UV-sensitivity in Kaluza-Klein Theories: Naturalness and Dark Dimension

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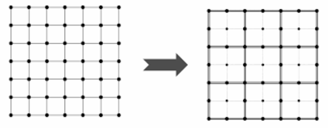
Problems with infinitely many DOFs

> There are a number of problems in science which have, as a common characteristic, that *complex microscopic behavior underlies macroscopic effects*. In simple cases the microscopic fluctuations average out when larger scales are considered, and the averaged quantities satisfy classical continuum equations. [...]
>
> Unfortunately, there is a much more difficult class of problems where fluctuations persist out to macroscopic wavelengths, and *fluctuations on all intermediate length scales are important* too. In this last category are the problems of fully developed turbulent fluid flow, critical phenomena, and elementary particle physics.

Wilson Nobel lecture

What is Wilson's lesson all about?

Wilson's Lesson: dealing with all scales fluctuations



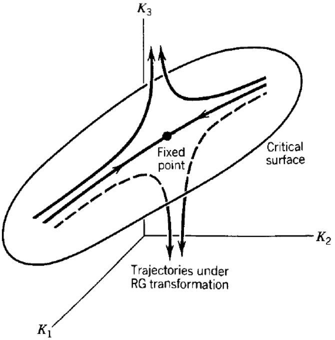
Theory at Λ: S_Λ $\rightarrow$ Theory at $\Lambda/2$: $S_{\Lambda/2}$ $\rightarrow$ ...

There is no cut-off in the sense some find disturbing .. rather a physical running scale $\Lambda \rightarrow \Lambda/2 \rightarrow \Lambda/4 \rightarrow \Lambda/8 \rightarrow \ldots$

> Theoretical foundation of EFT paradigm: any QFT is an EFT

- Contain an ultimate UV scale Λ
- $E > \Lambda$: UV completion (microscopic fluctuations)
- $E < \Lambda$: QFT effective, EFT (persistent fluctuations on all scales)

Renormalized theory

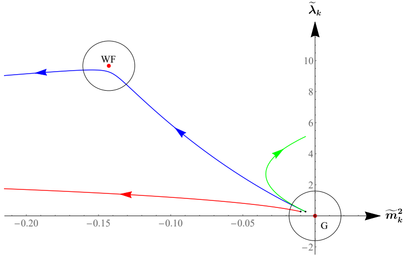


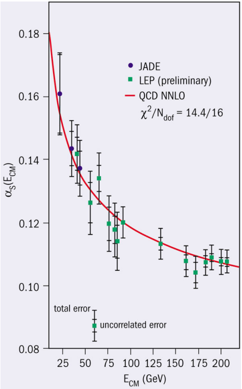
Renormalized theory: defined around a fixed point (critical surface)

In any dimesion: ..., $d = 3$, $d = 4$, $d = 4 + n$ ...

$d = 3$ dimensions : Wilson-Fisher

$d = 4$ dimensions : AF





Also for theories with $d > 4$ dimesions ... in particular...

Kaluza-Klein theories: $d = 4 + n$

EFTs with compact dimensions : $d = 4 + n$

- Field Theories with compact extra dimensions are ubiquitous
- Typically studied as $4D$ theories with infinite* towers of $4D$ states:

$$m_n = f_n \, \mu_{\rm tow}$$

- Surprising UV-softness :

Vacuum Energy / Effective Potential @ 1l $\sim \mu_{\rm tow}^4$

V_{1l} with cutoff Λ for $\widehat{p}^2$: controlled approximation of running potential $U_k(\phi)$, $k \to 0$ in LPA

How is this possible?

* Sometimes truncated according to the $4D$ interpretation: equivalent (see later)

Example : Scherk-Schwarz

$5D$ SUSY theory $\mathcal{S}_{(5)}$ defined on multiply connected spacetime $\mathcal{M}^4 \times S^1$

- Different R-charges for superpartners $(i = b, f)$

$$\Psi_i(x, z + 2\pi R) = e^{2\pi i R q_i}\Psi_i(x, z) \;\Rightarrow\; \Psi_i(x, z) = \sum_{n=-\infty}^{+\infty} \frac{\psi_{i,n}(x) e^{i(\frac{n}{R}+q_i)z}}{\sqrt{2\pi R}}$$

$\int dz\, \mathcal{L}_{(5)} \to \mathcal{L}_{(4)} \leftarrow$ infinite tower of $4D$ KK fields, $m^2_{i,n} \propto \left(\frac{n}{R} + q_i\right)^2$

- 4D "masses" mismatch: effective 4D non-local soft SUSY breaking

Higgs field ϕ : ϕ_0 , or $4D$ brane field , or ...

