2024. 1. 24 High1 workshop on Particle, String and Cosmology

Wave Nature of Gravitational Lensing and its Applications

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Based on

- "Small-scale shear: peeling off diffuse subhalos with gravitational waves"
  HGC, Chanung Park and Sunghoon Jung, Phys. Rev. D **104**, 063001 (2021)
- "Co-Existence test of Primordial black holes and Particle Dark Matter"
  HGC, Sunghoon Jung, Philip Lu, and Volodymyr Takhistov, arXiv:2311.17829

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1. Diffractive Gravitational lensing
2. Identifying dressed primordial black holes
3. Probing small dark matter halos with weak diffractive lensing

Why diffraction?

- Gravitational lensing of gravitational wave (GW lensing)

- Long wavelength of Gravitational waves
  - 100 Hz -> 3000 km, 1 GHz -> 0.3 m

- Small diffraction effects can be measured.
  - Smaller systematic errors compared to Electromagnetic waves
  - Only detector noise and GW source modeling matters

- We can study ultra-small-scale structures of our universe
  - Length scale of diffraction effects can be parsec order

Wave optics of GW

- Background metric in weak gravity

$$ds^2 = -(1+2U(\mathbf{x}))dt^2 + (1-2U(\mathbf{x}))d\mathbf{x}^2 = g^{(B)}_{\mu\nu} dx^\mu dx^\nu$$

$$\nabla^2 U = 4\pi\rho$$

- Propagation of GW (with appropriate gauge fixing)

$$g_{\mu\nu} = g^{(B)}_{\mu\nu} + h_{\mu\nu} \qquad \Box^{(B)} h_{\mu\nu} + 2R^{(B)}_{\gamma\mu\delta\nu} h^{\gamma\delta} = 0$$

- Wavelength << Background Curvature length scale

$$\Box^{(B)} h_{\mu\nu} \simeq 0$$

- Negligible changes(~U) in the polarizations

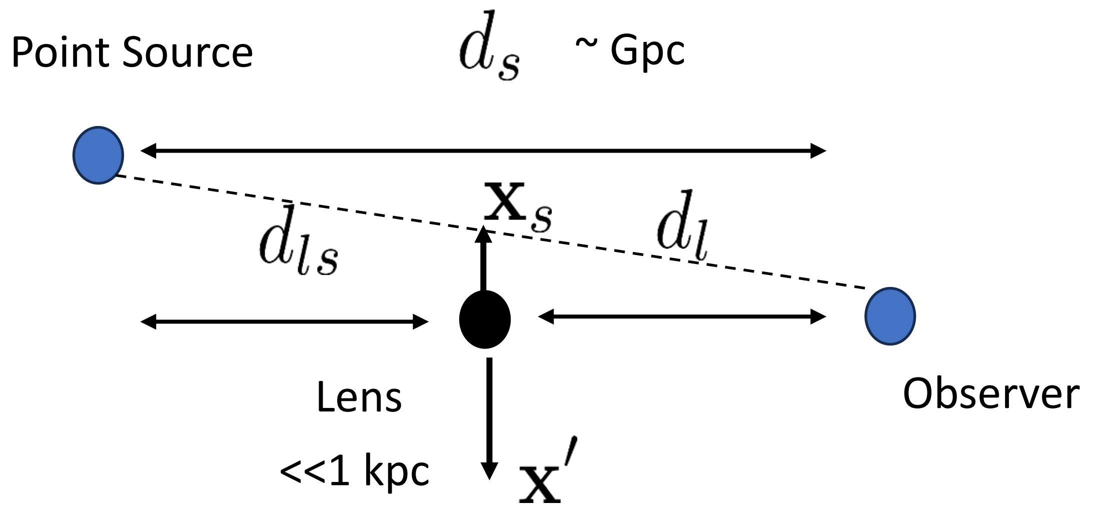
$$h_{\mu\nu}(t,\mathbf{x}) \simeq \phi(t,\mathbf{x}) e_{\mu\nu}$$

$$(\nabla^2 + w^2)\phi(w,\mathbf{x}) = 4w^2 U(\mathbf{x})\phi(w,\mathbf{x})$$

* Fourier transform

Wave optics of GW

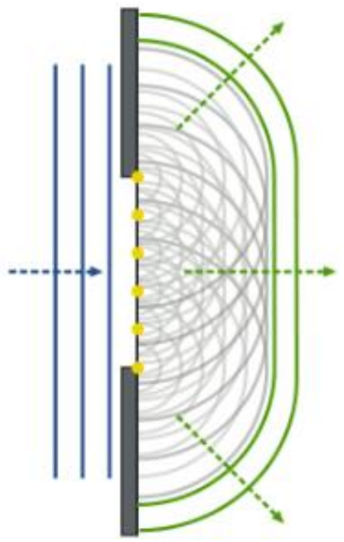
- Thin lens approximation
- Small angle approximation



