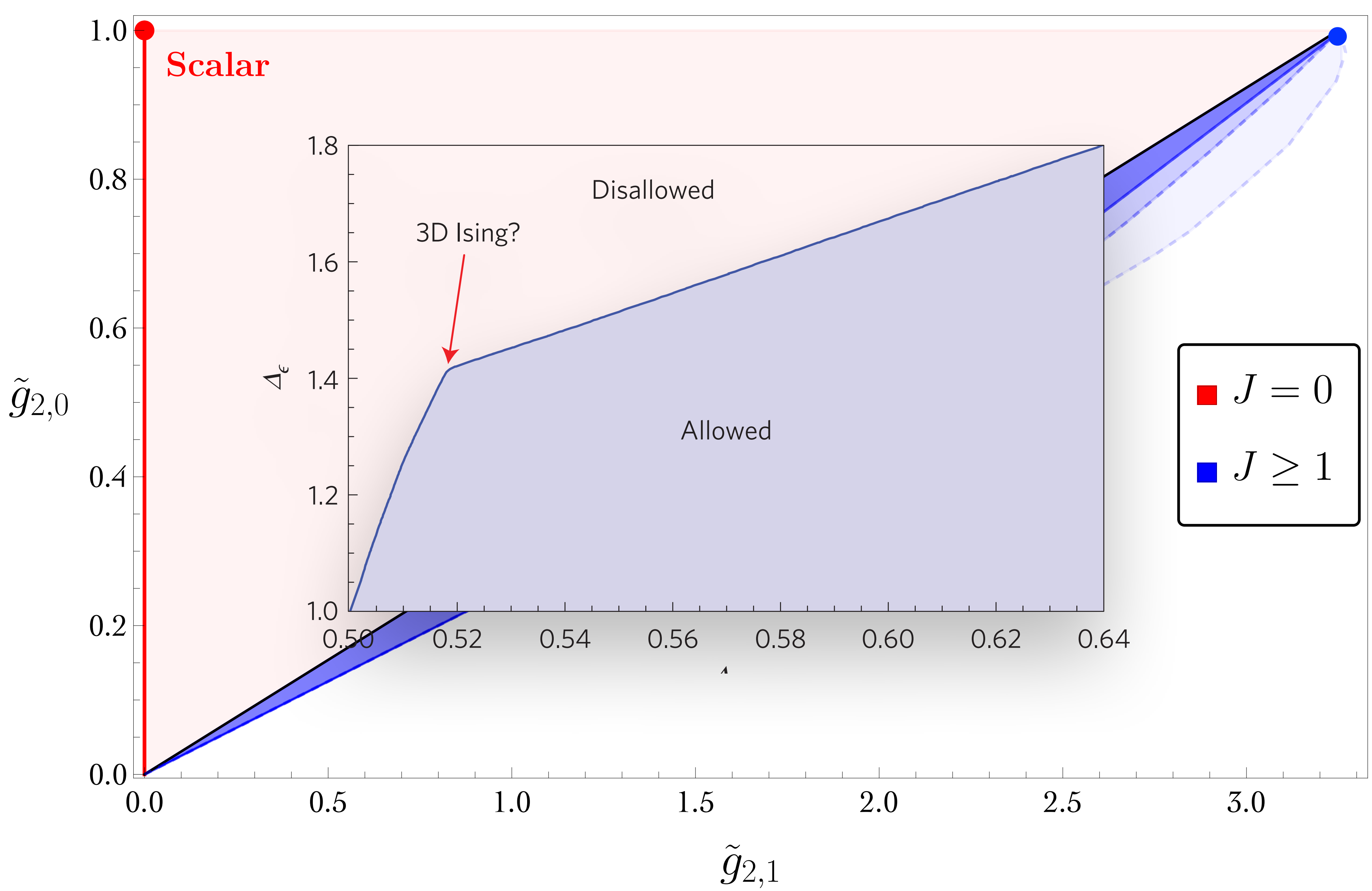

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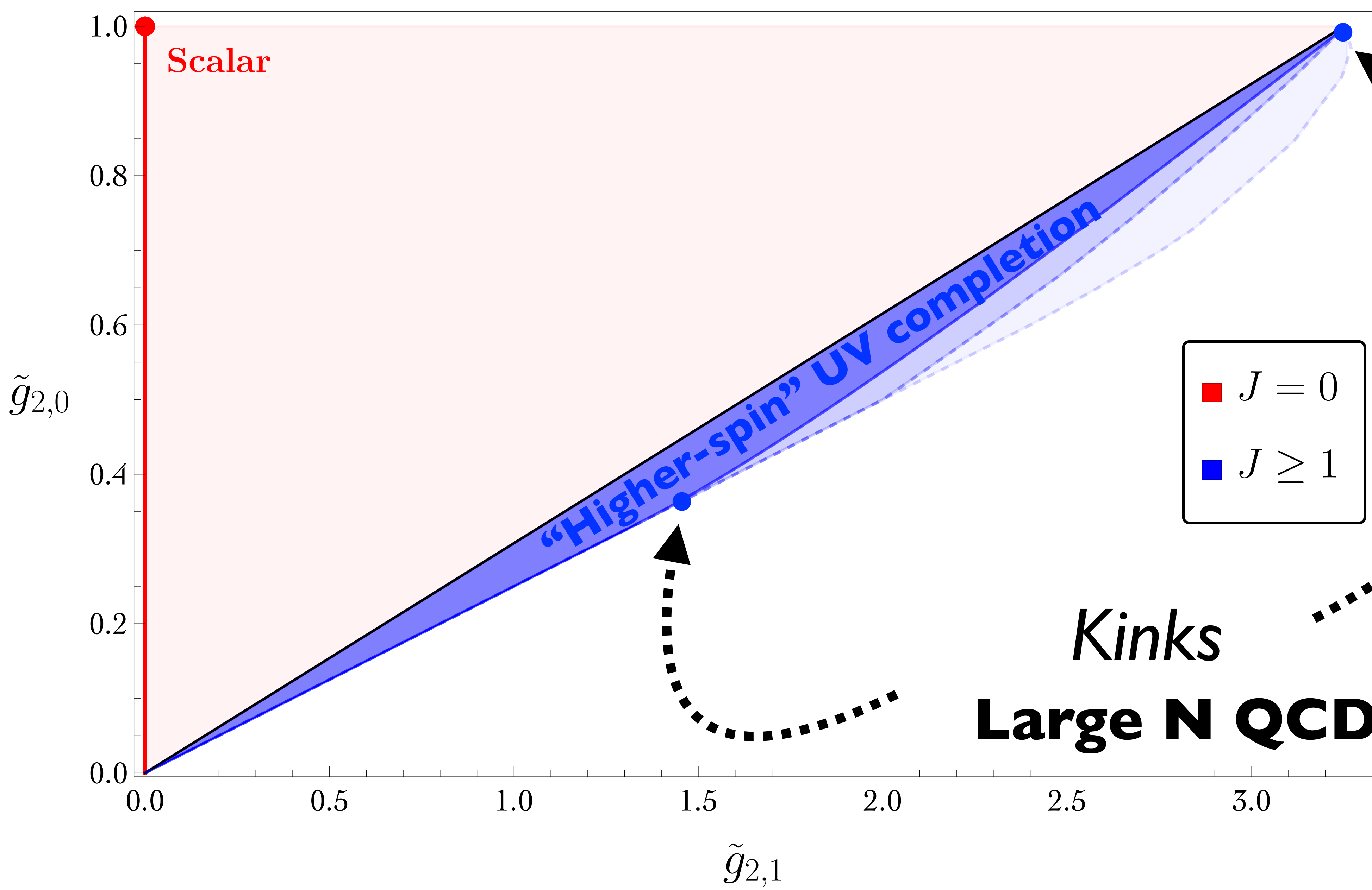

Kinks

Large N QCD?

Constraints on Wilson coefficients

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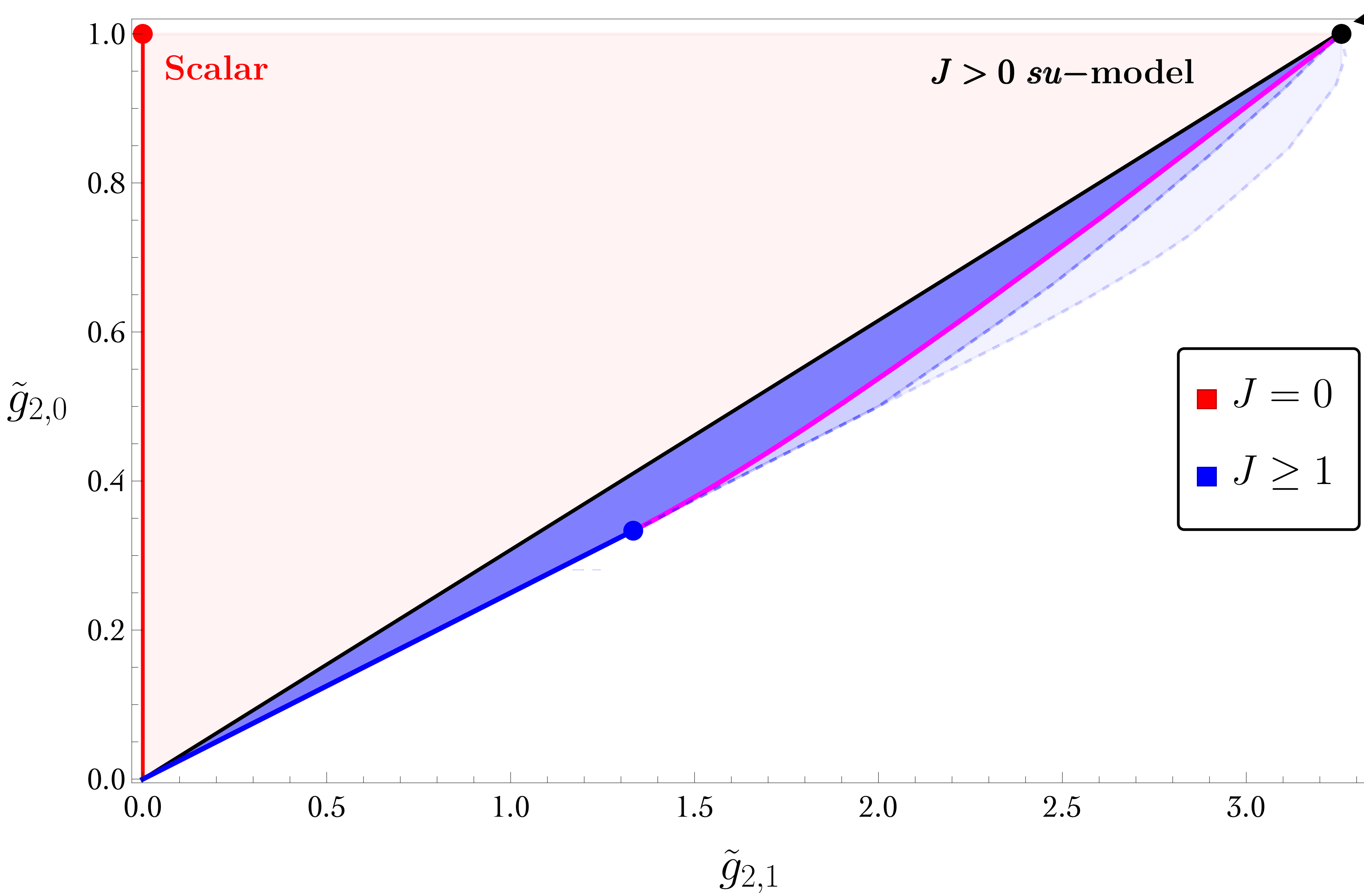
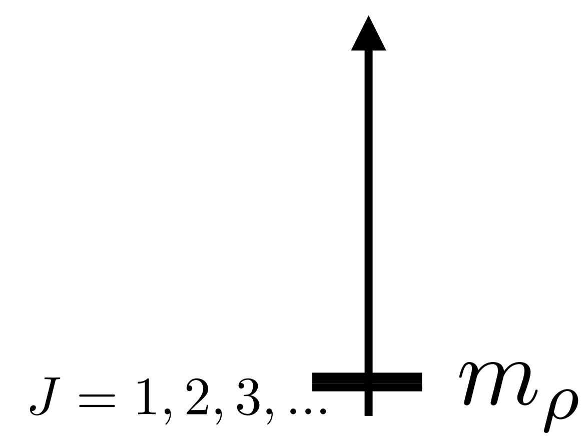


degenerate model:



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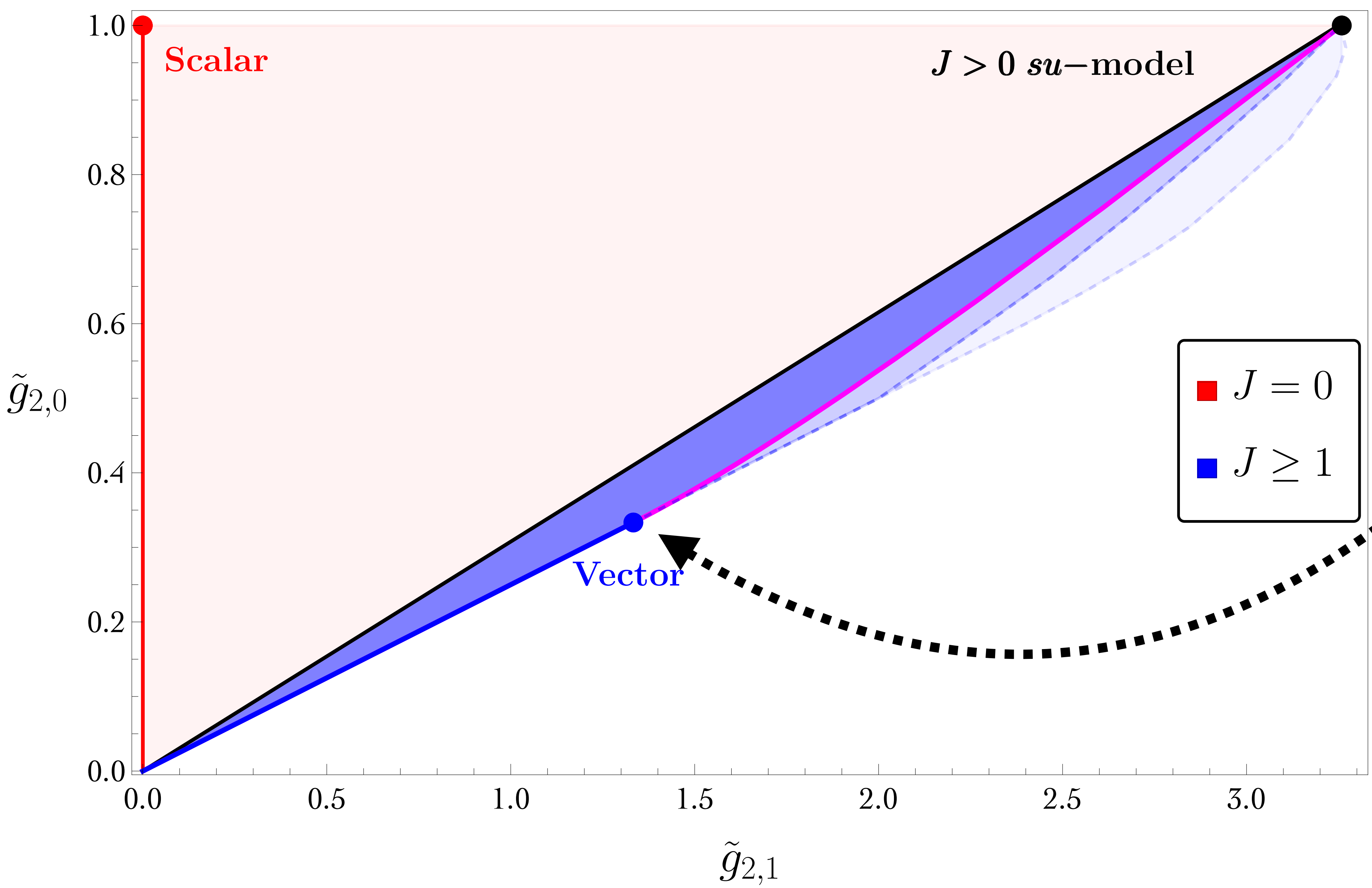
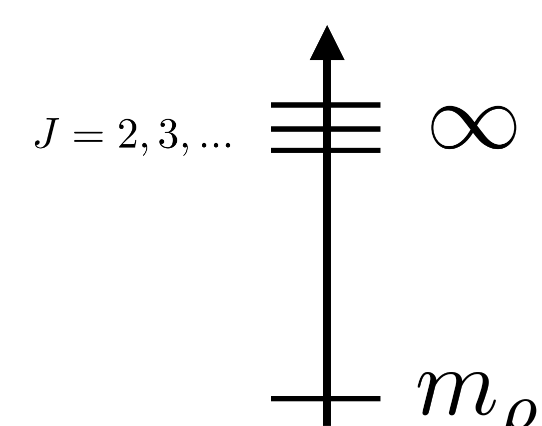


spin-1 model



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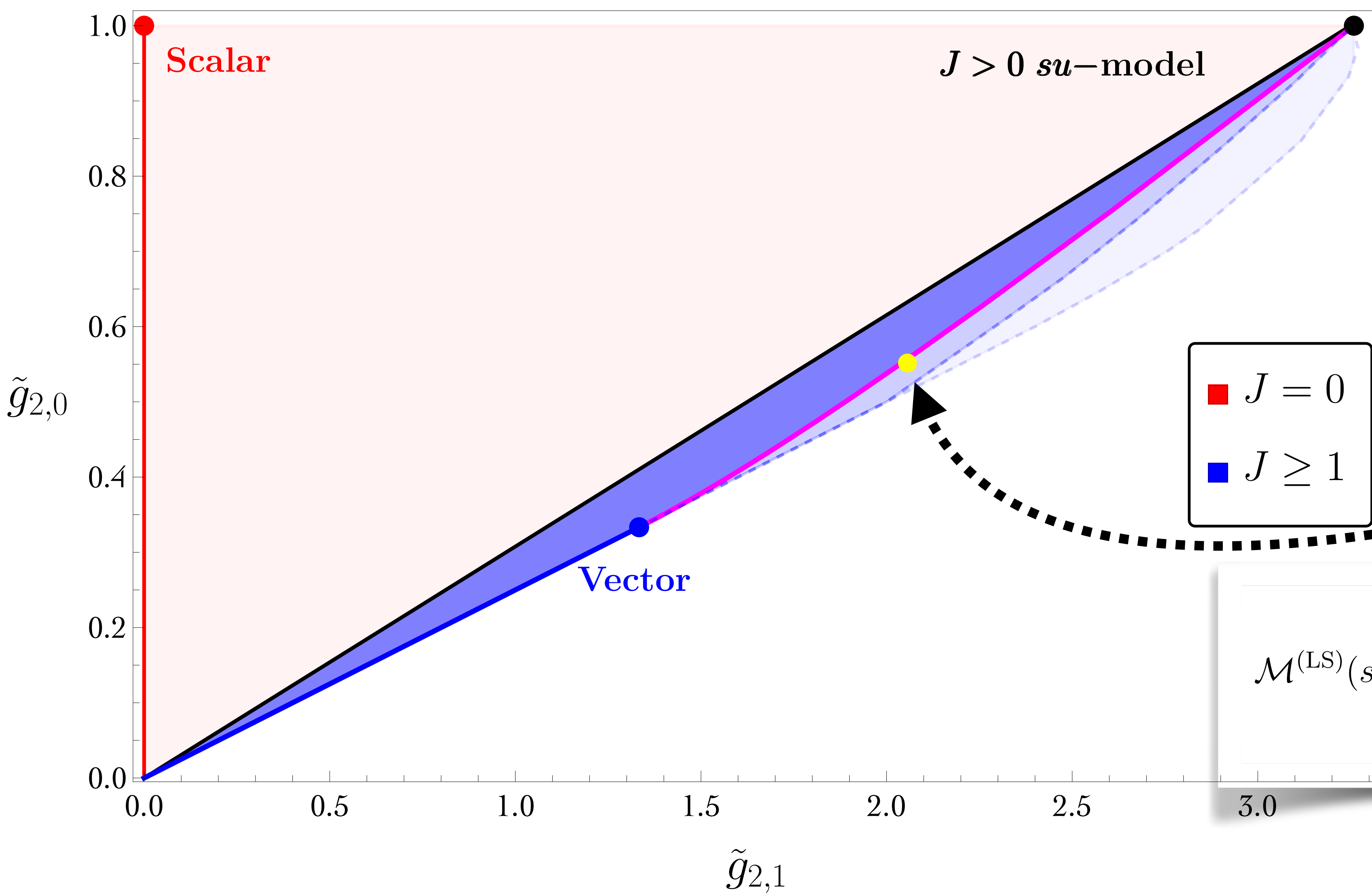
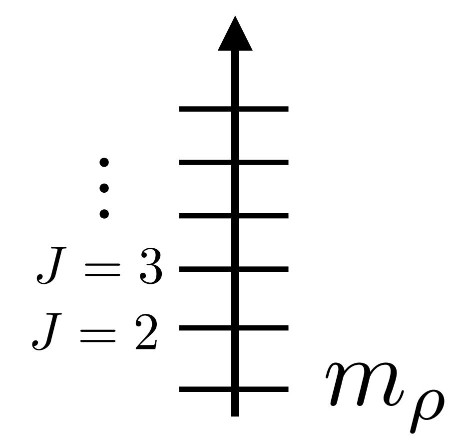


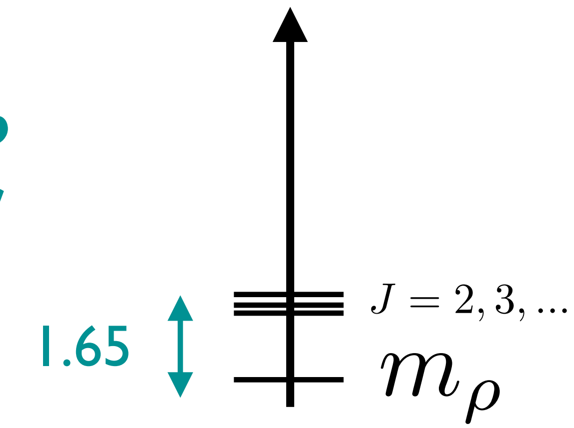


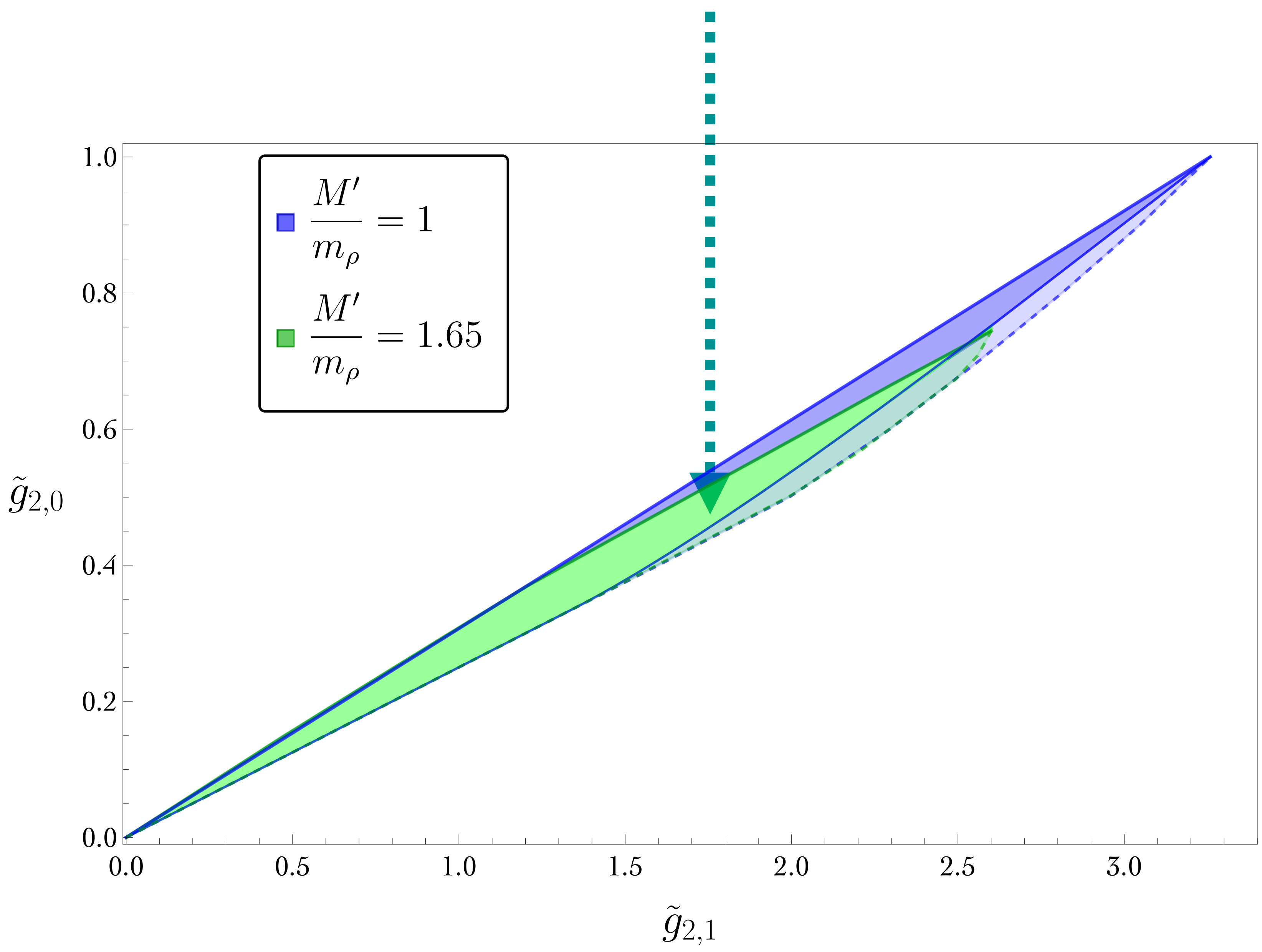
Lovelace-Shapiro:

$$\mathcal{M}^{(\mathrm{LS})}(s,u) = \frac{\Gamma\left(\frac{1}{2} - \frac{s}{2m_\rho^2}\right)\Gamma\left(\frac{1}{2} - \frac{u}{2m_\rho^2}\right)}{\Gamma\left(\frac{t}{2m_\rho^2}\right)}$$