Effective 4D quadratic operator

$$M^2_{i,n}(\phi) = m^2(\phi) + \left(\frac{n}{R} + q_i\right)^2$$

One-loop Higgs Effective Potential (4D calculation)

$$V_{1l}^{(4)}(\phi) = \frac{1}{2}\sum_a \sum_{i_a} (-1)^{\delta_{i_a,f_a}} \sum_{n=-\infty}^{\infty} \int \frac{d^4 p}{(2\pi)^4} \log\left(p^2 + m_a^2(\phi) + \left(\frac{n}{R} + q_{i_a}\right)^2\right)$$

One way of doing the calculation (not the only one)*:

- (First) infinite sum; (then) integrate $d^4 p$ with cutoff Λ

Antoniadis, Dimopoulos, Pomarol, Quiros/Delgado, Pomarol, Quiros/Barbieri, Hall, Nomura/Arkani-Hamed, Hall, Nomura, Smith, Weiner

Each tower contributes :

$$V_{1l}^{(4)}(\phi) = R\left(\frac{m^2\Lambda^3}{48\pi} - \frac{m^4\Lambda}{64\pi} + \frac{m^5}{60\pi}\right) - \sum_{k=1}^{\infty} \frac{e^{-2\pi k m R}(2\pi k m R(2\pi k m R + 3) + 3)\cos(2\pi k q)}{64\pi^6 k^5 R^4}$$

* Other methods, Proper time (Antoniadis, Quiros), Pauli-Villars (Contino, Pilo), Thick brane (Delgado, von Gersdorff, John, Quiros), all give the same result, see later

One-loop Higgs Effective Potential ("4D" calculation)

... Let's have a closer look ...

From each tower the Higgs Potential receives the contribution :

$$V_{1l}^{(4)}(\phi) = R\left(\frac{m^2\Lambda^3}{48\pi} - \frac{m^4\Lambda}{64\pi} + \frac{m^5}{60\pi}\right) - \sum_{k=1}^{\infty} \frac{e^{-2\pi k m R}(2\pi k m R(2\pi k m R + 3) + 3)\cos(2\pi k q)}{64\pi^6 k^5 R^4}$$

- Power UV-sensitivity through $m \implies$ canceled by SUSY
- No UV-sensitivity through q

$\implies$ Naturally UV-insensitive (finite) Higgs potential

Old Times $\sim$ 2000

- UV-insensitive Higgs mass!
- UV-insensitive Higgs potential!

Criticism : sum $[-L, L] \to$ UV-sensitive terms — Ghilencea, Nilles/Kim

... Heated debate! ...

Calculations done in a different setup, proper time, thick brane, Pauli-Villars, dimensional regularization all seem(ed) to confirm UV-insensitive result

Debate closed in favour of UV-insensitiveness* ... but ...

* In the absence of FI terms

Ghilencea, Groot-Nibbelink, Nilles

$5D$ calculation from the outset

$$\mathcal{S}_{(5)} = \int dz\, d^4x \left(\frac{1}{2}\,\partial_a\widehat{\Phi}\,\partial^a\widehat{\Phi} + \partial_a\widehat{\chi}\,\partial^a\widehat{\chi}^\dagger + \frac{m_\Phi^2}{2}\,\widehat{\Phi}^2 + m_\chi^2\,\widehat{\chi}\widehat{\chi}^\dagger + \frac{\widehat{\lambda}}{4!}\,\widehat{\Phi}^4 + \frac{\widehat{g}}{2}\,\widehat{\Phi}^2\widehat{\chi}\widehat{\chi}^\dagger \right)$$

$$\widehat{\Phi}(x, z + 2\pi R) = \widehat{\Phi}(x, z) \quad ; \quad \widehat{\chi}(x, z + 2\pi R) = e^{2\pi i R\, q}\,\widehat{\chi}(x, z)$$

