Analytic Formulae for Inflationary Correlators with Dynamical Mass

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Based on **arXiv:2312.09642** with

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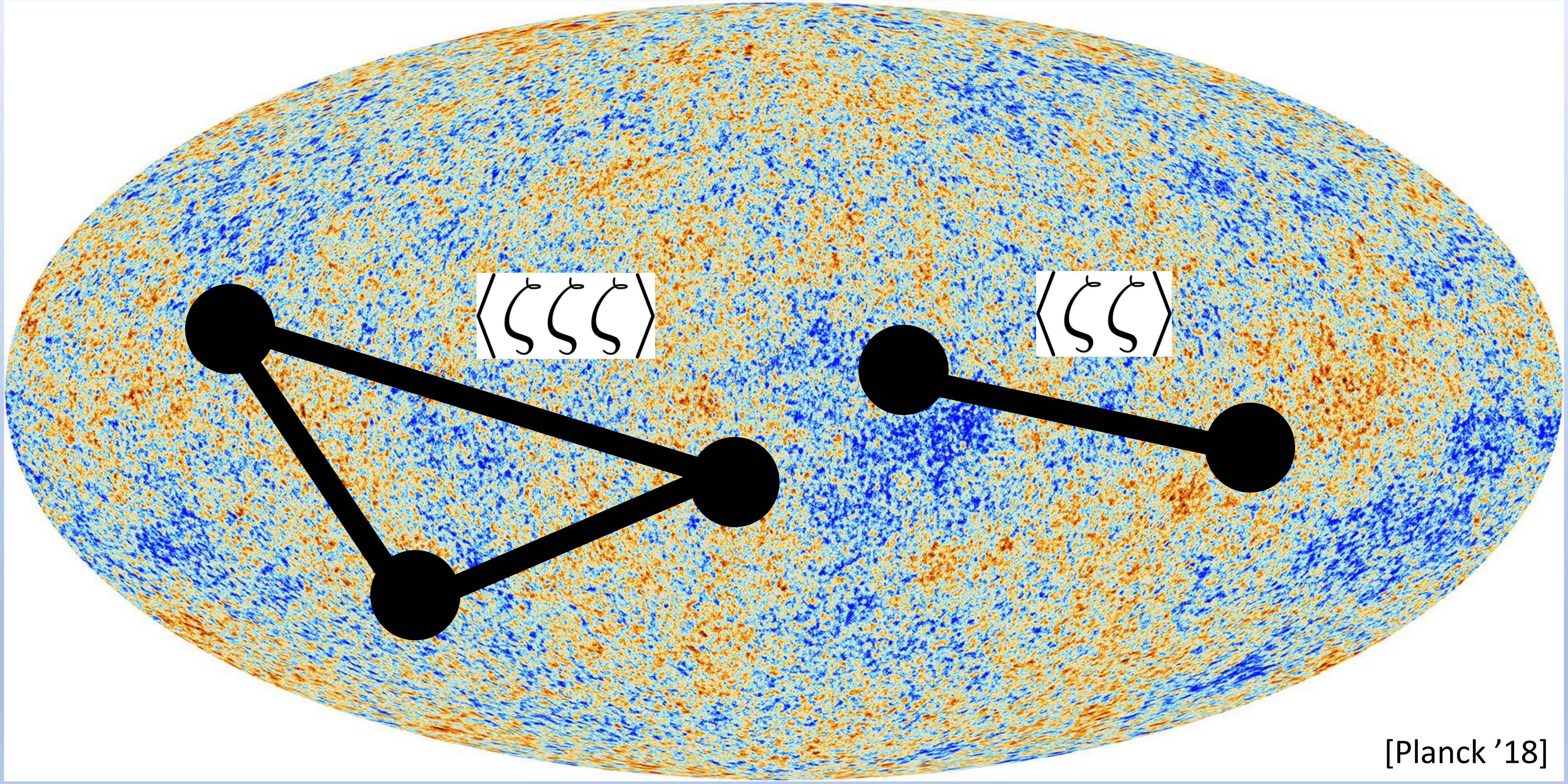
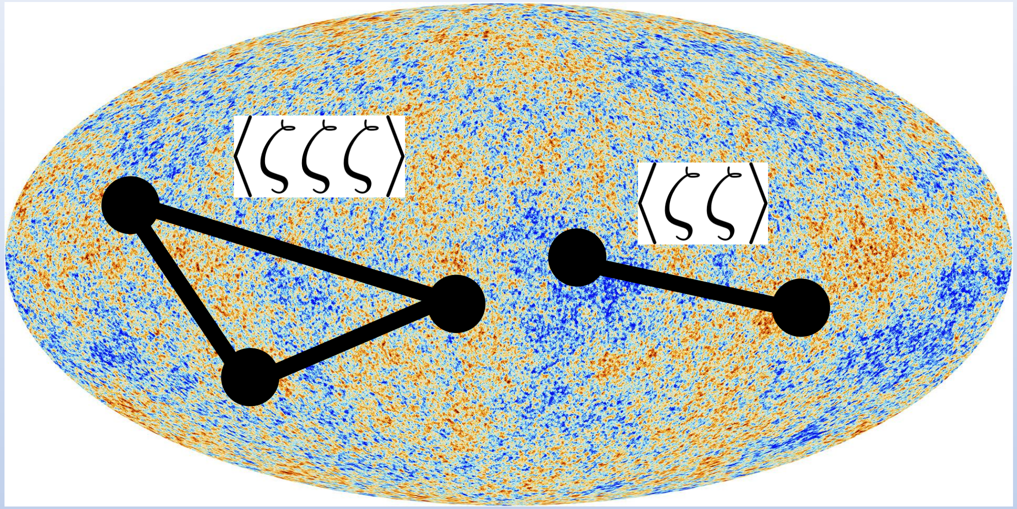
Observables for Inflationary Cosmology



ζ : Scalar curvature perturbation $\langle \zeta \cdots \zeta \rangle$: Correlation functions

Observables for Inflationary Cosmology



2pt. correlation function (power spectrum)

$$\langle\zeta_{\boldsymbol{k}}\zeta_{\boldsymbol{k}'}\rangle_{\text{inf. end}} = (2\pi)^3\delta^3(\boldsymbol{k}+\boldsymbol{k}')\frac{2\pi^2}{k^3}P_\zeta$$

$$P_\zeta \simeq \frac{H^2}{8\pi^2\epsilon}\left(\frac{k}{k_*}\right)^{n_s-1} \quad , \quad n_s \simeq 0.965 \quad \frac{dn_s}{d\log k} \simeq 0.002$$

[Planck '18]

➡ Degeneracy of inflation models

3pt. correlation function (bispectrum) ⟵ Not yet observed in sufficient accuracy

$$\langle\zeta_1\zeta_2\zeta_3\rangle_{\text{inf. end}} = (2\pi)^7\delta^3(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)\frac{P_{\zeta*}^2}{(k_1k_2k_3)^2}S\left(\frac{k_1}{k_3},\frac{k_2}{k_3}\right)$$

[Chen and Wang '09]

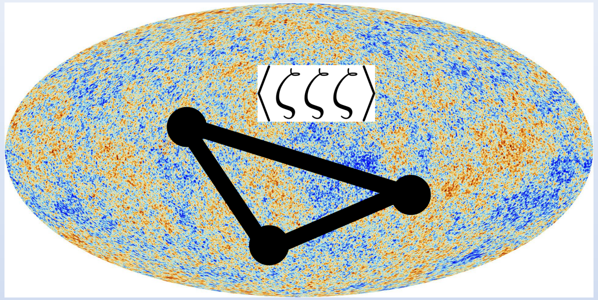
S : dim. less, model dependent shape function

$\langle\zeta\zeta\zeta\rangle$: effects of interactions in perturb. theory of QFT

➡ Probe for inflation models and BSM physics

Cosmological Collider Project

[Chen and Wang ’09, Noumi et al. ’12, Arkani-Hamed and Maldacena, ’15, Lee et al. ’16 etc.]