$m_n^2 = m_\rho^2(2n+1)\,, \quad n = 0, 1, 2, \ldots$

stronger bounds if we assume that, as in QCD, J>1 mesons are heavier





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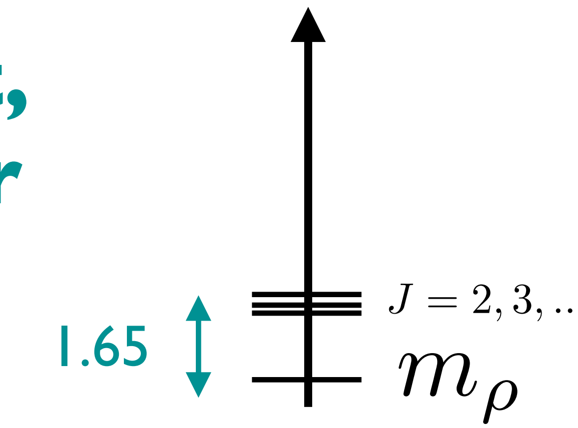
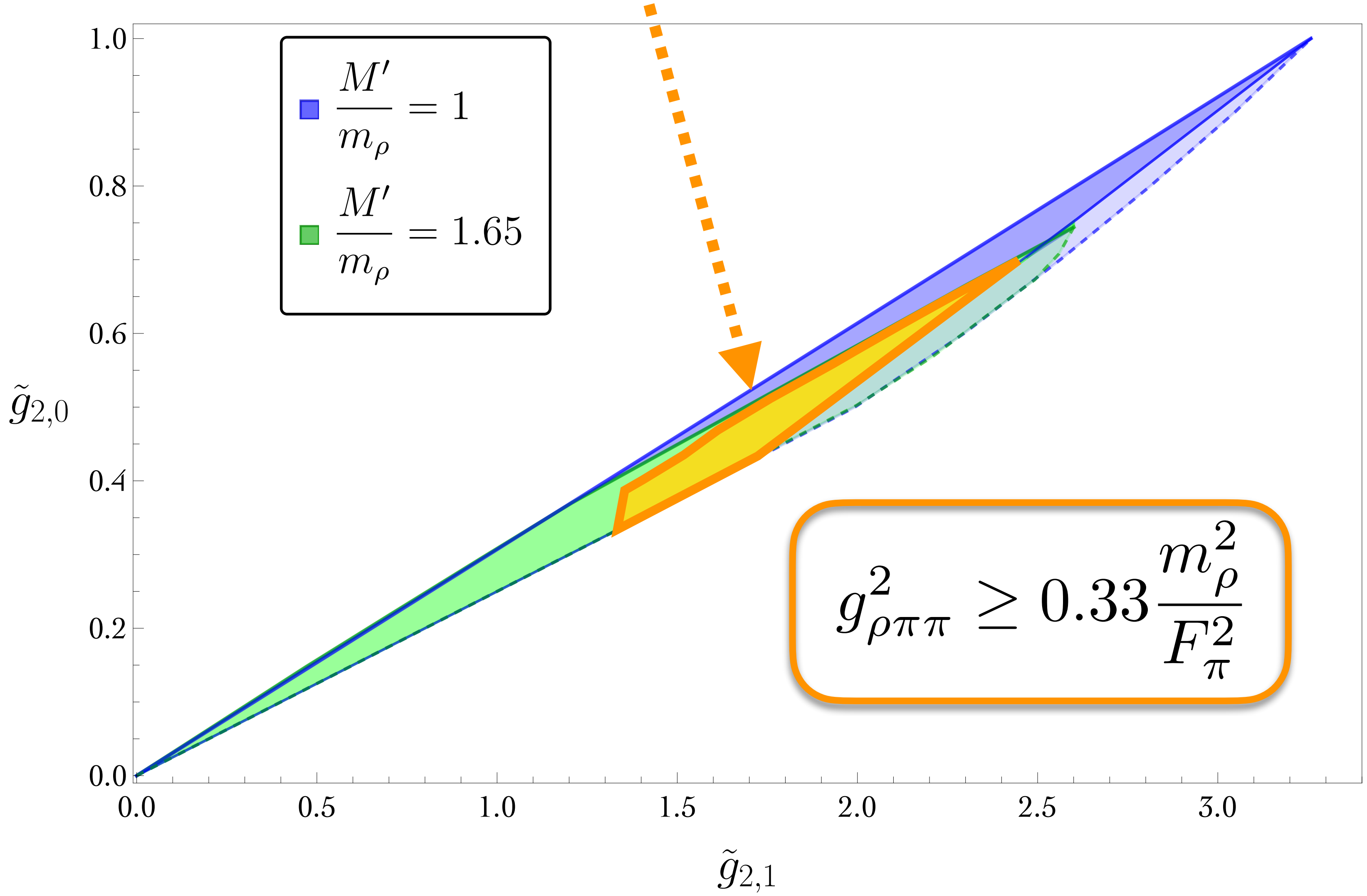


+ the spin-1 meson, the ρ, has a non-zero coupling to ππ



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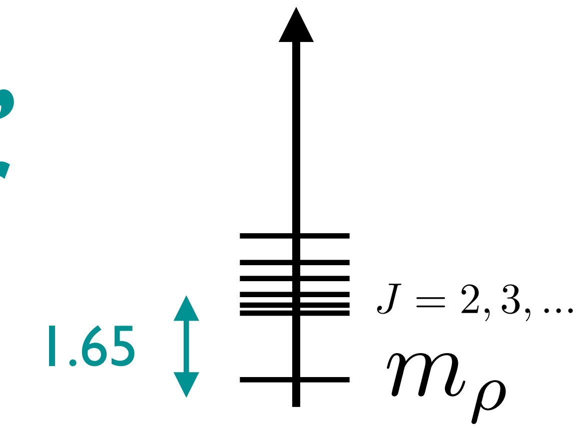

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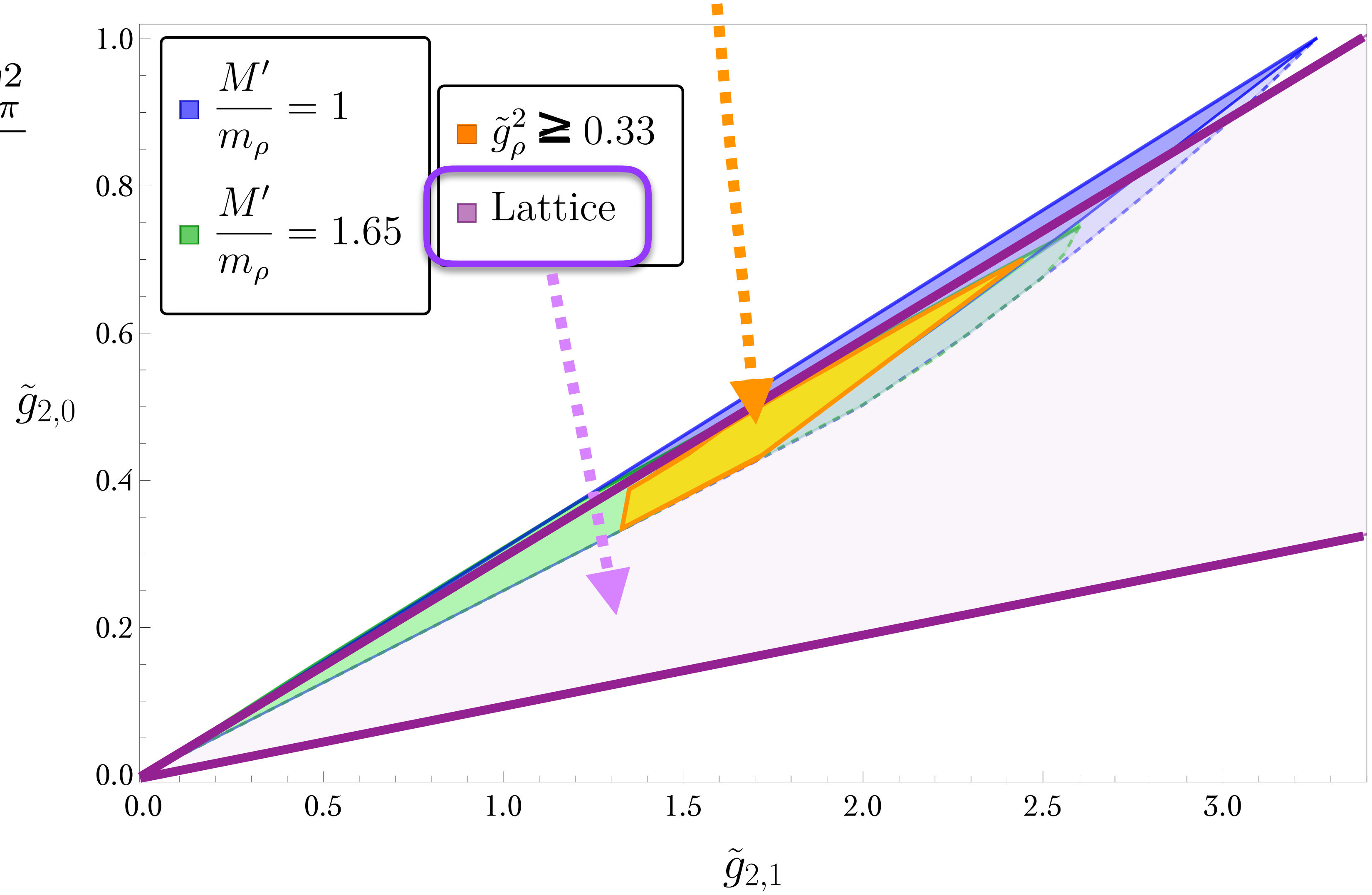

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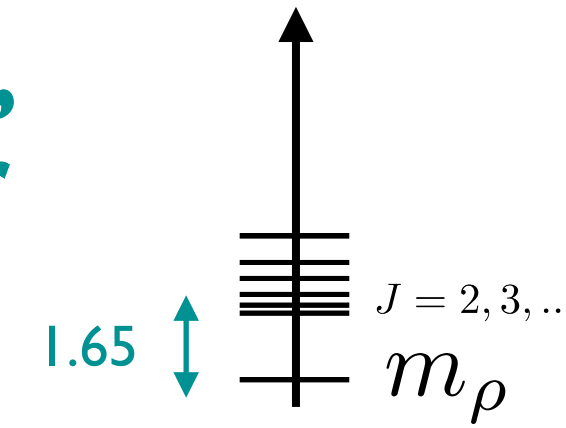

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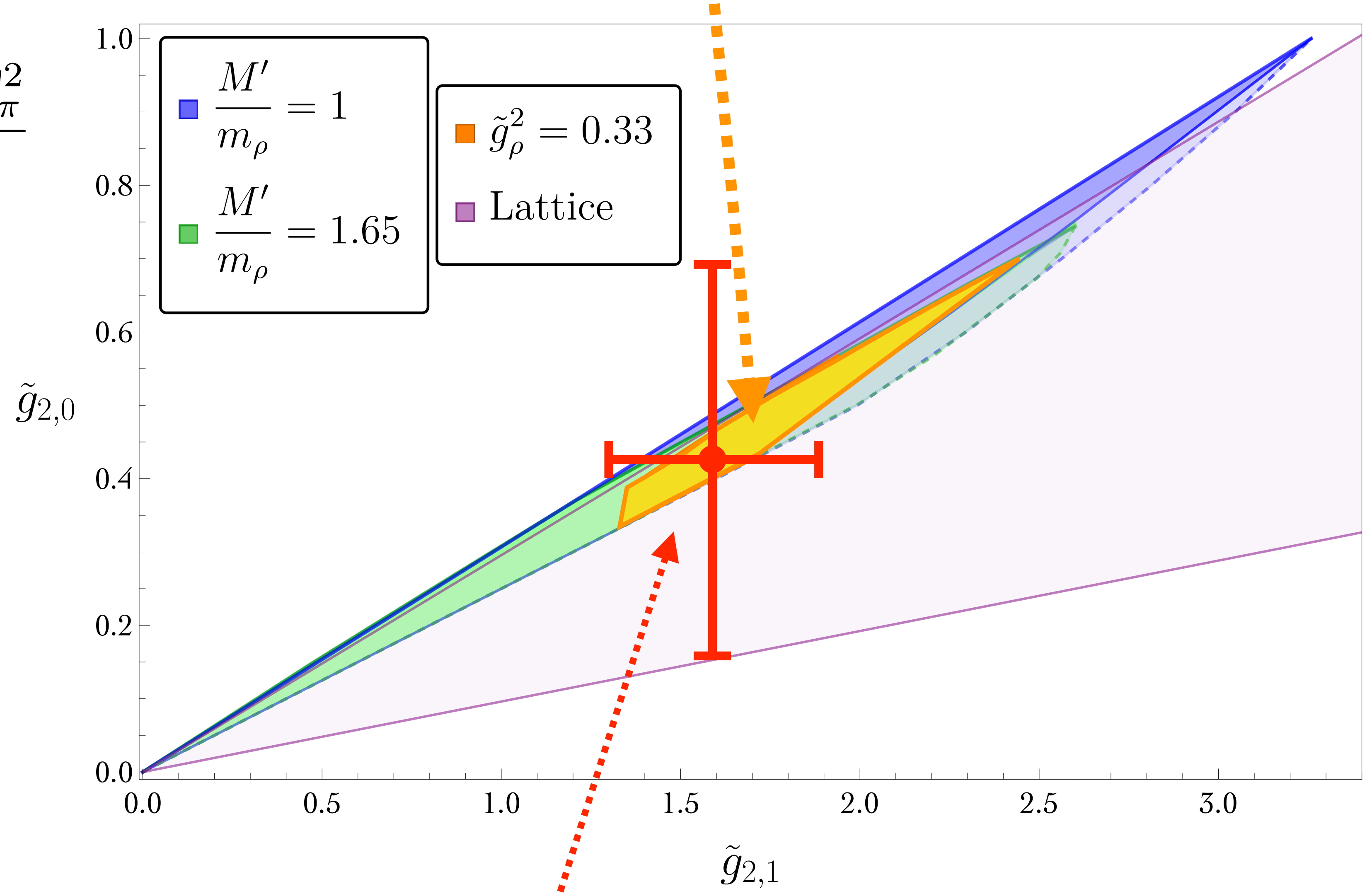

Experimental QCD data

Explaining the success of holography

AdS/QCD: 5D model for QCD mesons

$SU(2)_L \times SU(2)_R$ model with only s=0,1 fields:

Erlich+Katz+Son+Stephanov 05
Da Rold+Pomarol 05

$$\mathcal{L}_5 = \frac{M_5}{2} Tr \left[-L_{MN}L^{MN} - R_{MN}R^{MN} + |D_M \Phi|^2 + 3|\Phi|^2 \right]$$