$R\,q \equiv R\,q' - [R\,q'] \to q \in [0, R^{-1}]$

Fourier expansion of $\widehat{\chi}(x, z)$: EFT up to Λ (similar for $\widehat{\Phi}$)

$$\widehat{\chi}(x, z) = e^{iqz} \left(\sum_n \int \frac{d^4p}{(2\pi)^5 R} \right)' \widehat{\chi}_{n,p}\, e^{i\left(p\cdot x + n\frac{z}{R}\right)}$$

$$\left(\frac{1}{2\pi R} \sum_n \int \frac{d^4p}{(2\pi)^4} \right)' \equiv \frac{1}{2\pi R} \sum_{n=-[R\Lambda]}^{[R\Lambda]} \int^{C_\Lambda^n} \frac{d^4p}{(2\pi)^4}, \quad C_\Lambda^n \equiv \sqrt{\Lambda^2 - \frac{n^2}{R^2}}$$

$$\widehat{\chi}(x, z) = e^{iqz} \sum_{n=-[R\Lambda]}^{[R\Lambda]} \frac{\chi_n^\Lambda(x)\, e^{in\frac{z}{R}}}{\sqrt{2\pi R}}; \quad \chi_n^\Lambda(x) \equiv \frac{1}{\sqrt{2\pi R}} \int^{C_\Lambda^n} \frac{d^4p}{(2\pi)^4}\, \widehat{\chi}_{n,p}\, e^{ip\cdot x}$$

4D Effective Potential from 5D Effective Potential

$$\mathcal{V}_{1l}^{(5)}(\widehat{\Phi}) = \frac{1}{2}\mathrm{Tr}_5 \log \frac{p^2 + \frac{n^2}{R^2} + m_\phi^2 + \frac{\widehat{\lambda}}{2}\,\widehat{\Phi}^2}{p^2 + \frac{n^2}{R^2}} + \frac{1}{2}\mathrm{Tr}_5 \log \frac{p^2 + \left(\frac{n}{R} + q\right)^2 + m_\chi^2 + \frac{\widehat{g}}{2}\,\widehat{\Phi}^2}{p^2 + \frac{n^2}{R^2}}$$

- p & n intertwined: NO hierarchy when including asymptotics

$$\mathrm{Tr}_5 = \left(\frac{1}{2\pi R}\sum_n \int \frac{d^4p}{(2\pi)^4}\right)' = \frac{1}{2\pi R}\sum_{n=-[R\Lambda]}^{[R\Lambda]} \int^{C_\Lambda^n} \frac{d^4p}{(2\pi)^4}$$

Performing z integration $\rightarrow$ effective $V_{1l}^{(4)}(\phi)$ with $\phi = \phi_0$

$$V_{1l}^{(4)}(\phi) = \frac{1}{2}\sum_{n=-[R\Lambda]}^{[R\Lambda]} \int^{C_\Lambda^n} \frac{d^4p}{(2\pi)^4}\left(\log \frac{p^2 + \frac{n^2}{R^2} + m_\phi^2 + \frac{\lambda}{2}\,\phi^2}{p^2 + \frac{n^2}{R^2}} + \log \frac{p^2 + \left(\frac{n}{R} + q\right)^2 + m_\chi^2 + \frac{g}{2}\,\phi^2}{p^2 + \frac{n^2}{R^2}}\right)$$

$\lambda \equiv \frac{\widehat{\lambda}}{2\pi R}$; $g \equiv \frac{\widehat{g}}{2\pi R}$; $\widehat{\Phi} = \frac{\phi}{\sqrt{2\pi R}}$

$$V_{1l}^{(4)}(\phi) = 2\pi R\,\mathcal{V}_{1l}^{(5)}(\widehat{\Phi})$$

only if we respect the asymptotics

UV-sensitivity and non-trivial topology

$$V_{1l}(\phi) = \frac{5m^2 + 3q^2}{180\pi^2} R\Lambda^3 - \frac{35m^4 + 14m^2q^2 + 3q^4}{840\pi^2} R\Lambda + \frac{m^5 R}{60\pi} - \sum_{k=1}^{\infty} \frac{e^{-2\pi k m R}(2\pi k m R(2\pi k m R + 3) + 3)\cos(2\pi k q)}{64\pi^6 k^5 R^4}$$

New q-dependent UV-sensitive terms:

- NOT canceled by SUSY! $\propto \boxed{(q_b^2 - q_f^2)}\, m^2(\phi)\Lambda$
- Topological origin
  1. $= 0$ for $q = 0$ ($q\,\exists$ in multiply connected spacetime ($\not{R}$))
  2. UV-insensitive terms: $\neq 0$ for $q = 0$