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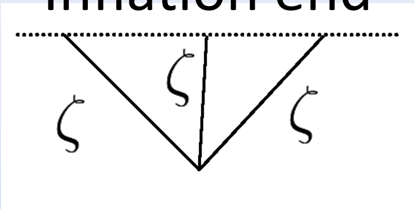
Inflation end



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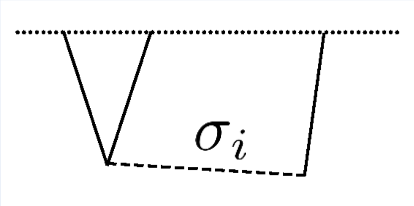

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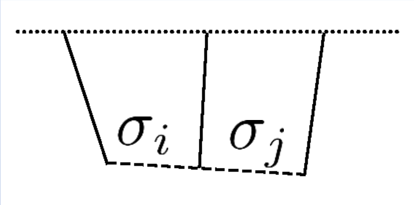

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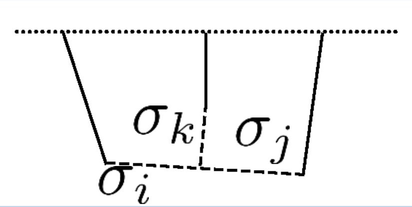


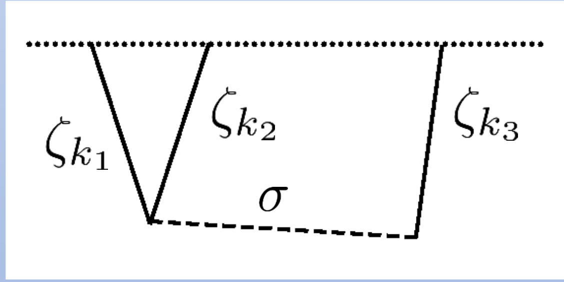
$+ \cdots$

Signals for massive particles

$$S \sim \left(\frac{k_3}{k_1}\right)^{1/2} e^{-\pi\mu} \underline{\underline{\cos\left(\mu \log\frac{k_1}{k_3} + \delta\right)}}$$

$k_3 \ll k_1 \simeq k_2$

$$\mu = \sqrt{\left(\frac{m_\sigma}{H}\right)^2 - \frac{9}{4}}$$



Mass: wavelength of the shape function

Dictionary for particles of BSM in high energy scale $\rho_{\rm inf}^{1/4} \lesssim 10^{15}\ \mathrm{GeV}$

Supersymmetry, gauge symmetry, CP violation, swampland, ...

[Baumann and Green ’12] [Maru and Okawa ’21] [Liu et al. ’21] [Reece et al. ’22]

Q. Distinction of Interactions?

Some Directions:

- Phase of oscillation [Qin and Xianyu '22]

$$S \sim \left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{1/2} e^{-\pi\mu} \cos\left(\mu \log \frac{k_{\rm L}}{k_{\rm S}} + \underline{\underline{\delta}}\right)$$

Expected to be uniquely determined by $\mu,\ \frac{k_{\rm L}}{k_{\rm S}},\ \text{spin},\ \text{diagram}$

- Non-unity sound speed in EFT [Jazayeri et al. '22, Jazayeri and Renaux-Petel '23]

A peak in not-so-squeezed region $\frac{k_{\rm L}}{k_{\rm S}} \sim c_s$ (sound horizon crossing)

- Beyond scale invariant approx. (our work)

Scale dependence $\Leftrightarrow$ De Sitter sym. breaking $\Leftrightarrow$ Non-der. ints.

Derivative ints.: $f(\partial_\mu\phi, \sigma, \partial_\mu\sigma)$

- respect shift sym. of ϕ (de Sitter)
- EFT, SUGRA, etc.

Non-derivative ints.: $f(\phi, \sigma, \partial_\mu\sigma)$

- break shift sym. (slow-roll effects)
- Higgs, axion, extra dim., etc.

$\phi\bar{\psi}\psi \quad \phi F^{\mu\nu}\tilde{F}_{\mu\nu} \quad e^{\alpha\phi/M_{\rm Pl}}\sigma^2$

Demonstration of Scale Dependence (our calculation)

Approximated / numerical results: [Wang '19, Reece et al. '22]

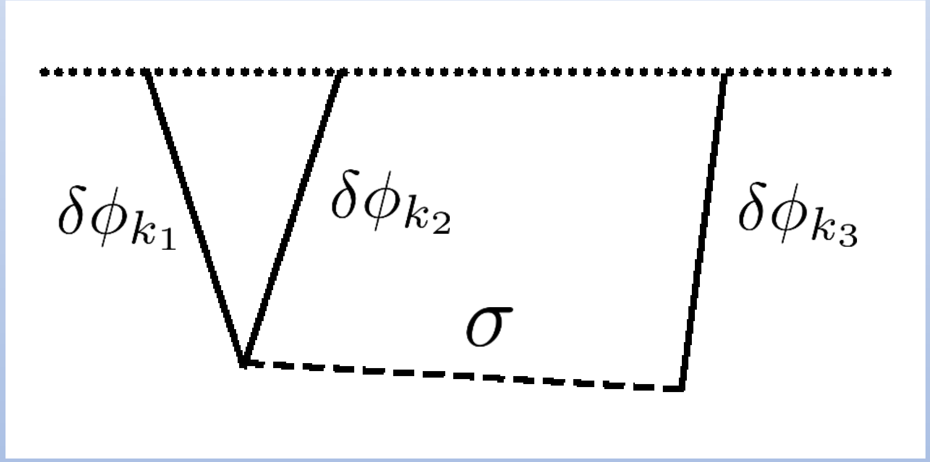
Action for Inflaton ϕ + massive scalar spectator σ

$$S = \int dx^4 \sqrt{-g} \left[\frac{M_{\rm pl}^2}{2} R - \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi) - \frac{1}{2}(\partial_\mu \sigma)^2 - \frac{1}{2} m_0^2 \sigma^2 - M_{\rm pl} y \phi \sigma^2 + \mathcal{L}_{\rm diag} \right]$$

Sym. breaking interaction

Interactions for the diagram

$$\mathcal{L}_{\rm diag} \supset c_2 (-\tau)^{-3} \sigma \delta\phi' + c_3 (-\tau)^{-2} \sigma (\delta\phi')^2$$



Time dependent mass (excursion of inflaton)

$$m_{\rm eff}^2 = m_0^2 + 2y M_{\rm pl} \phi_0$$

$$\phi_0' = \frac{\sqrt{2\epsilon} M_{\rm pl}}{\tau}$$

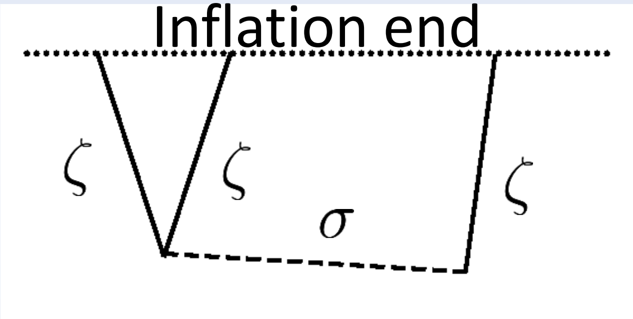
Slow-roll approx. $\phi_0 = \sqrt{2\epsilon} M_{\rm pl} \log \frac{\tau}{\tau_0}$

Linear approx. $\phi_0(\tau) \simeq \phi_{*0} - \sqrt{2\epsilon} M_{\rm pl} \left(1 - \frac{\tau}{\tau_*}\right)$

Initial condition $\phi_{*0} \simeq \sqrt{2\epsilon} M_{\rm pl} \log \frac{\tau_*}{\tau_0}$

Additional scale τ_0, τ_*

Analytic Calculation for Cosmo. Collider



$$\sim \int_{-\infty}^{0} d\tau_2 \int_{-\infty}^{0} d\tau_1 (-\tau_1)^{p_1} (-\tau_2)^{p_2} \underbrace{\theta(\tau_1 - \tau_2)}_{\text{Time-ordered}} e^{i(k_1\tau_1 - (k_2+k_3)\tau_2)} \underbrace{H^{(1)}_{i\mu}(-k_1\tau_1) H^{(2)}_{-i\mu}(-k_1\tau_2)}_{\text{Massive propagator}}$$

$$\mu^2 = \frac{m_\sigma^2}{H^2} - \frac{9}{4}$$

Analytical solution: convenient for extracting parameter dependence

But, the time-ordered integration is difficult (divergence $\Rightarrow$ $i\epsilon$, resum, etc.)