	Experiment	AdS$_5$	Deviation
m_ρ	775	824	+6%
m_{a_1}	1230	1347	+10%
m_ω	782	824	+5%
F_ρ	153	169	+11%
F_ω/F_ρ	0.88	0.94	+7%
F_π	87	88	+1%
$g_{\rho\pi\pi}$	6.0	5.4	−10%
L_9	$6.9 \cdot 10^{-3}$	$6.2 \cdot 10^{-3}$	−10%
L_{10}	$-5.2 \cdot 10^{-3}$	$-6.2 \cdot 10^{-3}$	−12%
$\Gamma(\omega \to \pi\gamma)$	0.75	0.81	+8%
$\Gamma(\omega \to 3\pi)$	7.5	6.7	−11%
$\Gamma(\rho \to \pi\gamma)$	0.068	0.077	+13%
$\Gamma(\omega \to \pi\mu\mu)$	$8.2 \cdot 10^{-4}$	$7.3 \cdot 10^{-4}$	−10%
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AP+Wulzer 08

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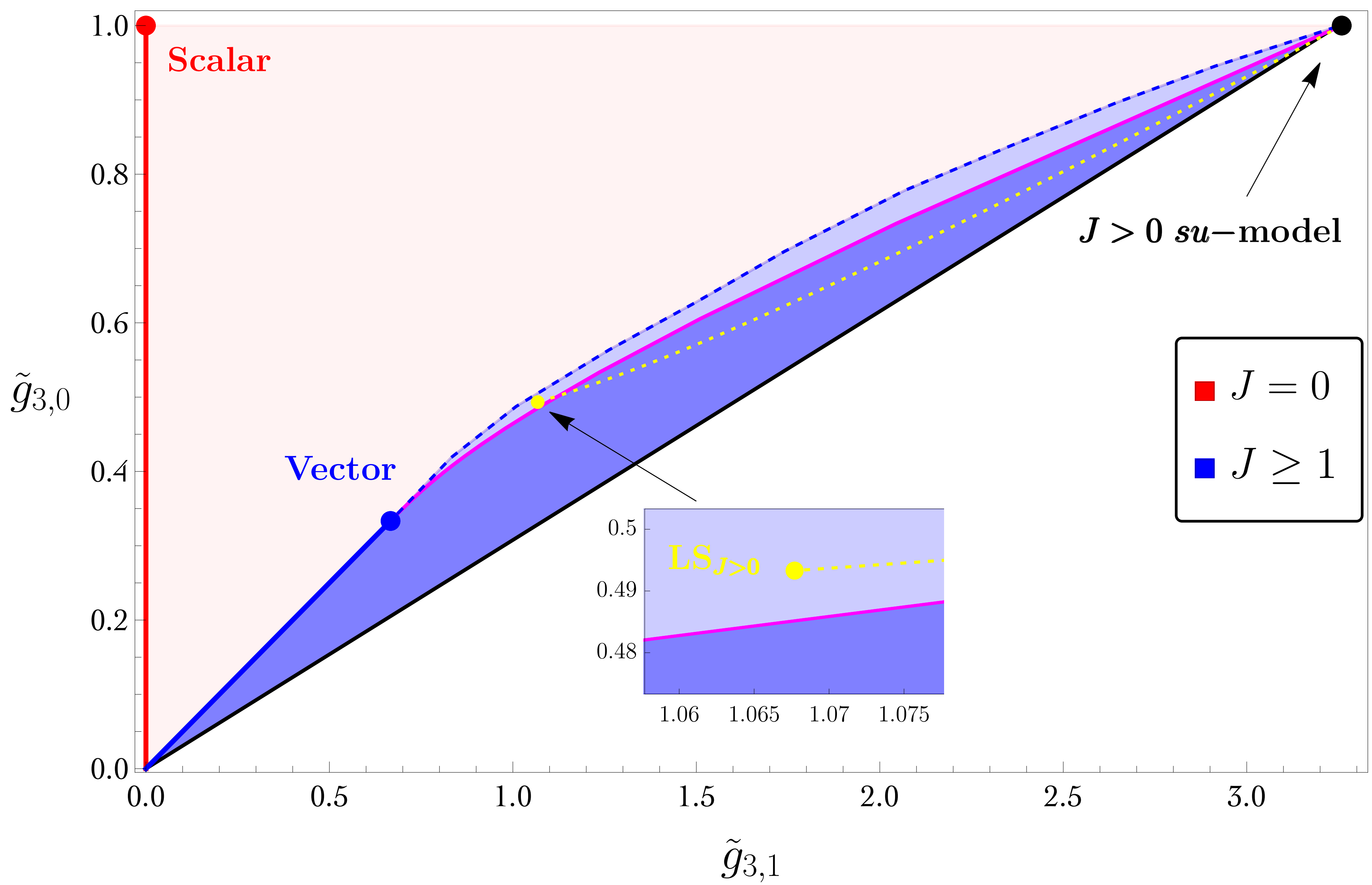
Success understood
from the above analysis:
J>1 mesons
contribute little
to low-energy observables

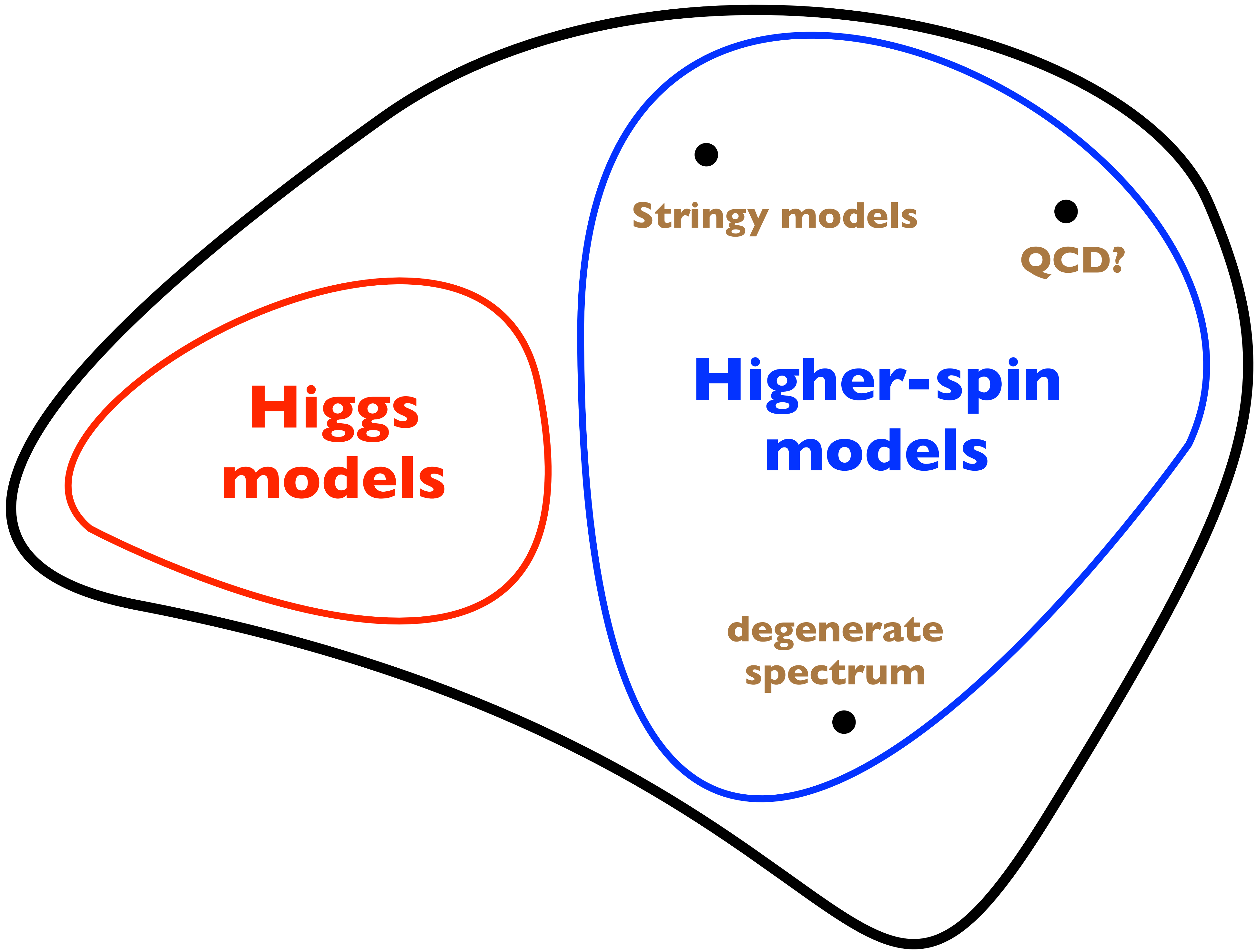
AP+Wulzer 08

Similar structure for higher-order Wilson coeff.

$O(s^3)$:



UV completions for models of Goldstones



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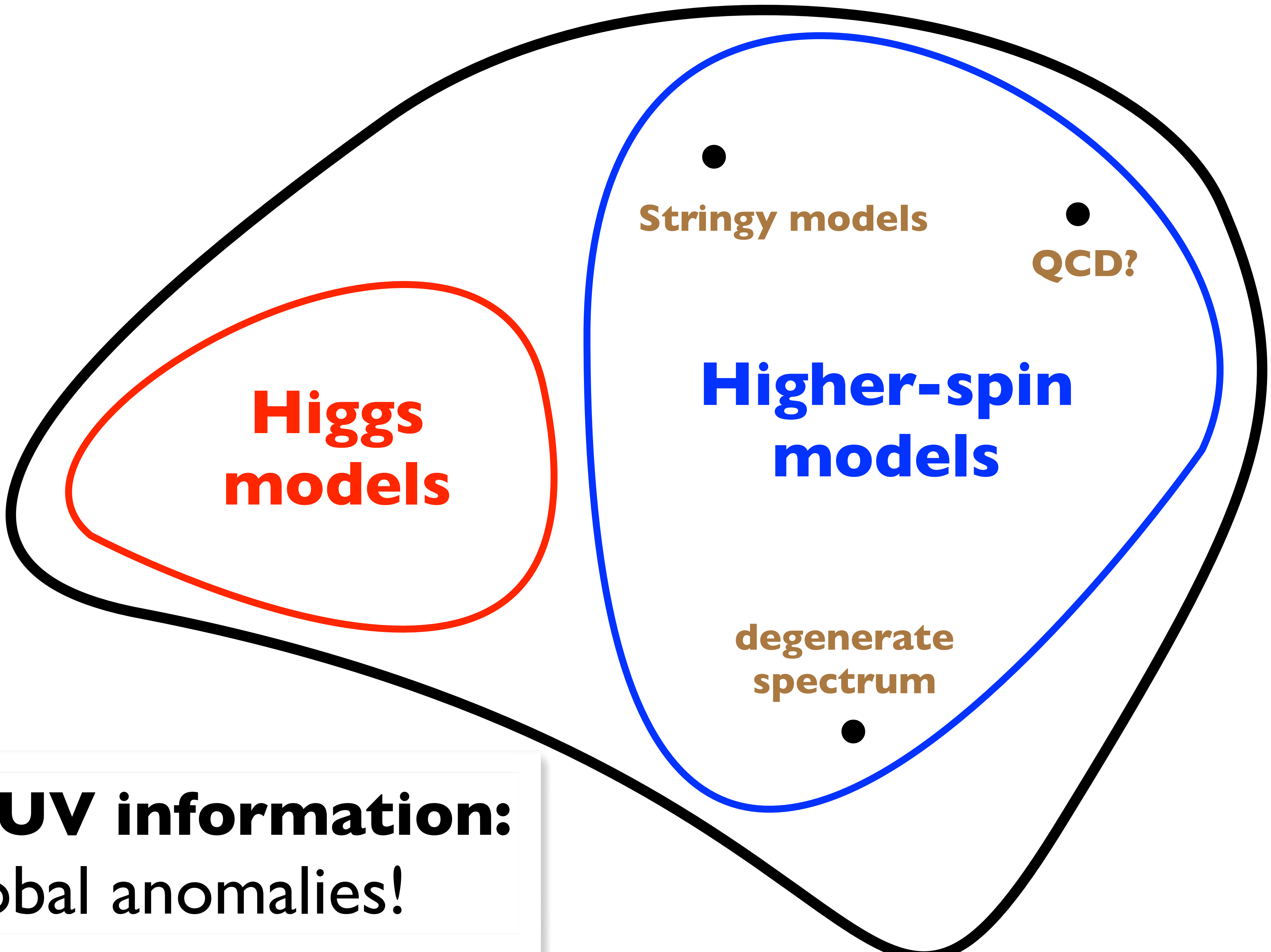

Extra UV information:
Global anomalies!