Alternatively : Infinite sum & Smooth cut

Typical argument: cut on sum → spurious "divergences" ... But ...

$$V_{1l}(\phi) = \frac{1}{2} \sum_{n=-\infty}^{\infty} \int \frac{d^4 p}{(2\pi)^4} \log \left(\frac{p^2 + m^2 + \left(\frac{n}{R} + q\right)^2}{p^2 + \frac{n^2}{R^2}} \right) e^{-\frac{p^2 + \frac{n^2}{R^2}}{\Lambda^2}}$$

⇒ **Same result** is found

UV-sensitive terms are **NOT** due to the sharp cut of the sum!
They come from a **correct treatment of $\widehat{p}$ asymptotics**

So ... why do "Proper time", "Thick brane" and "Pauli-Villars"

give UV-insensitive results ?

Secret liaison between proper time , thick brane & PV

Thick brane: $\sum_{n=-\infty}^{\infty} \int^{(\Lambda)} \frac{d^4p}{(2\pi)^4} \frac{e^{-\frac{\left(\frac{n}{R}+q\right)^2}{\Lambda^2}}}{p^2+m^2+\left(\frac{n}{R}+q\right)^2}$ — Delgado, von Gersdorff, John, Quiros

Pauli-Villars: $\sum_{n=-\infty}^{\infty} \int \frac{d^4p}{(2\pi)^4} \frac{(\Lambda R)^4}{(\Lambda R)^4+p^2+(\frac{n}{R}+q)^2} \frac{1}{p^2+m^2+\left(\frac{n}{R}+q\right)^2}$ — Contino, Pilo

Proper Time: — Antoniadis, Quiros

$$V_{1l}^{(4)}(\phi) = -\sum_{n=-\infty}^{\infty} \int \frac{d^4p}{(2\pi)^4} \int_{\frac{1}{\Lambda^2}}^{\infty} \frac{ds}{s} e^{-s\left(p^2+m^2+\left(\frac{n}{R}+q\right)^2\right)}$$

$$= -\sum_{n=-\infty}^{\infty} \int \frac{d^4p}{(2\pi)^4} \Gamma\left(0, \frac{p^2+m^2+\left(\frac{n}{R}+q\right)^2}{\Lambda^2}\right)$$

> Cut function of $\left(\frac{n}{R}+q\right)$ instead of $\frac{n}{R}$: artificial re-absorption of q

Equivalent to introduce a hierarchy between (p_1, p_2, p_3, p_4) and p_5

$\Rightarrow$ **Again : artificial wash-out of UV-sensitive terms**

First take-home message

$V_{1l}(\phi)$ is UV-sensitive even with SUSY

Due to the non-trivial topology of the spacetime

This conclusion is independent of the specific cutoff

Now ... we're ready for the Cosmological Constant ...

The Dark Dimension

Vacuum Energy and Dark Dimension

Ingredients: Montero, Vafa, Valenzuela

- (A)dS distance conjecture: when $\Lambda_{cc} \to 0$ Lüst, Palti, Vafa

$$\mu_{tow} \sim |\Lambda_{cc}|^{\alpha}$$

Λ_{cc} is ρ in Planck units

- Emergent string conjecture: $\mu_{tow} = m_{\rm KK}$ or $\mu_{tow} = M_s$ Lee, Lerche, Weigand
- 1l string calculations: $\rho_4 \sim M_s^4$ $(\to \rho_4 \sim \mu_{\rm tow}^4)$
- Higuchi bound $\alpha \le 1/2$ Higuchi

$\Rightarrow \frac{1}{4} \le \alpha \le \frac{1}{2} \Leftarrow$ Assumed as starting point for DD proposal

Experimental bounds on violations of $\frac{1}{r^2}$ Newton's law : $\mu_{tow} \gtrsim 6.6$ meV

Energy scale associated to Λ_{cc}: $\Lambda_{cc}^{1/4} \sim 2.31$ meV

$\Rightarrow \alpha = \frac{1}{4}$, "experimental value": $\mu_{tow}^{exp} \sim \mathrm{meV}$ ($\sim$ neutrino scale)

Vacuum Energy and Dark Dimension

In principle $\mu_{tow} = M_s$ possible, but ... "ruled out experimentally":

"we can describe physics above the neutrino scale with EFT", no sign of string excitations at these scales

Only possibility left: EFT decompactification scenario

$$m_{\rm KK} \sim \mu_{tow}^{exp} \sim {\rm meV}$$

This conclusion takes us to EFT: DD takes place in the (deep) EFT realm

Assuming the DD, i.e. $\rho \sim m_{\rm KK}^4$ true prediction of string theory

- EFT reproduces it: ✓
- EFT does not: Tension!!!
    1. Can we put the pieces together? How?
    2. Is there really a string theory realizing the DD in our Universe?