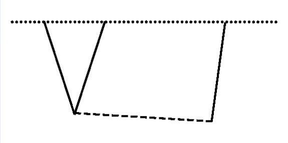
Newly developed approaches: Bootstrapping, Mellin-Barnes rep., cutting rule etc.

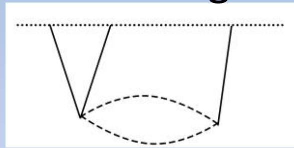
[Series of papers by Baumann, Lee, Pimentel et al. ’18, ’20, ’21, Pajer et al. ’20, Sleight ’19, Sleight and Taronna ’19, Melville and Pajer ’21 etc.]

Our work: bootstrapping and Mellin-Barnes representation

which is used for single exchange of scalar & vector, one-loop of scalar, …

[Qin and Xianyu ’22 and ’23] [Xianyu and Zhang ’22]





Overview of Bootstrap Method

[Series of papers by Baumann, Lee, Pimentel et al. ’18, ’20, ’21, Pajer et al. ’20, Qin and Xianyu ’22, ’23 etc.]

The mode functions are characterized by the value of $k\tau$

e.g., curvature perturbation $u_k = \frac{H}{\sqrt{4\epsilon k^3}}(1+ik\tau)e^{-ik\tau}$

$$\Rightarrow \left(\tau^2\frac{d^2}{d\tau^2} - 2\tau\frac{d}{d\tau} + k^2\tau^2\right)(\sqrt{k^3}u_k(\tau)) = 0 \quad \Longleftrightarrow \quad \left(k^2\frac{d^2}{dk^2} - 2k\frac{d}{dk} + k^2\tau^2\right)(\sqrt{k^3}u_k(\tau)) = 0$$

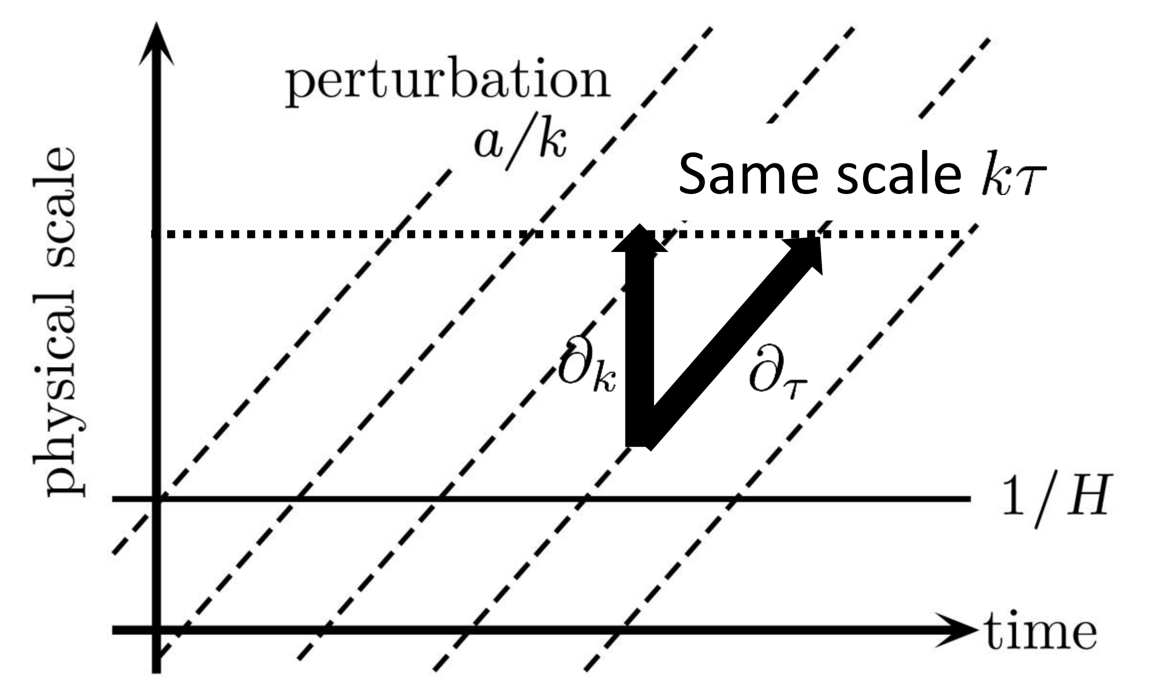
$\tau\partial_\tau \Leftrightarrow k\partial_k$

Thus, shape function

$$S(k_i) \sim \int d\tau_1 d\tau_2 (\text{propagators})$$

can be derived via diff. eq.

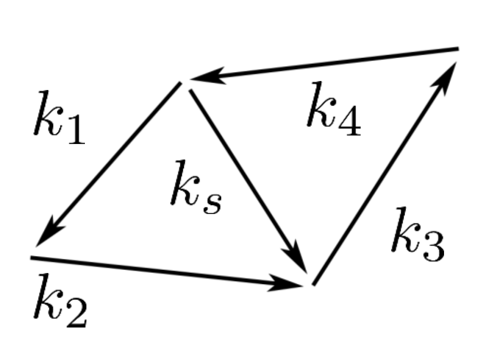
$$\mathcal{D}_{k_i}S \sim \int d\tau_1 d\tau_2 \mathcal{D}_{k_i}(\text{prop.}) \sim \int d\tau_1 d\tau_2 \delta(k_i\tau_1 - k_i\tau_2)$$

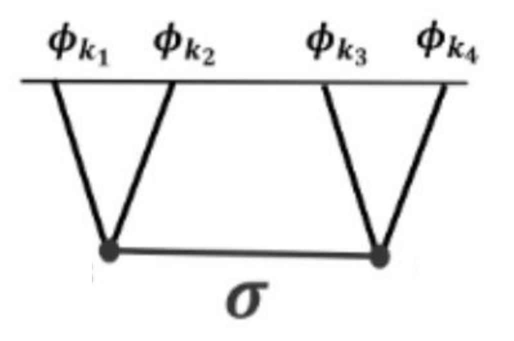


Constants of integration: matching with
direct integration in some limits

Bootstrap method

[Series of papers by Baumann, Lee, Pimentel et al. '18, '20, '21, Pajer et al. '20, Qin and Xianyu '22, '23 etc.]





Seed integrals

$$\mathcal{I}_{\mathrm{ab}}^{p_1p_2}(k_s,k_i) = -\mathrm{ab}k_s^{5+p_{12}}\int_{-\infty}^{0} \mathrm{d}\tau_1\ \mathrm{d}\tau_2\,(-\tau_1)^{p_1}\,(-\tau_2)^{p_2}\,e^{\mathrm{i}ak_{12}\tau_1+\mathrm{i}bk_{34}\tau_2}D_{\mathrm{ab}}\,(k_s;\tau_1,\tau_2)$$

Bispectrum $\langle\zeta^3\rangle \propto \dfrac{c_2c_3}{8k_1k_2k_3^4}\lim_{k_4\to 0}\sum_{\mathrm{a,b}=\pm}\mathcal{I}_{\mathrm{ab}}^{0,-2}+(k_3\to k_1,k_2)$

Strategy: Deriving equations of motion for the seed integrals

EoM for σ $\Rightarrow$ EoM for propagators $D_{\mathrm{ab}}\,(k;\tau_1,\tau_2)\sim\langle\sigma_k(\tau_1)\sigma_k(\tau_2)\rangle$