$U(1)_A$ axial anomaly

Introducing the η' (Goldstone of an anomalous symmetry):

$$U(2)\otimes U(2) \rightarrow U(2)$$

$$SU(2)\otimes SU(2)\otimes U(1)_A\otimes U(1)$$

$U(1)_A$ axial anomaly

Introducing the η' (Goldstone of an anomalous symmetry):

$$U(2)\otimes U(2) \to U(2)$$

$$\hookrightarrow SU(2)\otimes SU(2)\otimes U(1)_A \otimes U(1)$$

- WZW term: *5-goldstone int.*
- Adding external gauge-bosons:

$$\pi \to \gamma\gamma$$

$$\left.\right\} \propto \kappa$$

$$\kappa = \frac{N_c}{12\pi^2 F_\pi^3}$$

two q_L,q_R model

U(1)$_A$ axial anomaly

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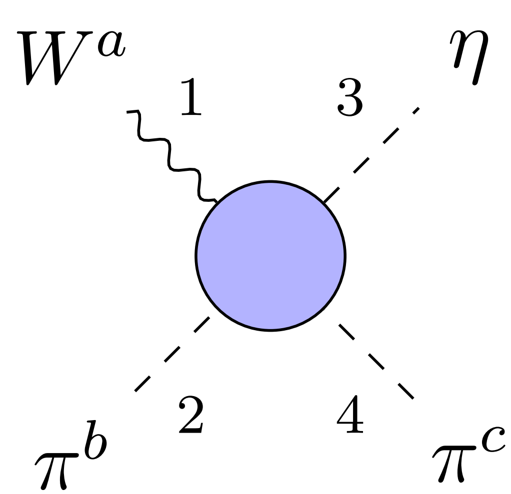
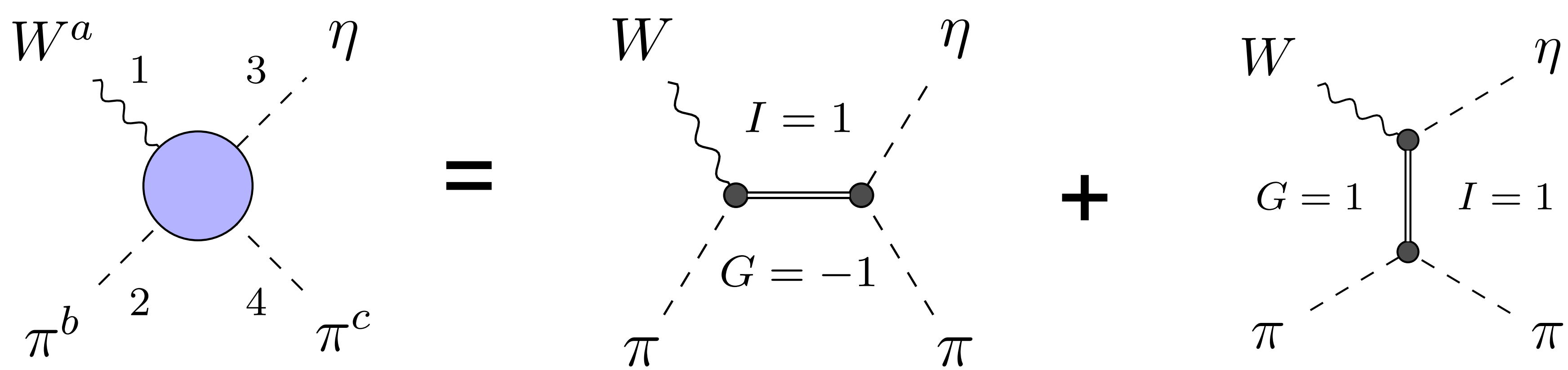
two q_L,q_R model

but also



low-energy $\longrightarrow$ κ

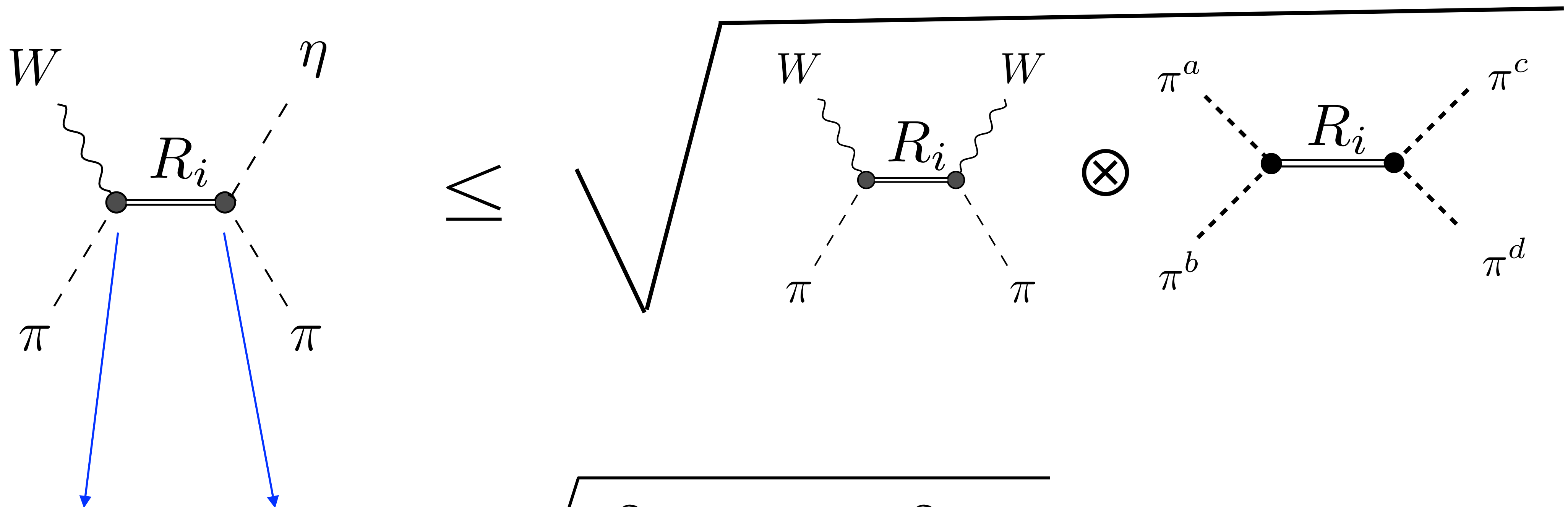


a) It cannot be mediated by scalars

☞ axial anomaly **discards** theories with **only** scalar resonances

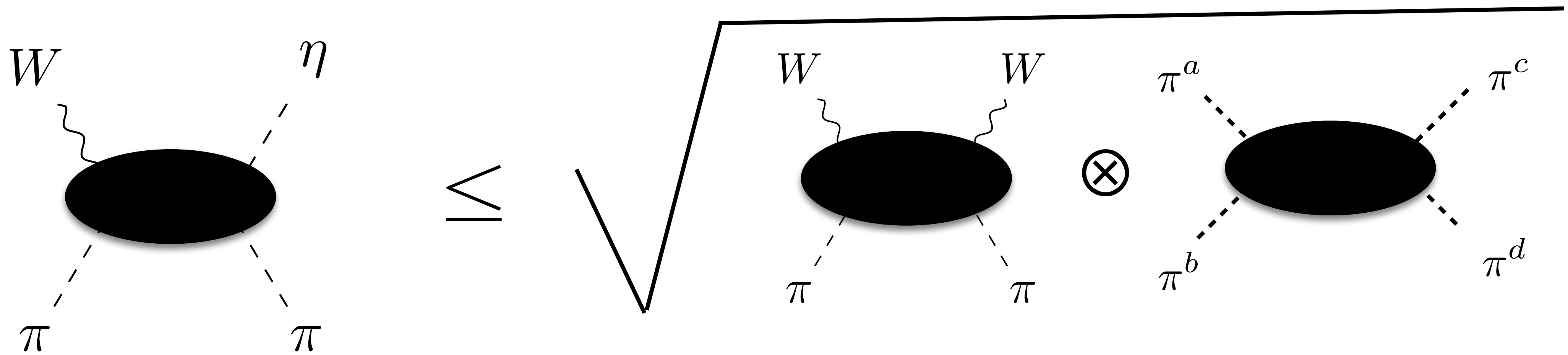
b) Bounded

How a bound on the anomaly arises:



$$g_{W\pi R_i} \times g_{\pi\pi R_i} \leq \sqrt{g^2_{W\pi R_i} \times g^2_{\pi\pi R_i}}$$

How a bound on the anomaly arises:



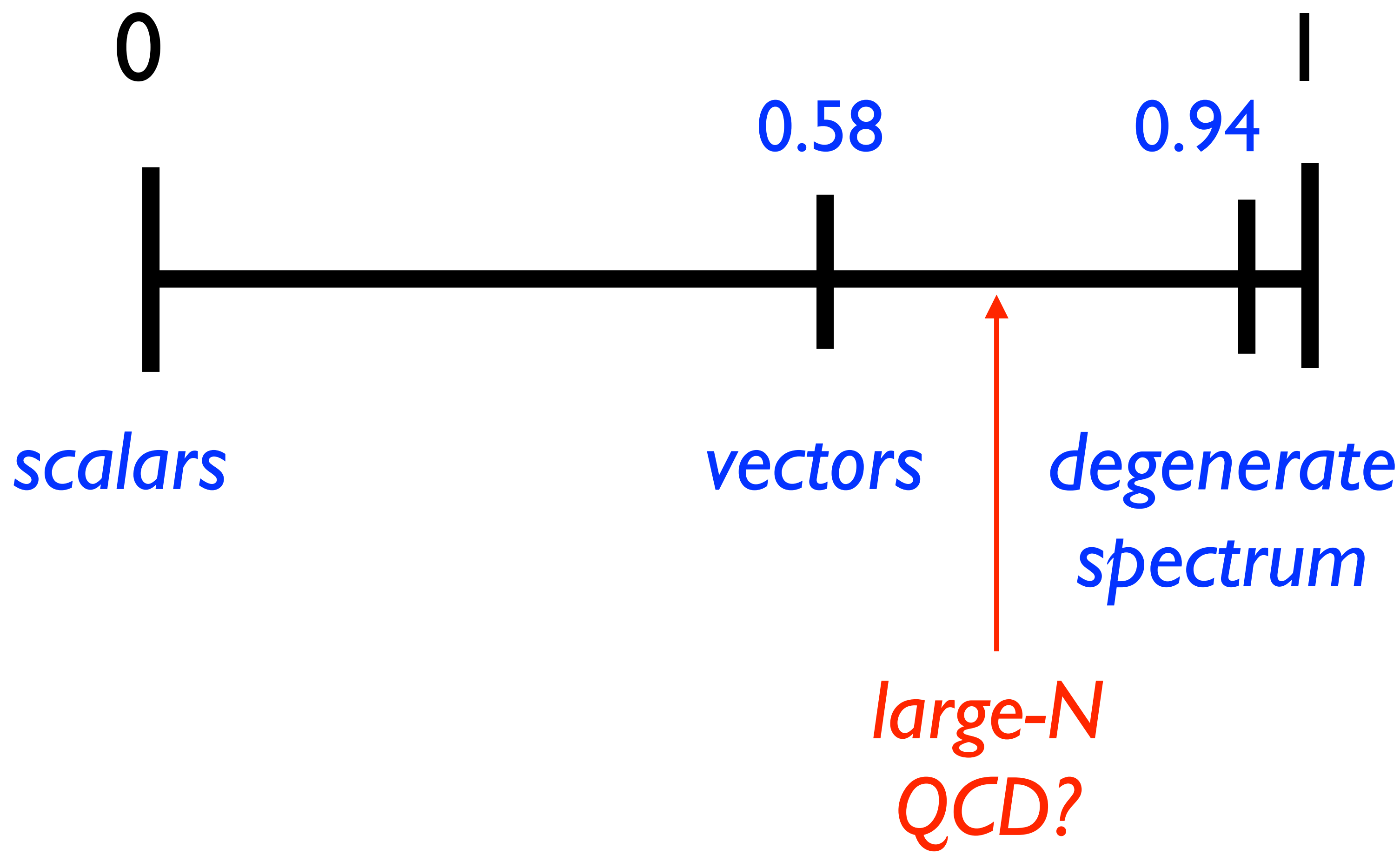
Amazingly, a bound can be extended in general (from positivity):

$$\kappa \leq \sqrt{\mathcal{P}\,\frac{1}{2F_\pi^2}}$$

pion polarizabilities

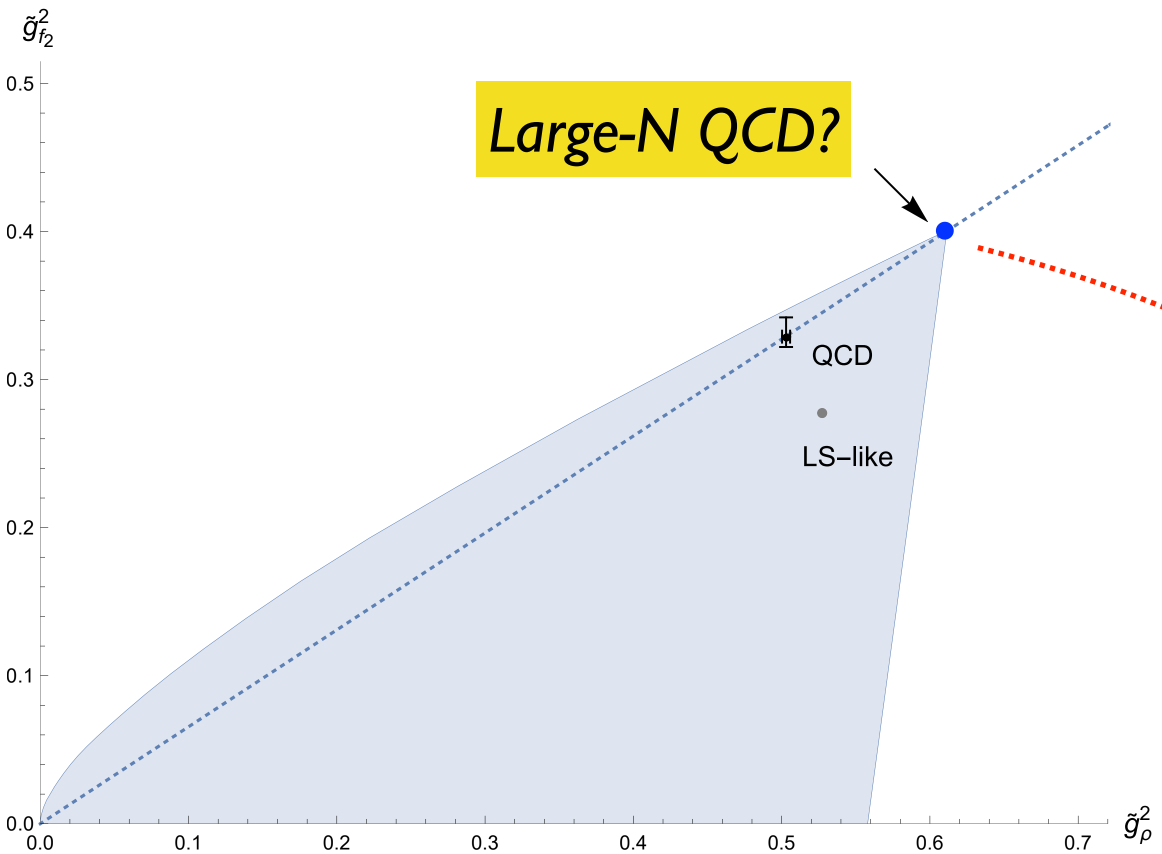
see also J. Albert and L. Rastelli, arXiv: 2307.01246