Vacuum Energy and Dark Dimension

Compactification with gravity $\widehat{g}_{MN} = \begin{pmatrix} e^{2\alpha\phi} g_{\mu\nu} - e^{2\beta\phi} A_\mu A_\nu & e^{2\beta\phi} A_\mu \\ e^{2\beta\phi} A_\nu & -e^{2\beta\phi} \end{pmatrix}$

Background configuration $g^0_{\mu\nu} = \eta_{\mu\nu}, A_\mu = 0, \phi = \phi_0$ (hereafter ϕ)

$$\rho_4 = \frac{5\log\frac{\Lambda^2 e^{2\alpha\phi}}{\mu^2} - 2}{300\pi^2} e^{2\alpha\phi} R\Lambda^5 + \frac{5m^2 + 3q^2 e^{4\alpha\phi}}{180\pi^2} e^{2\alpha\phi} R\Lambda^3 - \frac{35m^4 + 14m^2q^2e^{4\alpha\phi} + 3q^4 e^{8\alpha\phi}}{840\pi^2} e^{2\alpha\phi} R\Lambda + \frac{m^5}{60\pi} e^{2\alpha\phi} R + \frac{3\log\frac{\Lambda^2 e^{2\alpha\phi}}{\mu^2} + 2}{2880\pi^2 R^4} e^{10\alpha\phi} R\Lambda + R_4 + \mathcal{O}(\Lambda^{-1}) = 2\pi R e^{2\alpha\phi}\rho_5$$

$$R_4 = -\frac{x^2 \mathrm{Li}_3\left(re^{-x}\right) + 3x\mathrm{Li}_4\left(re^{-x}\right) + 3\mathrm{Li}_5\left(re^{-x}\right) + 6\zeta(5)}{128\pi^6 R^4} e^{12\alpha\phi} + h.c.$$

$$r \equiv e^{2\pi i q R} \quad , \quad x \equiv 2\pi e^{-2\alpha\phi} R\sqrt{m^2} \implies R_4 \propto \frac{e^{12\alpha\phi}}{R^4} = m^4_{KK}$$

Interlude: Direct calculation of the energy

Define $d+1$ theory $\rightarrow$ Quantize $\rightarrow$ Calculate the hamiltonian

$$\rho_{d+1} = \frac{1}{2}\frac{1}{2\pi R}\sum_{n=-[R\Lambda]}^{[R\Lambda]}\int^{C_\Lambda^n}\frac{d^{d-1}p}{(2\pi)^{d-1}}\sqrt{p^2+\left(\frac{n}{R}+q\right)^2+m^2}$$

Confirms q-dependent UV-sensitivity

Light tower limit and Dark Dimension

- SUSY: $\rho_4 \sim (q_b^2 - q_f^2)\, e^{6\alpha\phi} R\Lambda^3 = m_{KK}^2\, R\Lambda^3$
- NON-SUSY: $\rho_4 \sim e^{2\alpha\phi} R\Lambda^5 = m_{KK}^{\frac{2}{3}} \left(R^{\frac{1}{3}}\Lambda\right)^5$

Specific example of cutoffs:

1. $\Lambda = \widehat{M}_p \equiv (2\pi R)^{-1/3} M_p^{2/3}$, $\Lambda = M_s$: nothing changes
2. ~~$\Lambda = \Lambda_{sp} \sim m_{\mathrm{KK}}^{1/3} M_p^{2/3}$~~: same problems as PT, PV, ... backup slides