Effect of time dep. Mass introduced in our work

$$\mathcal{D}_{z_i}\widehat{D}_{ab}(r_1z_1,r_2z_2) = -ia\delta_{ab}H^2(r_1z_1)^2(r_2z_2)^2\delta(r_1z_1-r_2z_2)$$

$$\mathcal{D}_{z_i} = z_i^2\partial_{z_i}^2 - 2z_i\partial_{z_i} + r_i^2z_i^2 + \mu^2 + \frac{9}{4} - 2\gamma r_iz_i$$

$$z_1=-(k_1+k_2)\tau_1,\ z_2=-(k_3+k_4)\tau_2,\ r_1=\frac{k_s}{k_1+k_2},\ r_2=\frac{k_s}{k_3+k_4},\ \widehat{D}_{ab}=k_s^3D_{ab},\ \gamma=\pm\frac{y\sqrt{2\epsilon}M_{\mathrm{pl}}^2}{H^2}$$

Combinations r_iz_i ➡ Conversion of derivative $z_i\partial_{z_i}\leftrightarrow r_i\partial_{r_i}\quad(\mathcal{D}_{z_i}\leftrightarrow\mathcal{D}_{r_i})$

$$\mathcal{D}_{r_i}\mathcal{I}_{\mathrm{ab}}^{p_1p_2}(r_1,r_2) = \delta_{\mathrm{ab}}H^2e^{\mp\mathrm{i}p_{12}\pi/2}\Gamma\,(5+p_{12})\left(\frac{u_1u_2}{2\,(u_1+u_2-u_1u_2)}\right)^{5+p_{12}}$$

Solutions and the Constants of Integration

Solutions of bootstrap eq.

$$\mathcal{I}_{ab} = \sum_{c,d=\pm} A_{ab|cd} \mathcal{V}^{p_1}_{a|c}(u_1)\mathcal{V}^{p_2}_{b|d}(u_2) + \mathcal{G}^{p_1 p_2}_{ab}(u_1, u_2)$$

$A_{ab|cd}$: integration constant

$$\mathcal{V}^{p}_{a|c}(u) = \cdots \left(\frac{u}{2}\right)^{5/2+p+iac\mu} {}_2\mathcal{F}_1(\cdots; u) \qquad \mathcal{G}^{p_1 p_2}_{ab} = \cdots \sum_{n=0}^{\infty} \cdots u_1^{n+p_{12}+5}\left(1-\frac{1}{u_2}\right)^n {}_3F_2(\cdots; u_1)$$

Int. const.: matching with the direct integration

Method: Mellin-Barnes representation

[Sleight '19, Sleight and Taronna '19, Qin and Xianyu '22, '23]

$$W_{\kappa,i\mu}(z) = e^{z/2}\int_{-i\infty}^{i\infty} \frac{ds}{2\pi i}\frac{\Gamma(s-i\mu)\Gamma(s+i\mu)}{\Gamma(s-\kappa+1/2)} z^{-s+1/2}$$

Cf. $\sigma \propto W_{i\gamma,i\mu}(2ik\tau)$
In time dep. mass case

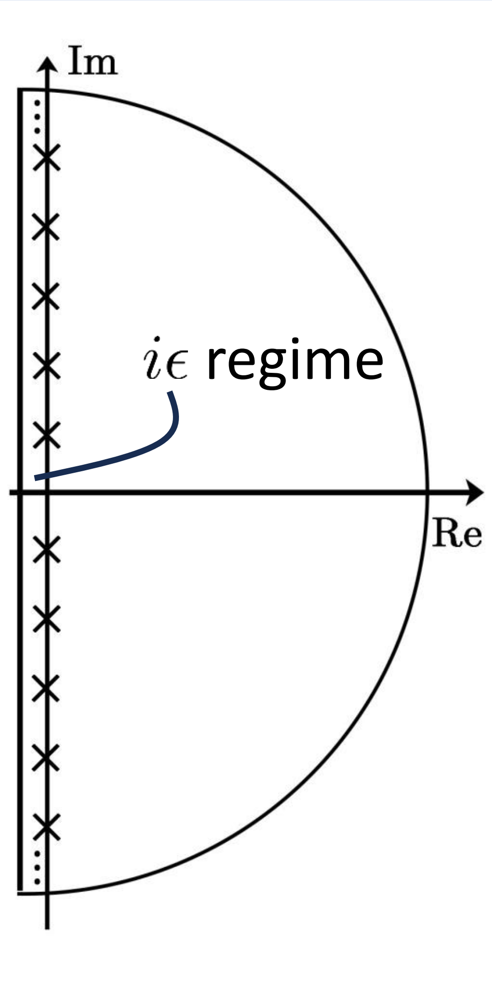
$$\int_{-\infty}^{0} \mathrm{d}\tau_1\, \mathrm{d}\tau_2 \cdots W_{i\gamma,i\mu}(2ik\tau_1) W_{-i\gamma,i\mu}(-2ik\tau_1) = \cdots \sum_n \cdots (\text{residues at } s \pm \imath\mu = -n)$$

Expression is obtained

E.g., $\displaystyle \lim_{u_1,u_2\to 0} \mathcal{I}^{p_1 p_2}_{\pm\pm} = \sum_{a,b=\pm} \tilde{A}_{\pm\pm} \tilde{\mathcal{V}}^{p_1}_{\pm|a}(u_1)\tilde{\mathcal{V}}^{p_2}_{\pm|b}(u_2) + \tilde{\mathcal{G}}^{p_1 p_2}_{ab}(u_1,u_2)$

coincides with $\displaystyle \lim_{u\to 0} \mathcal{V}^{p}_{\pm|a}(u)$ and $\displaystyle \lim_{u_1,u_2\to 0} \mathcal{G}^{p_1 p_2}_{ab}(u_1,u_2)$

Conditions $A_{ab|cd} = \tilde{A}_{ab|cd}$ fixes the coefficients



Analytical Results

$\boxed{}$: scale dependence
$\boxed{}$: same as const. mass signal

Bispectrum

$$S = \sum_{a,b=\pm} \left[\frac{k_1 k_2}{k_3^2} \mathcal{U}_{ab}^{0,-2}\left(\frac{2k_3}{k_{123}}, \boxed{\frac{k_3}{k_0}}\right) + \frac{k_2 k_3}{k_1^2} \mathcal{U}_{ab}^{0,-2}\left(\frac{2k_1}{k_{123}}, \boxed{\frac{k_1}{k_0}}\right) + \frac{k_3 k_1}{k_2^2} \mathcal{U}_{ab}^{0,-2}\left(\frac{2k_2}{k_{123}}, \boxed{\frac{k_2}{k_0}}\right) \right]$$

where

$$\mathcal{U}_{\pm\pm}^{p_1 p_2}(u, \boxed{v}) = D_1(p_1, p_2, \mu_v, \gamma) \boxed{u^{5+p_{12}}}\, {}_3F_2\left[\begin{matrix} 1, 3+p_2 \mp i\gamma, 5+p_{12} \\ \frac{7}{2}+p_2 - i\mu_v, \frac{7}{2}+p_2+i\mu_v \end{matrix} \,\middle|\, u \right]$$
$$\mp D_2(p_1, p_2, \mu_v, \gamma) \boxed{u^{5/2+p_1 \pm i\mu_v}}\, {}_2\mathcal{F}_1\left[\begin{matrix} p_1 + \frac{5}{2} \pm i\mu_v, \frac{1}{2} \pm i\mu_v \mp i\gamma \\ 1 \pm 2i\mu_v \end{matrix} \,\middle|\, u \right] + (\mu_v \to -\mu_v)$$

$$\mathcal{U}_{\pm\mp}^{p_1 p_2}(u, \boxed{v}) = C(p_1, p_2, \mu_v, \gamma) \boxed{u^{5/2+p_1 \pm i\mu_v}}\, {}_2\mathcal{F}_1\left[\begin{matrix} p_1 + \frac{5}{2} \pm i\mu_v, \frac{1}{2} \pm i\mu_v \mp i\gamma \\ 1 \pm 2i\mu_v \end{matrix} \,\middle|\, u \right] + (\mu_v \to -\mu_v)$$