$$\kappa\sqrt{\frac{2F_\pi^2}{\mathcal{P}}}$$

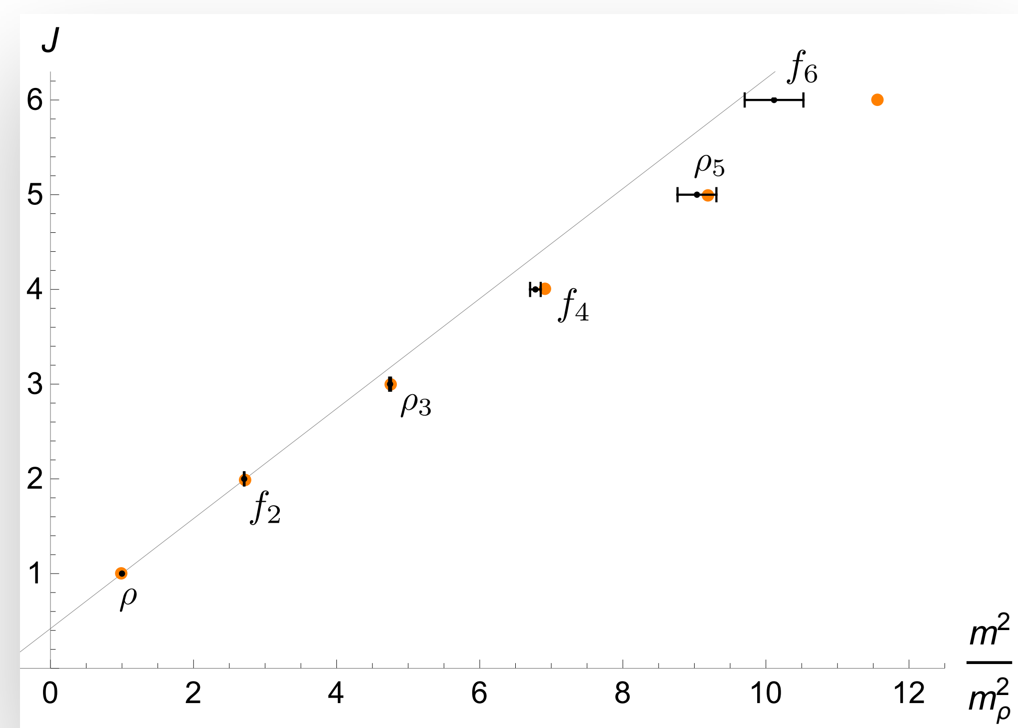


J.Albert, J.Henriksson, L.Rastelli, A.Vichi arXiv:2312.15013

spin-2 coupling to pions



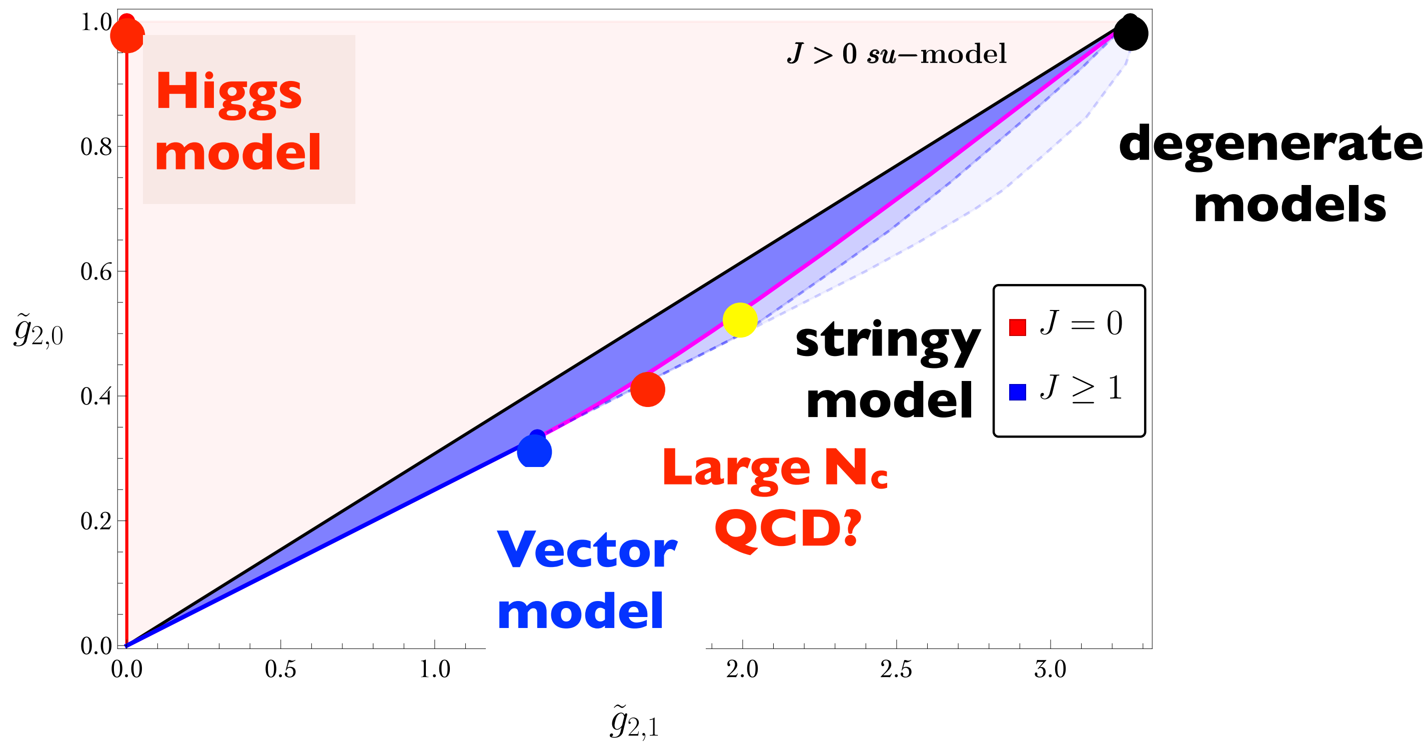
spin-1 coupling to pions



Conclusions

- ***Crossing* + *Analyticity* + *Unitarity*** allow to get information on possible **UV completions of theories of Goldstones**:

- Predicts a **“EFT-hedron”** structure



- Higher-spin (J>1) mesons are strongly constrained, giving a possible explanation for VMD & the success of holographic QCD

- **Axial anomaly** can discriminate between the possibilities

Bounded from above:

$$\frac{\kappa}{\sqrt{\mathcal{P}/F_\pi^2}} \leq \frac{1}{\sqrt{2}}$$

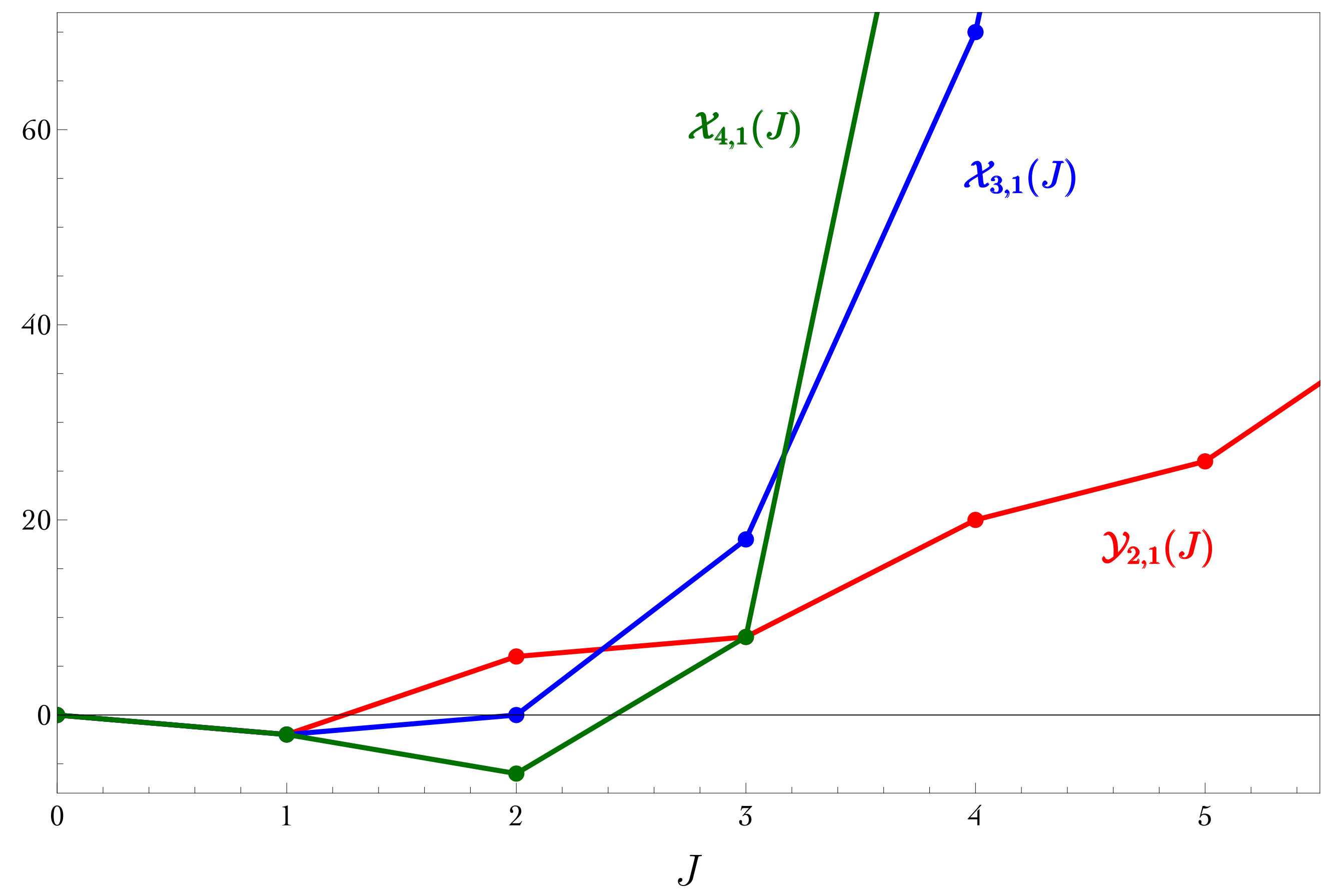
- **Gravitational anomaly?**

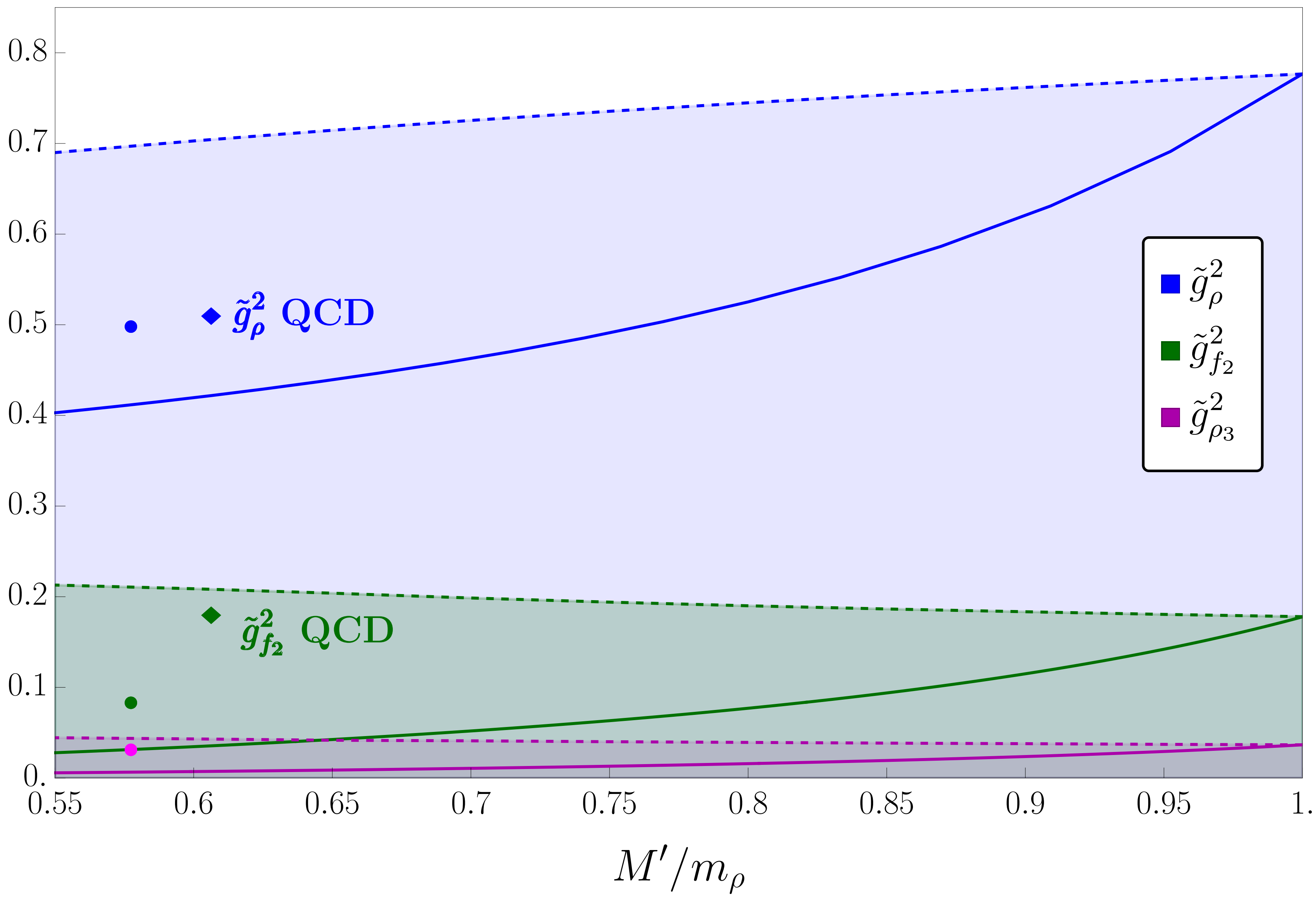

RESTRICTED AREA
MONITORED
BY VIDEO
CAMERA

$$0 = \sum_i \frac{|g_{\pi\pi\, i}|^2}{m_i^{2n}} \Bigg(\underbrace{\frac{2^{n-1}}{(n-1)!^3} P_{J_i}^{(n-1)}(1) - \mathcal{J}_i^2}_{\mathcal{X}_{n,1}} \Bigg)$$

n=2,3,4,...