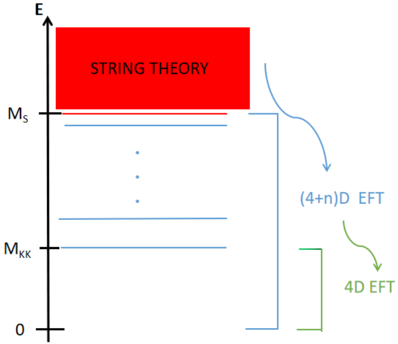
Even in the light tower limit $\phi \to -\infty$, R_4 cannot overthrow these contributions. No light tower regime where $\rho_4 \sim m_{KK}^4$

- In a $(4+1)$D EFT quantum fluctuations dress ρ_4: only a fine(r)-tuning (than in $4D$) might allow to reach $\rho_{\mathrm{measured}} \sim m_{KK}^4$
- The question of where does the zero-point energy of quantum fields end up is far from being settled

Global picture: From String Theory to 4D EFT

String theory $\Rightarrow$ EFT: M_s physical cutoff

Typical string result $\rho \sim M_s^d$ ... finite but (physical cutoff)^d



Global picture: $d+1$ EFT $\to$ d EFT

- Start: $\mathcal{S}^{(5)}_\Lambda$ w/ mode expansion

$$\widehat{\chi}(x,z) = e^{iqz}\left(\sum_n \int \frac{d^4p}{(2\pi)^5 R}\right)' \widehat{\chi}_{n,p}\, e^{i\left(p\cdot x + n\frac{z}{R}\right)}$$

- Integrating out modes in $[k,\Lambda] \to \mathcal{S}^{(5)}_k$ — k Wilsonian running scale

In LPA:

$$k\frac{\partial \mathcal{U}_k}{\partial k} = -\frac{\pi^{d/2}}{(2\pi)^d} k^2 \sum_{n=-[Rk]}^{[Rk]} g_n(k)$$

$$g_n(k) \equiv \left(k^2 - \frac{n^2}{R^2}\right)^{\frac{d-2}{2}} f_n\left(\sqrt{k^2 - \frac{n^2}{R^2}}\right) \qquad f_n(p) \equiv \log\frac{p^2 + \frac{(n+q)^2}{R^2} + \mathcal{U}''_k(\phi)}{p^2 + \frac{n^2}{R^2} + (\mathcal{U}''_k(0))}$$

Due to p_5 discreteness, p_5 eigenmodes contribution is "stepwise"

- $k < 1/R$: RG evolution becomes of $4D$ type

> In this sense, and **only in this sense**, the $4D$ theory emerges from the $5D$ one: **there is no infinite tower of states**

Example: ϕ^4 in $4+1$ D $(q=0)$

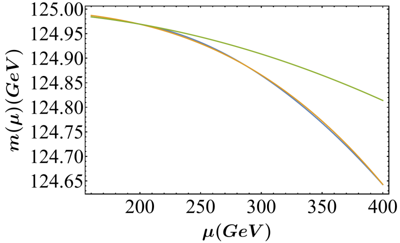
ϕ^4 truncation: $\mathcal{U}_k^{(5)}(\Phi) = \frac{\hat{m}_k^2}{2}\Phi^2 + \frac{\hat{\lambda}_k}{4!}\Phi^4$ $\quad \mathcal{U}_k^{(5)} = \frac{\mathcal{U}_k^{(4)}}{2\pi R},\ \hat{m}_k^2 = m_k^2,\ \lambda_k = \frac{\hat{\lambda}_k}{2\pi R},\ \Phi = \frac{\phi}{\sqrt{2\pi R}}$

$$k\frac{\partial}{\partial k} m_k^2 = -\frac{k^4}{16\pi^2}\frac{\lambda_k}{k^2+m_k^2} - \frac{k^4\lambda_k}{8\pi^2}\frac{[kR]}{k^2+m^2} + \frac{k^2\lambda_k}{16\pi^2 R^2}\frac{\frac{2}{3}[kR]^3+[kR]^2+\frac{1}{2}[kR]}{k^2+m^2}$$

4D running

$$k\frac{\partial}{\partial k}\lambda_k = \frac{3k^4}{16\pi^2}\frac{\lambda_k^2}{\left(k^2+m_k^2\right)^2} + \frac{3k^4}{8\pi^2}\frac{\lambda_k^2[kR]}{\left(k^2+m_k^2\right)^2} - \frac{\lambda_k^2 k^2}{16\pi^2 R^2}\frac{[kR]-3[kR]^2-2[kR]^3}{\left(k^2+m_k^2\right)^2}$$