$$k_{123} = k_1 + k_2 + k_3 \,, \quad \gamma = \pm \frac{y\sqrt{2\epsilon} M_{\rm pl}^2}{H^2} \,, \quad \mu_v^2 = \frac{1}{H^2}\left(m_0^2 + 2y\sqrt{2\epsilon} M_{\rm pl}^2 \boxed{\log v} \mp 2y\sqrt{2\epsilon} M_{\rm pl}^2 \right) - \frac{9}{4}$$

from evaluation at horizon crossing

Effects of Time-Dependent Mass

Evo. of perturb. ⬅ **Time of horizon crossing** ⬌ **Scales**

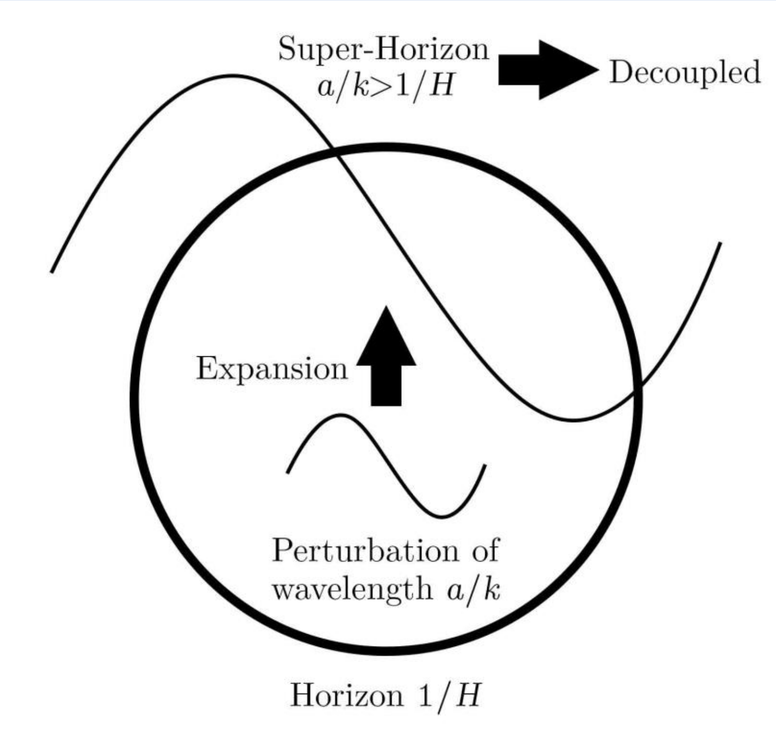
$k\tau = -1$

Constant mass: Scale invariant $S(k_1/k_3, k_2/k_3)$

Time dependent mass: Scale dependent

Different mass for each horizon crossing scale

➡ Dependence on values of scales itself



Fixing additional scales

Two additional scales τ_0, τ_*

$$\phi_0(\tau) \simeq \phi_{*0} - \sqrt{2\epsilon} M_{\rm pl}\left(1 - \frac{\tau}{\tau_*}\right)$$

$$\phi_{*0} \simeq \sqrt{2\epsilon} M_{\rm pl} \log \frac{\tau_*}{\tau_0}$$

$$\begin{cases} k_i \tau_* = -1 & \text{(expansion at horizon crossing)} \\ \phi_{*0} \simeq \sqrt{2\epsilon} M_{\rm pl} \log\left(\dfrac{k_0}{k_*}\right) \sim \mathcal{O}\left(N_{\rm CMB}\sqrt{2\epsilon} M_{\rm pl}\right) & \end{cases}$$

at the largest scale of CMB

τ_0, τ_* ⬌ k_0, k_*

$k_* \sim 10^{-4}\ {\rm Mpc}^{-1}$

Observational Signals

cf. const. mass

$$S \sim \left(\frac{k_3}{k_1}\right)^{1/2} e^{-\pi\mu} \cos\left(\mu \log \frac{k_3}{k_1}\right) \qquad \mu = \sqrt{\left(\frac{m_0}{H}\right)^2 - \frac{9}{4}}$$

Scale dependence : mass of short mode at the time of horizon crossing

$$S \sim \left(\frac{k_3}{k_1}\right)^{1/2} e^{-\pi\mu\left(v\frac{k_3}{k_1}\right)} \cos\left[\mu\left(v\frac{k_3}{k_1}\right) \log \frac{k_3}{k_1}\right] \quad \text{in} \quad k_3 \ll k_1 \simeq k_2$$

$$v = k_1/k_0 = 10^4 k_1, \qquad \mu^2 = \frac{1}{H^2}\left(m_0^2 + 2y\sqrt{2\epsilon}M_{\rm pl}^2 \log\left(v\frac{k_3}{k_1}\right) \mp 2y\sqrt{2\epsilon}M_{\rm pl}^2\right) - \frac{9}{4}$$

$$(vk_3/k_1 = k_3/k_0)$$

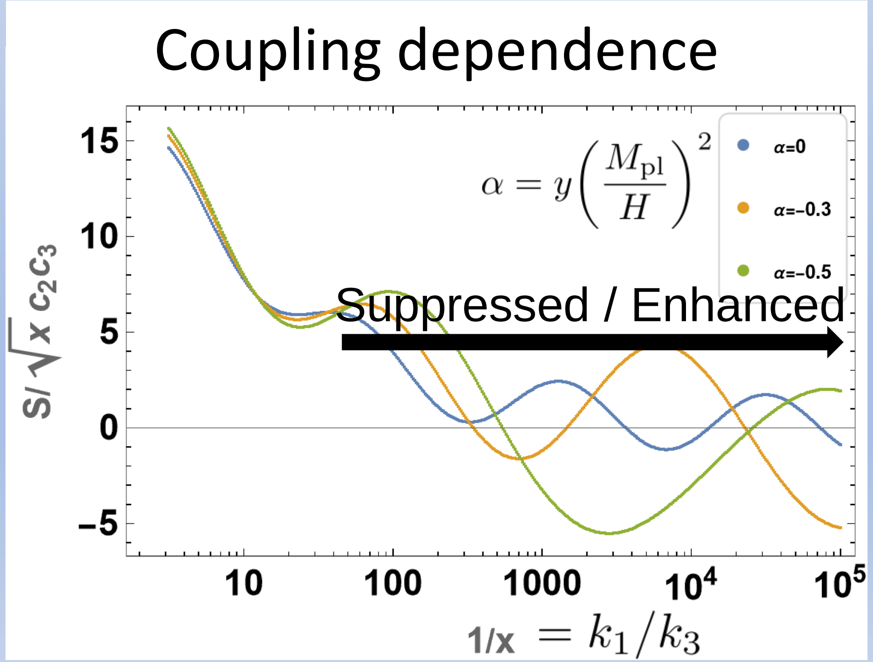
$$\Delta\phi \sim N\sqrt{\epsilon}M_{\rm pl}$$

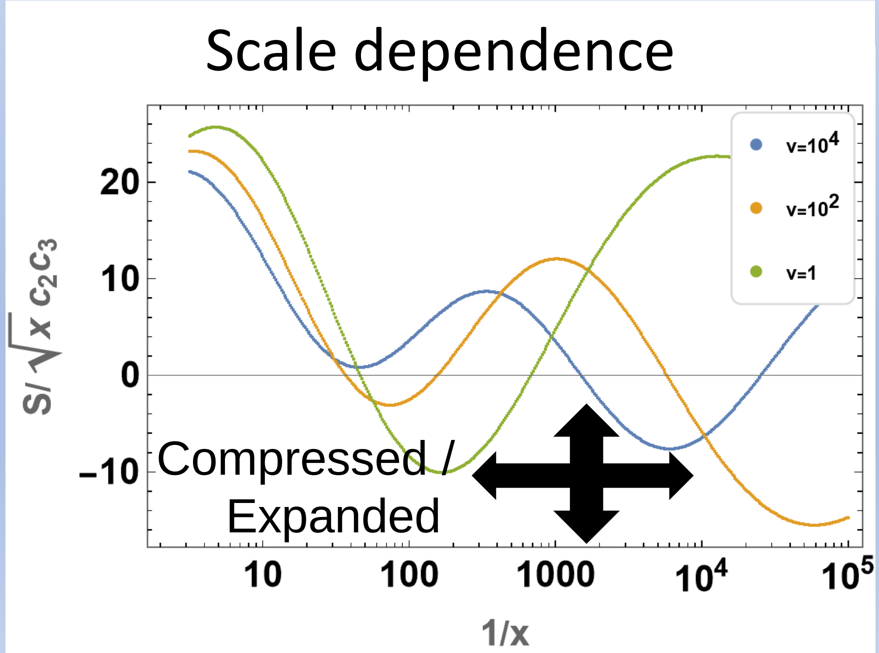
[Lyth '96]

Not slow-roll suppressed thanks to the hierarchy

$$M_{\rm pl}/H \gtrsim 10^5$$

in case of non-der. ints.