$\mathcal{J}^2 \equiv J(J+1)$

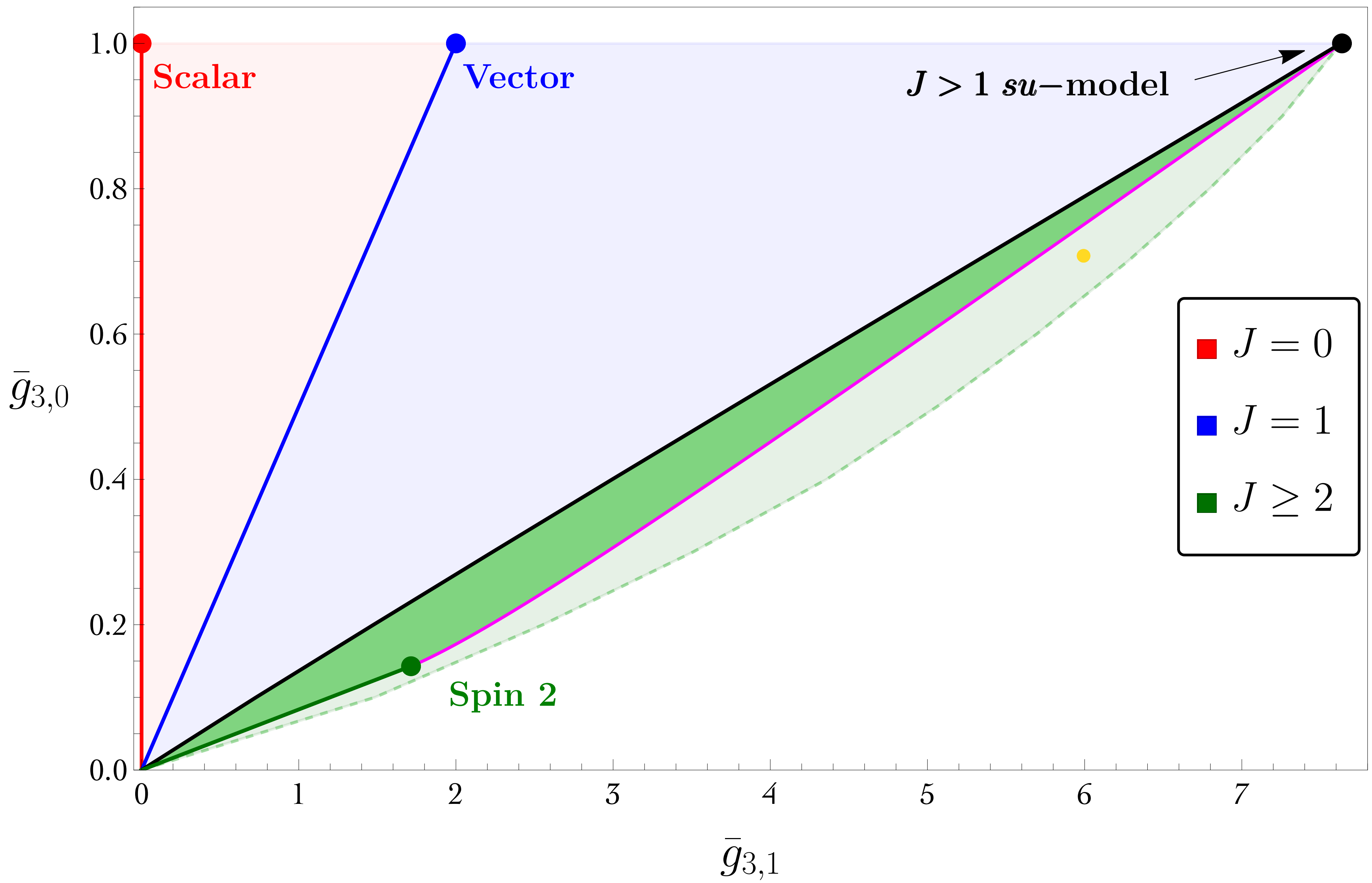




0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.
0.55
0.6
0.65
0.7
0.75
0.8
0.85
0.9
0.95
1.
M'/m_ρ
$\tilde{g}^2_\rho$ QCD
$\tilde{g}^2_{f_2}$ QCD
$\tilde{g}^2_\rho$
$\tilde{g}^2_{f_2}$
$\tilde{g}^2_{\rho_3}$

Lets assume at s$\rightarrow\infty$ and either t or u fixed:

$$\frac{\mathcal{M}_t^{I=2}(s,u)}{s^2} \to 0$$



Scalar
Vector
$J > 1$ $su-$model
Spin 2
$J = 0$
$J = 1$
$J \geq 2$
$\bar{g}_{3,0}$
$\bar{g}_{3,1}$

C The su-models

Let us consider the most general theory of a degenerate spectrum that contributes to the four-pion amplitude $\mathcal{M}(s,u)$ [7, 8]. This means that all states have equal mass m, and therefore the denominator of this amplitude is fixed to be $\mathcal{M}(s,u) \propto 1/((s-m^2)(u-m^2))$. If we further demand that Eq. (6a) and Eq. (6b) are satisfied for $k_{\min}=1$, we are led to

$$\mathcal{M}(s,u) = \frac{a_1 m^4 + a_2 m^2 (s+u) + a_3 su}{(s-m^2)(u-m^2)}\,, \tag{91}$$

where a_i are constants. The Adler's zero condition fixes $a_1 = 0$. Then, aside from a global multiplicative factor, the amplitude has only one free parameter. We can write it as

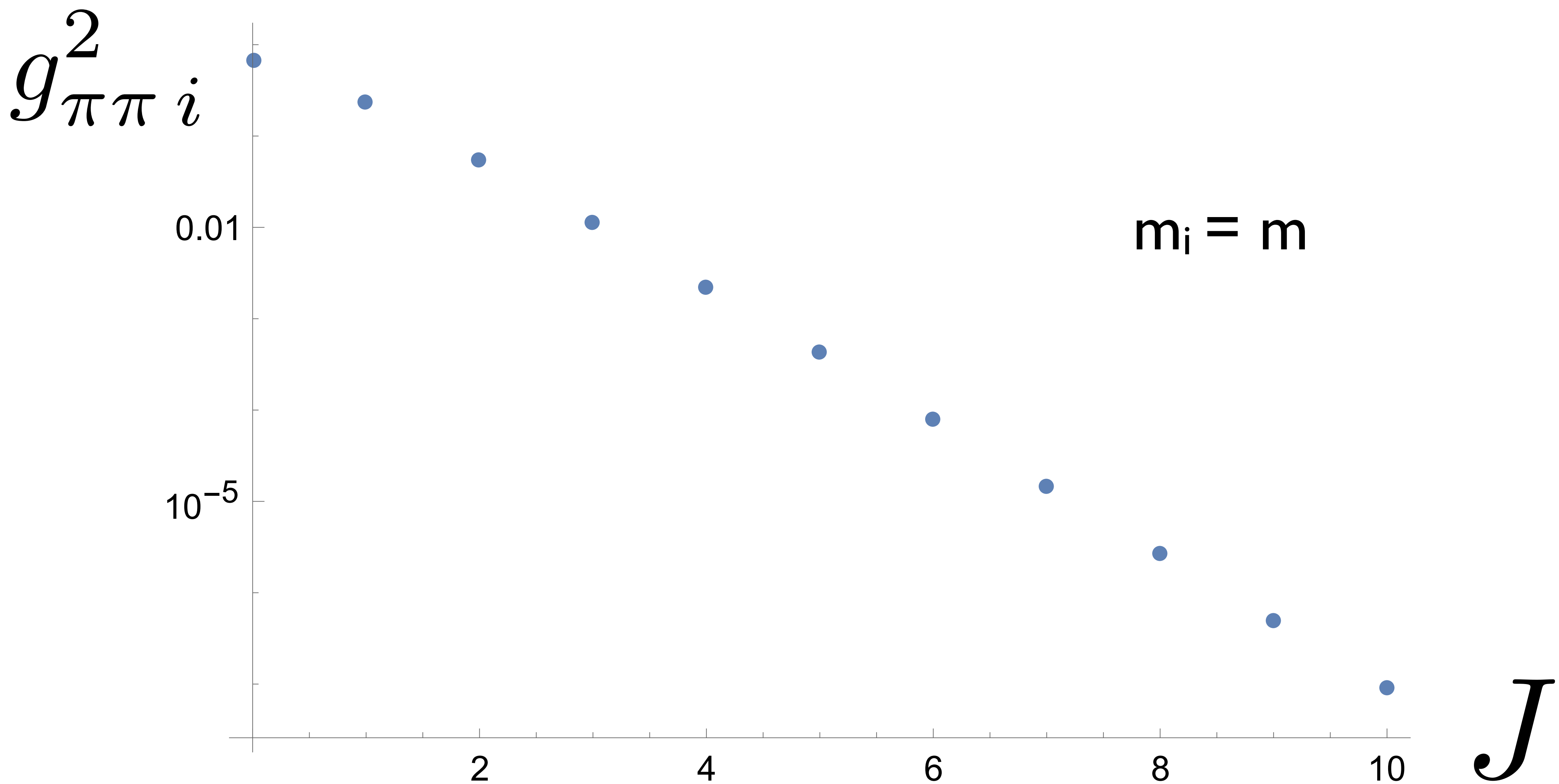
$$\mathcal{M}_1^{(su)}(s,u) = \frac{m^2(s+u) + \lambda su}{(s-m^2)(u-m^2)}\,, \tag{92}$$

where the possible values of λ are determined by unitarity. Indeed, imposing the positivity of the residues of Eq. (92), we obtain

$$-2 \leq \lambda \leq \frac{2\ln 2 - 1}{1-\ln 2}\,. \tag{93}$$

In the limiting case $\lambda = -2$, the residues of all $J>0$ states are zero, and we are left with the scalar amplitude Eq. (22). In the other limit,

$$\lambda = \frac{2\ln 2 - 1}{1-\ln 2} \simeq 1.26\,, \tag{94}$$



$g^2_{\pi\pi\, i}$
0.01
10^{-5}
$m_i = m$
2
4
6
8
10
J

D The Lovelace-Shapiro amplitude

The Lovelace-Shapiro (LS) amplitude for the scattering of four pions is defined as [26, 27]

$$\mathcal{M}^{(\mathrm{LS})}(s,u) = \frac{\Gamma(1-\alpha(s))\Gamma(1-\alpha(u))}{\Gamma(1-\alpha(s)-\alpha(u))} \, , \tag{105}$$

where $\alpha(s) = \alpha_0 + \alpha' s$ is referred as the Regge trajectory. We will fix the values of α_0 and α' by requiring that Eq. (106) satisfies the Adler zero condition, $\mathcal{M}^{(\mathrm{LS})}(s,u) \to 0$ for $s, u \to 0$, and that the first pole of Eq. (106) occurs for $s = m_\rho^2$. These two conditions lead to $\alpha_0 = 1/2$ and $\alpha' = 1/(2m_\rho^2)$ [66] and then we can write

$$\mathcal{M}^{(\mathrm{LS})}(s,u) = \frac{\Gamma\left(\frac{1}{2} - \frac{s}{2m_\rho^2}\right)\Gamma\left(\frac{1}{2} - \frac{u}{2m_\rho^2}\right)}{\Gamma\left(\frac{t}{2m_\rho^2}\right)} \, . \tag{106}$$

By looking at the poles of Eq. (106), one can see that the LS amplitude corresponds to a theory of higher-spin states with masses

$$m_n^2 = m_\rho^2(2n+1) \, , \quad n = 0, 1, 2, \ldots \, . \tag{107}$$

For a given n, there are at most $n+1$ states with spin $J = 0, 1, ..., n+1$. Furthermore, Eq. (106) satisfies the condition Eq. (6a) and Eq. (6b) with $k_{\min} = 1$.

E The Coon amplitude

The Lovelace-Shapiro amplitude presented in Appendix D can be generalized to a larger class of amplitudes depending on an additional parameter q. This is the so-called Coon amplitude, which was first proposed in [28][11]:

$$\mathcal{M}_q(s,u) = C(\sigma,\tau,q) \prod_{n=0}^{\infty} \frac{(1-q^{n+1})\,(\sigma\tau - q^{n+1})}{(\sigma - q^{n+1})\,(\tau - q^{n+1})} \ , \tag{118}$$

where $\sigma = 1 + (q-1)(\alpha_0 + \alpha' s)$ and $\tau = 1 + (q-1)(\alpha_0 + \alpha' u)$. As explained in Appendix D, we take $\alpha_0 = 1/2$ and $\alpha' = 1/(2m_\rho^2)$. The parameter q takes values between 0 and 1, and in the limit $q \to 1$ we recover the LS amplitude Eq. (106). There is some freedom in the choice of the prefactor C, as long as it satisfies $\lim_{q\to 1} C(\sigma,\tau,q) = 1$.