Decompactification limit, $R\to\infty$, $[kR]=kR$:
$$\begin{cases} k\frac{\partial}{\partial k}\hat{m}_k^2 = -\frac{k^5}{24\pi^3}\frac{\hat{\lambda}_k}{k^2+\hat{m}_k^2} \\ k\frac{\partial}{\partial k}\hat{\lambda}_k = \frac{3k^5}{24\pi^3}\frac{\hat{\lambda}_k^2}{\left(k^2+\hat{m}_k^2\right)^2} \end{cases}$$



Summary & Conclusions

- Usual calculations mistreat the asymptotics of the loop momenta
- Correct treatment of the loop momenta asymptotics unveils the presence of UV-sensitive terms of topological origin, missed in the usual calculations
- Interpretation of the $(4+n)$ D theory with compact extra dimensions as a $4D$ theory with an infinite number of 4D fields needs to be taken with a grain of salt
- Not a solution to the naturalness/hierarchy problem
- Not a solution to the CC problem
- Strong tension between swampland relation and EFT: is there really a dark dimension?

Physical tuning: Only way out?

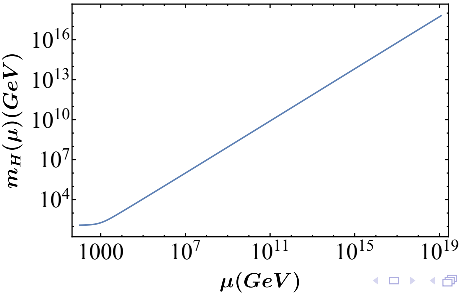
- A Physical tuning $\oplus$
- Evidence that supports $(4+1)D$ tuning over $4D$ or $(4+n)D$

We need to pledge our ignorance of the UV: no different than $4D$

String theory should give at Λ the extremely fine-tuned parameters $\rho_\Lambda, m^2_\Lambda, \lambda_\Lambda, y_{t_\Lambda}, \dots$ for the dynamical dressing to produce $\rho_k \sim m^4_{KK}$ at cosmological scales and the SM parameters $(m^2_h, \lambda, y_t, \dots)$ at μ_F

Ex: Higgs mass in the SM

CB, Branchina, Contino - "*Physical tuning and naturalness*"



List of criticisms

Anchordoqui, Antoniadis, Lüst, Lüst

- Cutoff dependence of the result
- Higuchi bound: the non-SUSY case might violate it
- Related to that, nonsensical to extract $\rho - m_{\mathrm{KK}}$ relationship without fixing the cutoff
- $T \neq 0$: same result T^4
- Result must vanish for $T \to 0$ in TFT, $R \to \infty$ in KK
- Modular invariance of string theory dictates regularization
- Λ_{sp} as a cutoff: violates Higuchi
- m_{KK} as a cutoff: DD is fine
- QG: cutoff removed

NB: $m_{\mathrm{KK}} = \frac{e^{3\alpha\phi}}{R}$, R is only a constant, the interest is on the ϕ-dependence, not on the constant dependence

Higuchi bound

For a spin 2 massive field in $4D$ dS: $m^2 \geq \frac{2}{3}\Lambda_{cc}$

- Relation between the physical parameters, not results of 1l calculation
- Comes from an instability in dS space due to the fact that for (massive and massless spin 2) CC plays the role of a negative mass
- Higuchi bound for KK gravitons should be carefully derived from the corresponding bound for the $5D$ graviton

In particular... application of the bound to the result of the 1l calculation requiring $\rho = \rho_{\text{measured}}$ before any renormalization is **misleading**:

Our point is precisely that the physical ρ_{measured} cannot be obtained without a renormalization

Cutting the tower with Λ_{sp}

Cut in tower typical in Swampland: Species scale Λ_{sp} (e.g. emergence proposal)

Grimm, Palti, Valenzuela

Calculation of the vacuum energy ρ_4 using the species scale cutoff $\Lambda_{\rm sp}$

$4D$ theory with N particle states, $\Lambda_{\rm sp} = M_p/\sqrt{N}$

$\Lambda_{\rm sp}$: $4D$ cutoff

$5D$ theory with one compact dimension: $\Lambda_{\rm sp}$ identified by counting the number of KK states such that $m_n^2 \leq \Lambda_{\rm sp}^2$