Probing Ints. 1: Der. vs Non-Der. Ints.

Scale dependence : mass at horizon crossing

(1) Non-derivative coupling, e.g., $\frac{\alpha}{M_{\rm pl}^{n-2}}\phi^n\sigma^2$

$$\frac{\Delta m_{\rm eff}^2}{H^2} \simeq \alpha \underline{\underline{\left(\frac{M_{\rm pl}}{H}\right)^2}} \epsilon^{n/2}\left(\log\left(v\frac{k_3}{k_1}\right)\right)^n$$

$\Rightarrow$ Large scale dependence

(2) Derivative coupling, e.g., $\frac{\beta}{M_{\rm pl}^{n(m+1)-2}}(\partial^m\phi)^n\sigma^2$ $\qquad nm$: even

$$\frac{\Delta m_{\rm eff}^2}{H^2} \simeq \beta \underline{\underline{\left(\frac{H}{M_{\rm pl}}\right)^{nm-2} \epsilon^{nm-n/2}}}\left(\log\left(v\frac{k_3}{k_1}\right)\right)^n$$

Stronger suppression because of slowroll $\partial_t^m\phi \sim \epsilon^{m-1/2}H^m M_{\rm pl}$

(same order as the signal)

➡ Large scale dependence $\Leftrightarrow$ Non-derivative coupling

Probing Ints. 2: Among Non-Der. Ints.

Boltzmann suppression of the signal $S \sim \left(\frac{k_3}{k_1}\right)^{1/2} \underline{\underline{e^{-\pi\mu}}} \cos\left(\mu \log \frac{k_3}{k_1}\right)$

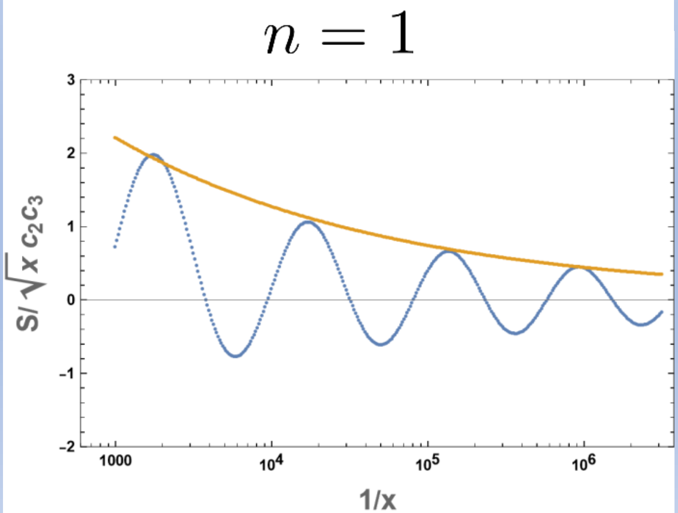
E.g., power function $\frac{\alpha}{\Lambda^{n-2}} \phi^n \sigma^2$ ➡ $\frac{\Delta m_{\rm eff}^2}{H^2} \simeq \alpha \left(\frac{M_{\rm pl}}{\Lambda}\right)^{n-2} \left(\frac{M_{\rm pl}}{H}\right)^2 \epsilon^{n/2} \left(\log\left(v\frac{k_3}{k_1}\right)\right)^n$

$$\Rightarrow \quad e^{-\pi\mu} \sim \exp\left[-\pi\sqrt{\alpha\left(\frac{M_{\rm pl}}{\Lambda}\right)^{n-2}\left(\frac{M_{\rm pl}}{H}\right)^2 \epsilon^{n/2} \underline{\underline{\left(\log\left(v\frac{k_3}{k_1}\right)\right)^n}}}\,\right]$$

Determination of n from the suppression

More generally, $\mathcal{L}_{\rm int} = g(\phi)\sigma^2$

$$\Rightarrow \quad e^{-\pi\mu} \sim \exp\left[-\frac{\pi}{H}\sqrt{\underline{\underline{g\left(M_{\rm pl}\sqrt{2\epsilon}\log\left(v\frac{k_3}{k_1}\right)\right)}}}\,\right]$$



Suppression / enhancement rate is uniquely characterized by $g(\phi)$

Summary

BSM

$$\frac{\alpha}{M_{\rm pl}^{n-2}}\phi^n\sigma^2$$
$$\frac{\beta}{M_{\rm pl}^{n(m+1)-2}}(\partial^m\phi)^n\sigma^2$$
$$\vdots$$

Cosmological collider project:

$k_3 \ll k_1 \simeq k_2$

- Dictionary for particles $S \sim \left(\frac{k_3}{k_1}\right)^{1/2} e^{-\pi\mu}\cos\left(\mu\log\frac{k_3}{k_1}\right)$
- Scale dependence: types of interactions

Signals: horizon crossing (e.g., $\mu \to \mu(vk_3/k_1)$)

Distinguishing ints. by scale dependence in $\Delta m_{\rm eff}$:

○ Derivative vs. Non-derivative interactions

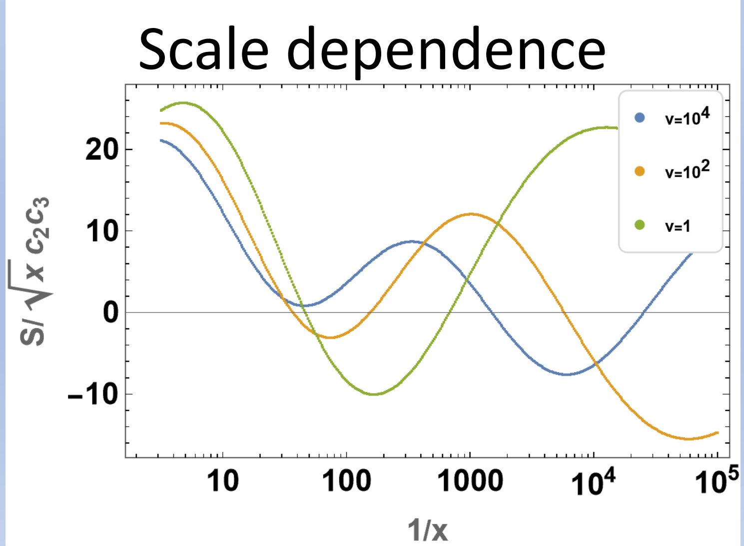
Derivative ints. Non-derivative ints.

$$\left(\frac{H}{M_{\rm pl}}\right)^{nm-2}\epsilon^{nm-n/2}\left(\log\left(v\frac{k_3}{k_1}\right)\right)^n << \left(\frac{M_{\rm pl}}{H}\right)^2\epsilon^{n/2}\left(\log\left(v\frac{k_3}{k_1}\right)\right)^n$$

Observably large thanks to $M_{\rm pl}/H \gtrsim 10^5$

○ Determining a non-der int. $g(\phi)\sigma^2$

$$e^{-\pi\mu} \sim \exp\left[-\frac{\pi}{H}\sqrt{g\left(M_{\rm pl}\sqrt{2\epsilon}\log\left(v\frac{k_3}{k_1}\right)\right)}\right]$$