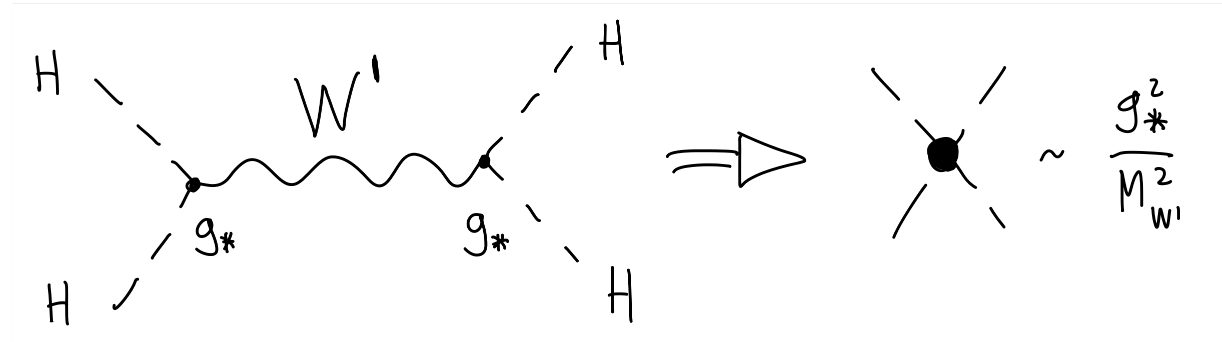
The Coon amplitude has an infinite number of simple poles at

$$s_n = m_\rho^2 \frac{1 + q - 2q^{n+1}}{1-q} \ , \qquad n = 0, 1, 2, \ldots \ . \tag{119}$$

Impact on BSM searches at the LHC

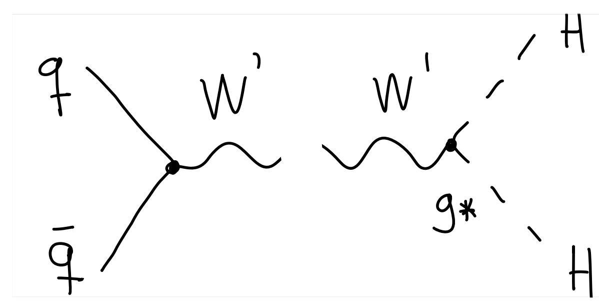
Higgs as a Pseudo-Goldtone boson:

Indirect probes:



deviations in Higgs coupling

Direct probes:



Impact on BSM searches at the LHC

Higgs as a Pseudo-Goldtone boson:

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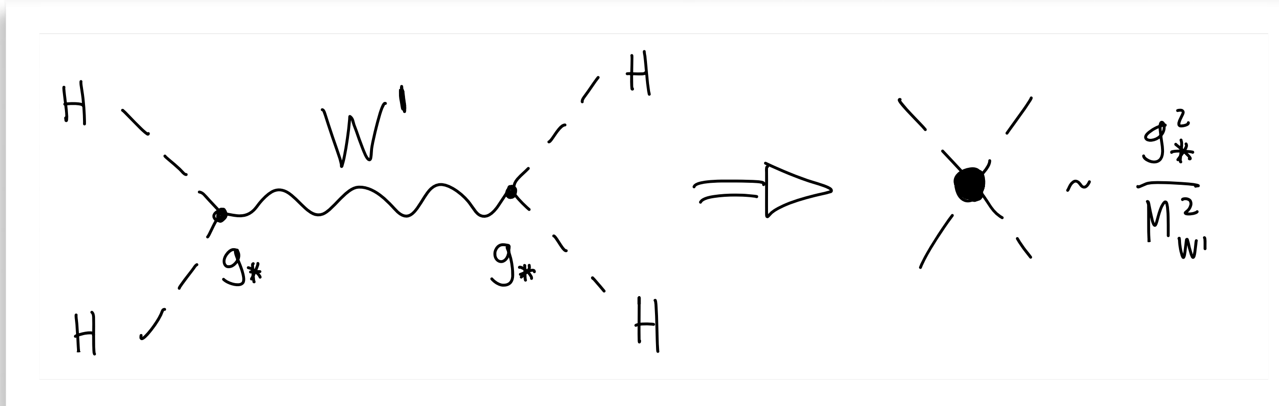
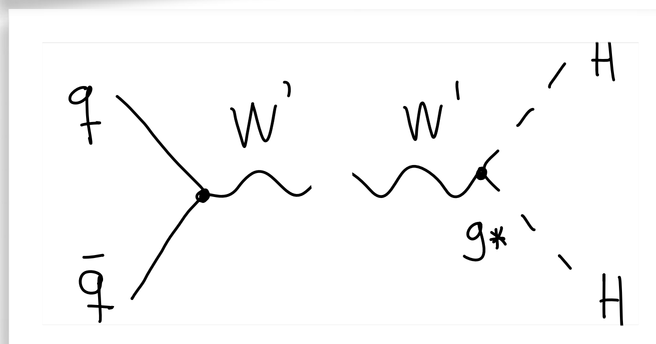
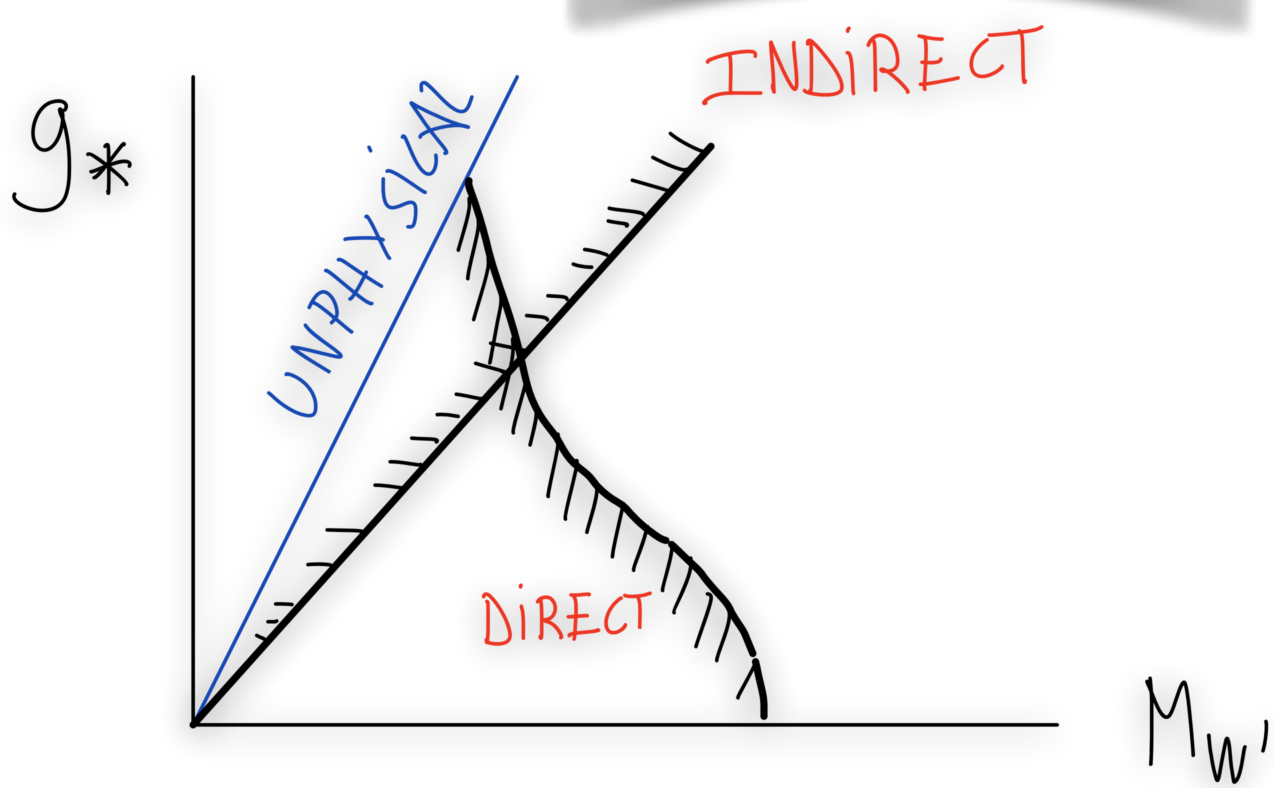


deviations in Higgs coupling

Direct probes:



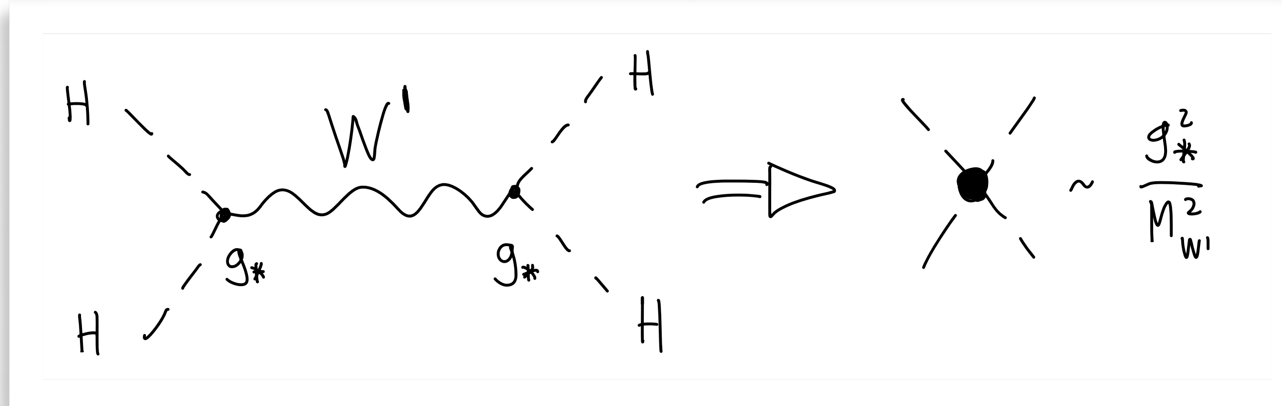


e.g. 1502.01701 [hep-th]

Impact on BSM searches at the LHC

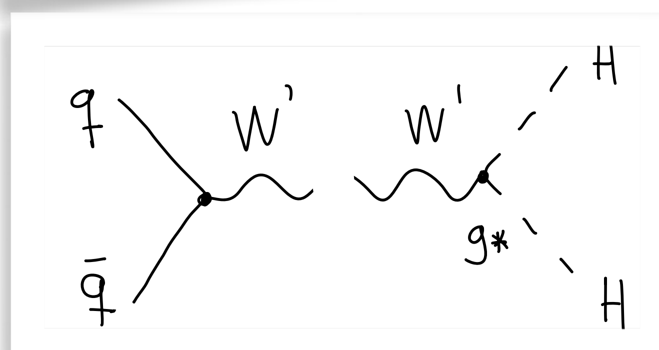
Higgs as a Pseudo-Goldtone boson:

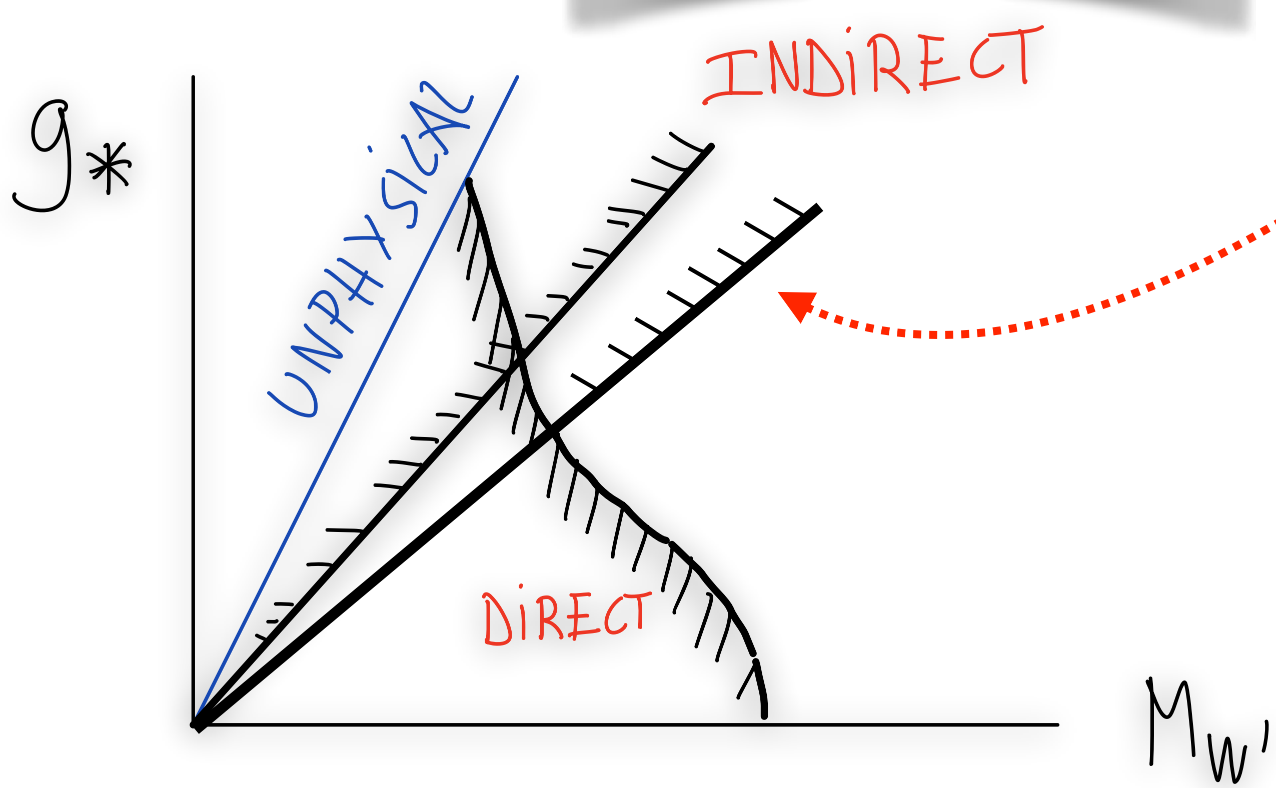
Indirect probes:



deviations in Higgs coupling

Direct probes:





J > 1 must **at least** contribute a **23%** to the Wilson coeff.

e.g. 1502.01701 [hep-th]