Inequality saturated: $m_\phi^2 + \frac{(n+q)^2}{R_\phi^2} = \Lambda_{\rm sp}^2 \to N = n_+ + |n_-| + 1$

$$\Lambda_{\rm sp}^2 = \frac{M_p^{4/3}}{(2R_\phi)^{2/3}} - \frac{M_p^{2/3}}{3\,(2R_\phi^4)^{1/3}} + \frac{m_\phi^2 + \frac{1}{4R_\phi^2}}{3} + \mathcal{O}(M_P^{-2/3}).$$

Cutting the tower with Λ_{sp}

$$
\begin{aligned}
\rho_4 = {} & \frac{20\log\frac{4M_p^2}{5\mu^3 R_\phi} + 12\pi - 57}{2^{-1/3}\cdot 3840\pi^2 R_\phi^{2/3}} M_p^{10/3} + \frac{-4\log\frac{4M_p^2}{\mu^3 R_\phi} - 6\pi + 27}{2^{-2/3}\cdot 2304\pi^2 R_\phi^{4/3}} M_p^{8/3} + \frac{12\pi - 35}{4608\pi^2 R_\phi^2} M_p^2 \\
& + \frac{\left(4\, m_\phi^2 R_\phi^2 + 1\right)\log\frac{M_p^2}{2\mu^3 R_\phi} - 3(5-4\pi) m_\phi^2 R_\phi^2}{1152\pi^2 R_\phi^2} M_p^2 \\
& + \frac{-20\log\frac{M_p^2}{\mu^3 R_\phi} - 120\pi + 309 + 104\log 2}{2^{-1/3}\cdot 124416\pi^2 R_\phi^{8/3}} M_p^{4/3} + \frac{3(19-8\pi) - 4\log\frac{4M_p^2}{\mu^3 R_\phi}}{2^{-1/3}\cdot 3456\pi^2 R_\phi^{8/3}} \left(m_\phi R_\phi\right)^2 M_p^{4/3} \\
& + \frac{525\pi + 367\log 2 - 1953 + 35\log\frac{M_p^2}{\mu^3 R_\phi}}{2^{-2/3} 1866240\pi^2 R_\phi^{10/3}} M_p^{2/3} + \frac{9\log\frac{M_p^2}{\mu^3 R_\phi} + 135\pi - 432 + 99\log 2}{2^{-2/3} 46656\pi^2 R_\phi^{10/3}} m_\phi^2 R_\phi^2 M_p^{2/3} \\
& + \frac{2\log\frac{M_p^2}{\mu^3 R_\phi} + 3\pi - 30 - 14\log 2}{2^{-2/3} 1728\pi^2 R_\phi^{10/3}} m_\phi^4 R_\phi^4 M_p^{2/3} \\
& + \frac{61 - 18\pi + 40(17-6\pi) m_\phi^2 R_\phi^2 + 80(33-9\pi) m_\phi^4 R_\phi^4}{138240\pi^2 R_\phi^4} + \frac{m_\phi^5 R_\phi}{60\pi} + R_4 + \mathcal{O}(M_P^{-2/3})
\end{aligned}
$$

Cutting the tower with Λ_{sp}

NO UV-sensitive terms proportional to q: why?

Again, it arises from a physically illegitimate operation: rather than a cut on n implements a cut on $m_n^2 = m_\phi^2 + (n+q)^2/R_\phi^2$

$\Rightarrow \Lambda_{sp}$ inapplicable: too literal interpretation of KK states as $4D$ fields

$\widehat{M}_p$ is what should really be considered as the maximal QG cutoff

These warnings do not apply to the case of a $4D$ theory with a large number N of fields coupled to gravity (original $\Lambda_{\rm sp}$ framework):

- In this case $\Lambda_{\rm sp}$ is the true quantum gravity physical cutoff

Finite temperature & Casimir energy

Analogy with finite temperature should not be used:

- The infinite sum in TFT implements the ergodic theorem

  Thermal fluctuations (at equilibrium) $\times$ $3D$ quantum fluctuations

- Regularization and Renormalization **ARE** performed
- $T \to 0$ only quantum fluctuations (result: $\not\to 0$)

The infinite sum in this case is a MUST

The Casimir force is UV-insensitive ... not the vacuum energy

- Casimir energy is UV-finite only after subtraction!

The "mixed position-momentum" calculation approaches (and solves) the $1D$ dynamics first:

- It corresponds to perform the infinite sum first $\Rightarrow$ Untenable