Appendices

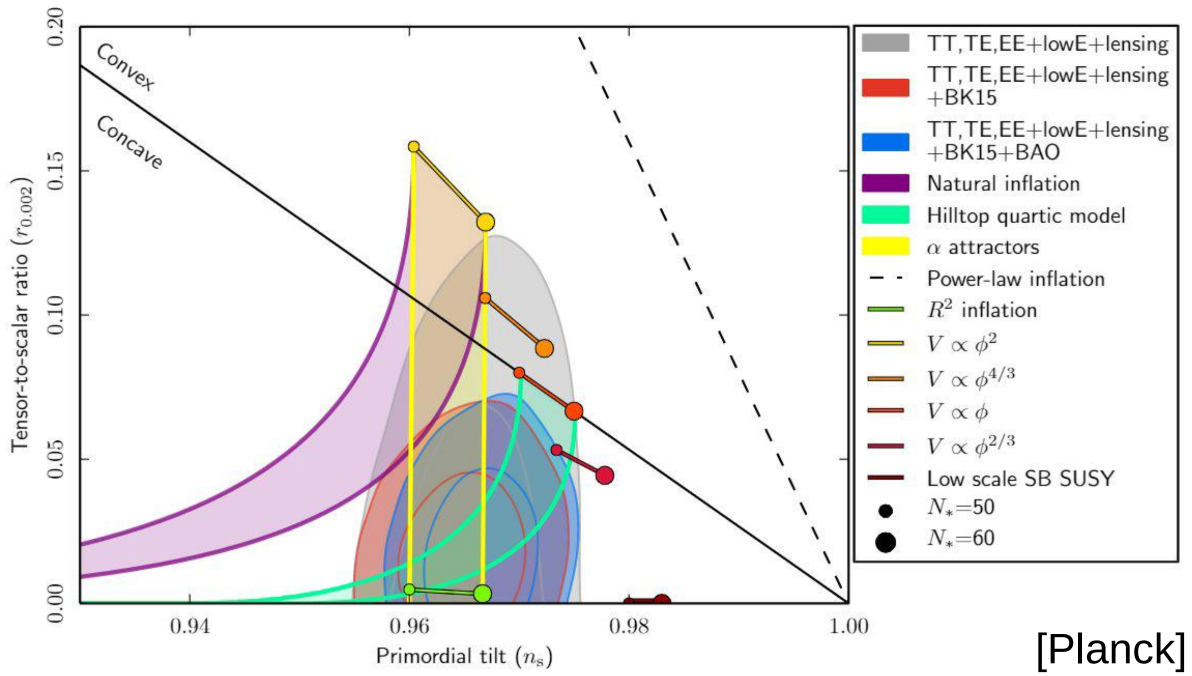
Planck 2018

Linear perturbations:

$$P_\zeta \simeq 2 \times 10^{-9}, \quad n_s \simeq 0.0965$$

Tensor: not yet detected

$$r = \frac{P_\gamma}{P_\zeta} < 0.056$$



[Planck]

Isocurvature perturbation: not detected

➡ Single field inflation is preferred.

Non-Gaussianities:

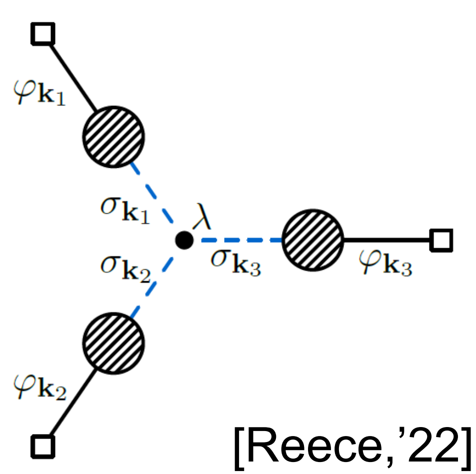
Squeezed: $f_{\rm NL}^{\rm local} = -0.9 \pm 5.1$, Equilateral: $f_{\rm NL}^{\rm equil} = -26 + 47$

Form factor: insufficient resolution

Future experiment: 21 cm line ➡ resolution $\mathcal{O}(10^{-2})$

Non-derivative Ints., Numerical

[Reece et al. ’22]



[Reece,’22]

Setup

Effective mass of the heavy particle: $m_{\text{eff}} = e^{\alpha\phi/M_{\text{pl}}} m_0^2$

Oscillatory feature

Wavelength, Amplitude } amplified or dumped

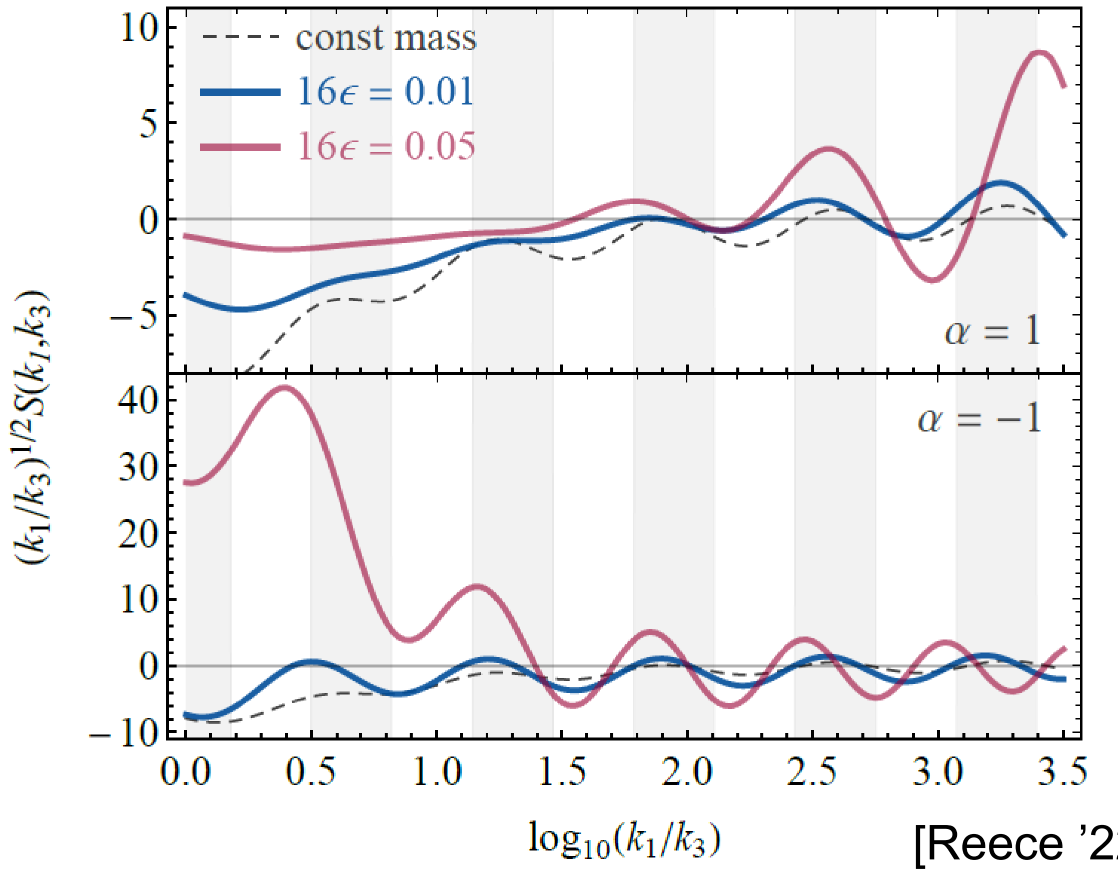
Things not clear in numerical work:

-physical interpretation

-model dependence

-scale dependence

Breaking down of de Sitter



[Reece ’22]

Mode Functions of the Heavy Field

Mode expansion

$$\sigma(x) = \int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\left(v_k(\tau)a_{\boldsymbol{k}} + v_k^*(\tau)a_{-\boldsymbol{k}}^\dagger\right)e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \quad ,$$

$[a_{\boldsymbol{k}}, a_{\boldsymbol{k}'}^\dagger] = (2\pi)^3\delta(\boldsymbol{k}-\boldsymbol{k}')$

v_k : Mode function

Equation of motion for σ

$$v_k'' - \frac{2}{\tau}v_k' + \left(k^2 + \frac{m_{\rm eff}^2}{H^2\tau^2}\right)v_k = 0 \quad ,$$

$$m_{\rm eff}^2 = m_0^2 + 2yM_{\rm pl}^2\left[\frac{\phi_{*0}}{M_{\rm pl}} \mp \sqrt{2\epsilon}\left(1 - \boxed{\frac{\tau}{\tau_*}}\right)\right]$$

