The 30th KIAS Combinatorics Workshop

Korea Institute for Advanced Study
Seoul, Korea
March 8–9, 2024

1 General Information

Title: The 30th KIAS Combinatorics Workshop

Date: March 8–9, 2024

Venue: Room 1503, 5th Fl, Bldg.1, KIAS, Seoul

Homepage `http://events.kias.re.kr/h/combinatorics/`

Invited Speakers

Yunhyung Cho (Sungkyunkwan University)

June Huh (Princeton University & KIAS)

Donggyu Kim (KAIST & IBS DIMAG)

Eunjung Kim (KAIST)

Hyunwoo Lee (KAIST & IBS ECOPRO)

Seung Jin Lee (Seoul National University)

Oliver Lorscheid (University of Groningen / IMPA)

Minho Song (Sungkyunkwan University)

Organizers

Jaehoon Kim (KAIST)

Jang Soo Kim (Sungkyunkwan University)

Jeong Han Kim (KIAS)

Seog-Jin Kim (Konkuk University)

Young Soo Kwon (Yeungnam University)

Sang June Lee (Kyung Hee University)

Jongyook Park (Kyungpook National University)

Seunghyun Seo (Kangwon National University)

2 Schedule and Abstracts

1st Day: March 8 (Friday)

13:00 - 14:00 Registration and Opening

——— **Session A** ——— Chair: Seog-Jin Kim

14:00 - 14:40 **Eunjung Kim**. *Flow-augmentation technique and its applications*

14:50 - 15:30 **Minho Song**. *Combinatorial reciprocity for Riordan arrays*

15:30 - 16:00 Coffee break

——— **Session B** ——— Chair: Seunghyun Seo

16:00 - 16:40 **Yunhyung Cho**. *Mutations of polytopes and related geometries*

16:50 - 17:30 **Hyunwoo Lee**. *Dense triangle-free* (n, d, λ)*-graphs for all orders*

18:00 - Dinner

2nd Day: March 9 (Saturday)

——— **Session C** ——— Chair: Sang-il Oum

10:00 - 11:00 **June Huh**. *Matroids over hyperfields, Lorentzian polynomials, and related topics*

11:10 - 12:10 **Oliver Lorscheid**. *Lorentzian polynomials and matroids over triangular hyperfields*

12:10 - 14:00 Lunch

——— **Session D** ——— Chair: Young Soo Kwon

14:00 - 14:40 **Seung Jin Lee**. *Chromatic symmetric functions and linked rook placements*

14:50 - 15:30 **Donggyu Kim**. *Orthogonal matroids over tracts*

Speaker: Eunjung Kim

Affiliation: KAIST

Title: Flow-augmentation technique and its applications

Abstract

We present a flow-augmentation algorithm in directed graphs: There exists a randomized polynomial-time algorithm, given a directed graph G, two vertices s and t, and an integer k, which adds a number of arcs so that for every minimal st-cut Z in G of size at most k, Z becomes an minimum st-cut in the resulting graph with probability $2^{-poly(k)}$. The algorithm can be made deterministic.

The directed flow-augmentation tool allows us to prove fixed-parameter tractability of a number of problems parameterized by the cardinality of the deletion set, whose parameterized complexity status was repeatedly posed as open problems: (1) Chain SAT, defined by Chitnis, Egri, and Marx [ESA'13, Algorithmica'17], (2) a number of weighted variants of classic directed cut problems, such as Weighted st-Cut or Weighted Directed Feedback Vertex Set.

Furthermore, we consider the problem of finding a boolean assignment of a satisfiability problem over a fixed finite boolean constraint language Γ so as to minimize the number of violated constraints, called MinSAT(Γ). This problem family MinSAT(Γ) captures classic (parameterized) problems such as st-Min Cut, Edge Bipartiation, Almost 2-Sat on the tractable side. On the intractable side, for certain constraint languages Γ, the problem MinSAT(Γ) is known not to allow a constant-factor fpt-approximation. Leveraging the directed flow-augmentation technique, we obtain a complete dichotomy theorem for MinSAT(Γ) parameterized by the number of unsatisfied constraints

The talk is based on a series of joint work with Stefan Kratsch, Marcin Pilipczuk, Magnus Wahlström.

Speaker: Minho Song

Affiliation: Sungkyunkwan University

Title: Combinatorial reciprocity for Riordan arrays

Abstract

Let $F(n)$ be a counting function for certain combinatorial objects. A result that states a relation between $F(n)$ and $F(-n)$ is called a *combinatorial reciprocity theorem*. An early instance of a combinatorial reciprocity theorem goes back to 1958 in the book Combinatorial Analysis written by J. Riordan. After, a deep and important generalization appeared in R. Stanley's landmark paper in 1974. More recent developments are discussed in the book of M. Beck and R. Sanyal.

If $F(n)$ satisfies a homogeneous linear recurrence relation, then we can find $F(-n)$ using the recurrence. In this talk, we extend this to so-called *Riordan arrays*, or Riordan matrices, such that entries satisfy a certain recurrence relation. Our focus is on calculating the negative territory using these recurrence relations and investigating their combinatorial properties in connection with the original Riordan arrays. We take this negative part, flip it, and change signs on alternate diagonals. When this new matrix coincides with the original array, we call it a *bogus-involution*. We give a general way of constructing bogus-involutions and introduce some interesting examples and related problems. This is joint work with Jihyeug Jang and Louis W. Shapiro.