Mode functions for σ (Bunch-Davies vacuum)

$$v_k = \frac{\boxed{e^{\pi\gamma/2}}}{\sqrt{2k}}(-H\tau)W_{\boxed{-i\gamma,i\mu}}(2ik\tau)$$

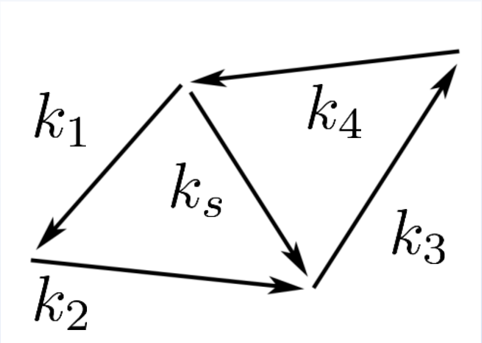
$$\mu^2 = \frac{1}{H^2}\left(m_0^2 + \underline{\underline{2yM_{\rm pl}\phi_{*0} \mp 2y\sqrt{2\epsilon}M_{\rm pl}^2}}\right) - \frac{9}{4}$$

$$\gamma = \pm\frac{y\sqrt{2\epsilon}M_{\rm pl}^2}{H^2}$$

$y \to 0$: const. mass mode function

$$v_k = e^{-\pi\mu/2}\frac{\sqrt{\pi}}{2}H(-\tau)^{3/2}H_{i\mu}^{(1)}(-k\tau)$$

Soft Limit to Obtain Bispectrum



Bispectrum: $\langle\zeta^3\rangle \propto \frac{c_2 c_3}{8k_1k_2k_3^4}\lim_{k_4\to0}\sum_{\mathrm{a,b}=\pm}\mathcal{I}_{\mathrm{ab}}^{0,-2} + (k_3\to k_1,k_2)$

$$u_1 = \frac{2k_s}{k_{12}+k_s} \to \frac{2k_3}{k_{123}} \qquad u_2 = \frac{2k_s}{k_{34}+k_s} \to 1$$

$$\mathcal{I}_{ab}^{p1,p2} = \sum_{c,d=\pm} A_{ab|cd}\mathcal{V}^{p_1}_{a|c}(u_1)\mathcal{V}^{p_2}_{b|d}(u_2) + \mathcal{G}^{p_1p_2}_{ab}(u_1,u_2)$$

$$\mathcal{V}^p_{a|c}(u) = iac2^{iac\mu}\pi\,\mathrm{csch}(2\pi\mu)\left(\frac{u}{2}\right)^{5/2+p+iac\mu}{}_2\mathcal{F}_1\left[\begin{matrix}\frac52+p+iac\mu,\frac12-ia\gamma+iac\mu\\1+2iac\mu\end{matrix}\middle| u\right]$$

$$\mathcal{G}^{p_1p_2}_{ab} = \delta_{ab}\frac{H^2 e^{-a\frac{\pi}{2}ip_{12}}\Gamma(p_{12}+5)}{2^{p_{12}+5}}\sum_{n=0}^{\infty} u_1^{n+p_{12}+5}\left(1-\frac{1}{u_2}\right)^n\binom{n+p_{12}+4}{n}$$
$$\times \frac{1}{\mu^2+\left(\frac52+n+p_2\right)^2}{}_3F_2\left[\begin{matrix}1,3+n+p_2-ia\gamma,5+n+p_{12}\\ \frac72+n+p_2-i\mu,\frac72+n+p_2+i\mu\end{matrix}\middle|u_1\right]$$

Infinite sum. in $\mathcal{G}^{p_1p_2}_{ab}$

Only $n=0$ contributes

➡ resolved

Divergences in $\mathcal{V}^p_{a|c}$

$${}_2\mathcal{F}_1\left[\begin{matrix}\frac52+p+iac\mu,\frac12-ia\gamma+iac\mu\\1+2iac\mu\end{matrix}\middle| u\right] \xrightarrow[u\to1-0]{} (1-u)^{-2+ia\gamma}\Gamma(-2+ia\gamma)+\cdots\longrightarrow\infty$$

is canceled after the summation $\sum_{c,d=\pm} A_{ab|cd}\mathcal{V}^{p_1}_{a|c}(u_1)\mathcal{V}^{p_2}_{b|d}(u_2)$

➡ Final expression: next slide

Equilateral limit $k_1 = k_2 = k_3$

$v = k_1/k_0$ dependence

$$\frac{\partial S}{\partial v} = f(m_0)\frac{\sqrt{\epsilon}\alpha}{v} + \mathcal{O}(\epsilon)$$

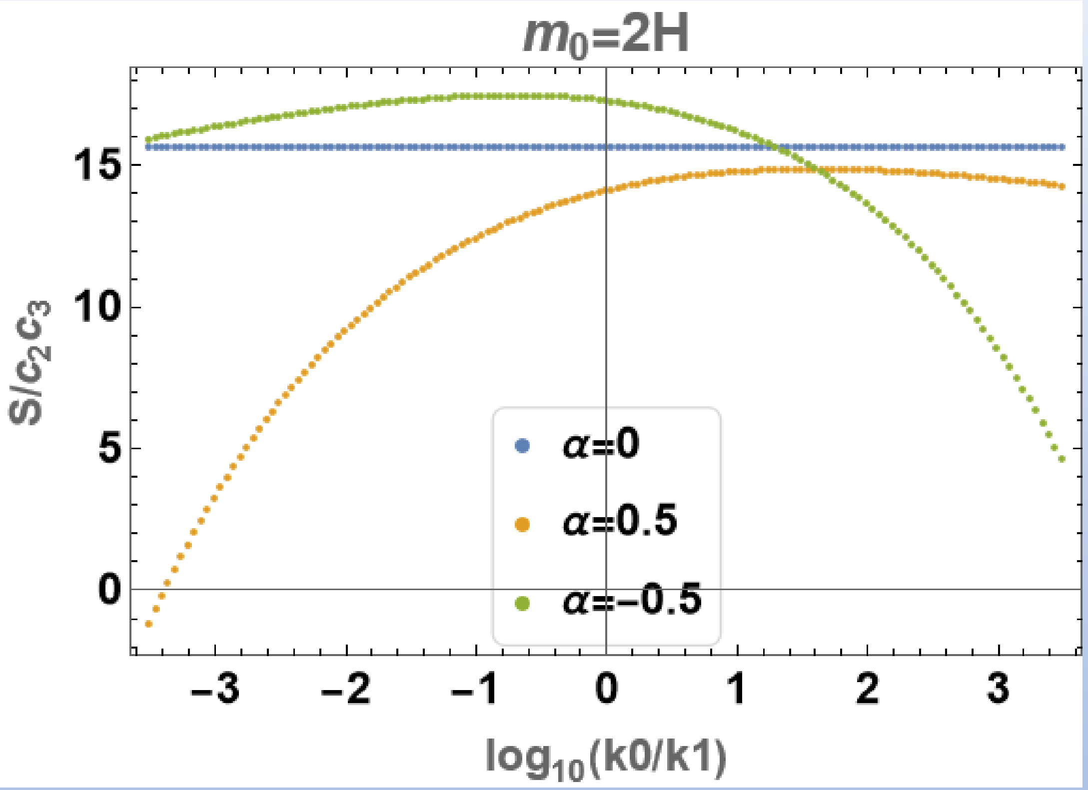
The same scale dependence as

the general single field inflation

[Chen, '07]

(Consistent to EFT description

integrating out heavy field)



Amplitude

$$S_{\rm eq}(\approx f_{\rm NL}^{\rm eq}) \sim c_2 c_3 \mathcal{O}(10)$$

c_2c_3 : dim. less ➡ $\mathcal{O}(1)?\quad \mathcal{O}(\epsilon)?$