Speaker: Yunhyung Cho

Affiliation: Sungkyunkwan University

Title: Mutations of polytopes and related geometries

Abstract

A convex polytope arises in many area of mathematics such as algebraic geometry, combinatorics, representation theory, and symplectic geometry. A mutation is an operation acting on the set of convex polytopes that preserves Ehrhart polynomials. In this talk, I will explain how the theory of lattice polytopes and their mutations affect various algebras and geometries.

Speaker: Hyunwoo Lee

Affiliation: KAIST & IBS ECOPRO

Title: Dense triangle-free (n, d, λ)-graphs for all orders

Abstract

In 1994, Alon constructed a triangle-free (n, d, λ)-graph with $d = \Omega(n^{2/3})$ and $\lambda = O(d^{1/2})$ for an exponentially increasing sequence of integers n. Using his ingenious construction, we deduce that there exist triangle-free (n, d, λ)-graphs with $d = \Omega(n^{2/3})$ and $\lambda = O((d \log n)^{1/2})$ for all sufficiently large n.This is joint work with Jaehoon Kim.

Speaker: June Huh

Affiliation: Princeton University & KIAS

Title: Matroids over hyperfields, Lorentzian polynomials, and related topics

Abstract

This presentation aims to provide an accessible introduction to topics needed to understand Oliver Lorscheid's talk. The focus will be on hyperfields and matroids over them, along with the process of dequantization, and the theory of Lorentzian polynomials.

Speaker: Oliver Lorscheid

Affiliation: University of Groningen / IMPA

Title: Lorentzian polynomials and matroids over triangular hyperfields

Abstract

Lorentzian polynomials were introduced by Branden and Huh as a tool to prove unimodality. A particular feature of Lorentzian polynomials is that their support is a matroid. Conversely, every valuated matroid is naturally a Lorentzian polynomial, which exhibits a tight relation between Lorentzian polynomials and matroid theory. Matroids over hyperfields were introduced by Baker and Bowler as a general framework for matroids with coefficients.

In joint work (in progress) with Baker, Huh and Kummer, we extend the relation between Lorentzian polynomials and valuated matroids to matroids over so-called triangular hyperfields, as introduced by Viro as a mean for a better understanding of tropicalizations: geometry over a triangular hyperfield T_p are amoebas (where p expresses the hyperbolicity of the metric), and its limit $p = 0$ reflects tropical geometry. Matroids over T_0 (the tropical hyperfield) are valuated matroids and therefore Lorentzian polynomials. Conversely, every Lorentzian polynomial is a T_2-matroid. In turn, this yields insights into the topological nature of the space of all Lorentzian polynomials.

For the purpose of this talk, we use this connection as a motivation to showcase some aspects of Lorentzian polynomials and matroids over hyperfields.

Speaker: Seung Jin Lee

Affiliation: Seoul National University

Title: Chromatic symmetric functions and linked rook placements

Abstract
Stanley-Stembridge and Shareshian-Wachs conjectured that for a unit interval graph the chromatic quasisymmetric function is e-positive. In this talk, we introduce linked q-hit numbers which are refined notion of q-hit numbers defined by Garsia-Remmel. We describe all e-coefficients of a chromatic symmetric function with the bounce number ≤ 3 as (unique) finite linear combinations of linked q-hit numbers. If time permits, I will present the algorithm to calculate coefficients in the linear combination without the bounce number condition for $q = 1$

Speaker: Donggyu Kim

Affiliation: KAIST & IBS DIMAG

Title: Orthogonal matroids over tracts

Abstract

Orthogonal matroids (which are equivalent to even delta-matroids) generalize matroids, as they are defined by a certain basis exchange axiom weaker than that of matroids. One natural example of orthogonal matroids comes from a skew-symmetric matrix over a given field K, and we say such an orthogonal matroid is representable over the field K. Interestingly, a matroid is representable over K in the usual manner if and only if it is representable over K in the sense of orthogonal matroids. The representability of matroids got much interest and was extensively studied such as excluded minors and representability over more than one field. Recently, Baker and Bowler (2019) integrated the notions of K-representable matroids, oriented matroids, and valuated matroids using tracts that are commutative ring-like structures in which the sum of two elements may output no element or two or more elements.

We generalize Baker-Bowler's theory of matroids with coefficients in tracts to orthogonal matroids. We define orthogonal matroids with coefficients in tracts in terms of Wick functions, orthogonal signatures, circuit sets, and orthogonal vector sets, and establish basic properties on functoriality, duality, and minors. Our cryptomorphic definitions of orthogonal matroids over tracts provide proofs of several representation theorems for orthogonal matroids. In particular, we give a new proof that an orthogonal matroid is regular (i.e., representable over all fields) if and only if it is representable over $\mathbb{F}_2$ and $\mathbb{F}_3$, which was originally shown by Geelen (1996), and we prove that an orthogonal matroid is representable over the sixth-root-of-unity partial field if and only if it is representable over $\mathbb{F}_3$ and $\mathbb{F}_4$.

This is joint work with Tong Jin.