Wave optics of GW

- The solution is given by Kirchhoff's diffraction formula
  - Huygens' principle on lens plane

cf) Diffraction by a single Slit



Slit -> Lens plane

$$\frac{\phi(w;\mathbf{x}_o)}{\phi_0(w;\mathbf{x}_o)} = F(w;\mathbf{x}_s) = \frac{1}{2\pi i}\int \frac{dx'^2}{r_F^2(w)} e^{i\left[\frac{1}{2}|\mathbf{x}'-\mathbf{x}_s|^2-\psi(\mathbf{x}')\right]/r_F^2(w)}$$

$$r_F = \sqrt{\frac{d_{\rm eff}}{w}}$$ **Fresnel length**

$$d_{\rm eff} = \frac{d_l d_{ls}}{d_s}$$ Effective lens distance

$$\psi(\mathbf{x}) = 2d_{\rm eff}\int dz U(\mathbf{x}, z)$$ Line-of-sight gravitational potential

Wave optics of GW

Diffraction regime: Large Fresnel length (low Freq.) $\mathbf{x}_s/r_F \,,\; \psi(\mathbf{x})/r_F^2 \ll 1$

Geometric optics regime: When the Fresnel length is the smallest (high Freq.)

(Geo.) $F(w) \simeq \sum_{j=1}^{N} |\mu_j|^{1/2} e^{iwT_j + in_j}$ Interference between images

(Diff.) $F(w) \simeq \text{const.} + \int dx'^2 e^{i(\cdots)} \frac{\psi(\mathbf{x})}{r_F^2}$ $r_F = \sqrt{\frac{d_{\text{eff}}}{w}}$

$\propto \overline{\kappa}(r_F)$ Average mass density within radius r_F

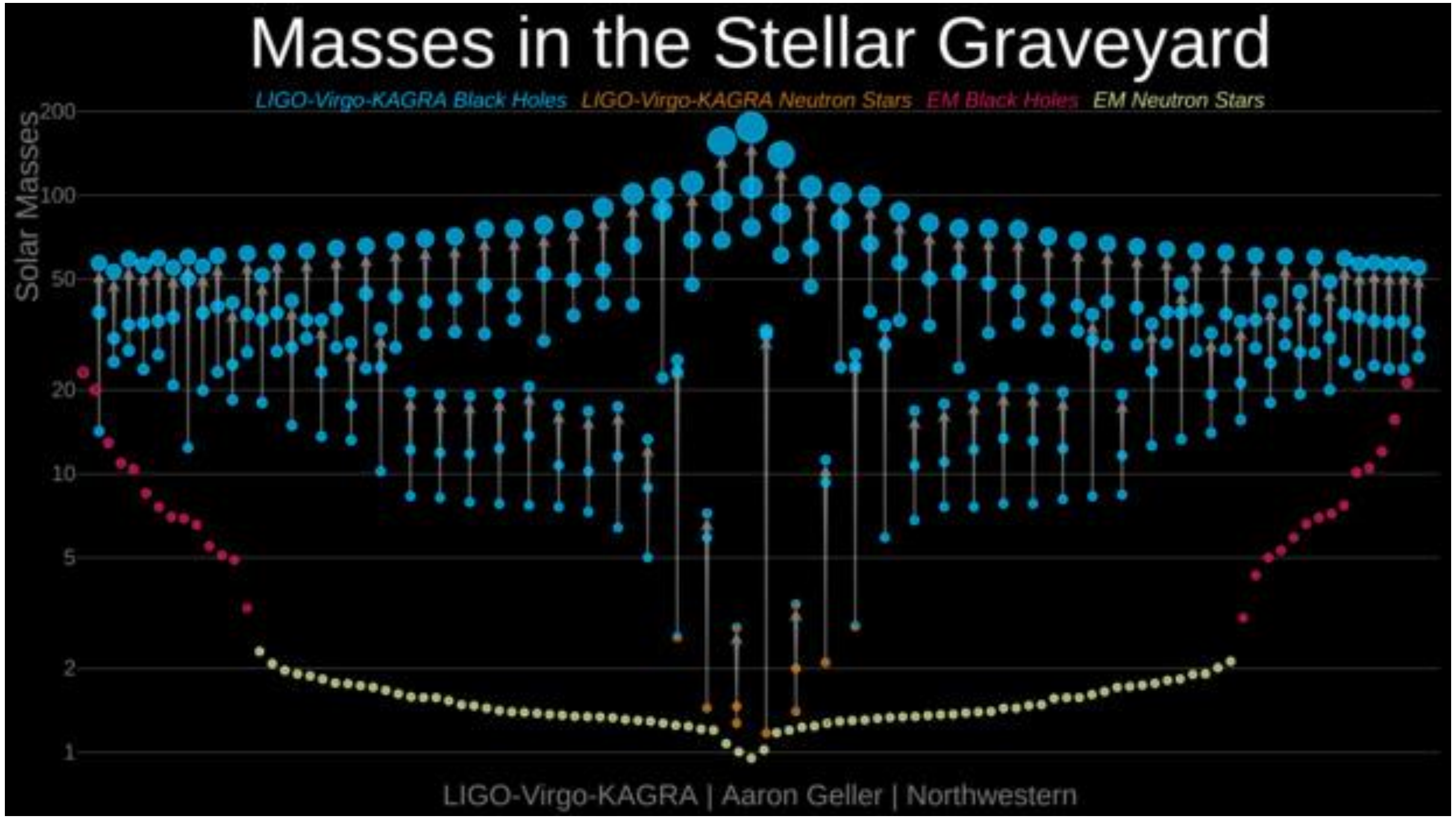
F(w) is directly related to lensing profile by Fresnel length!

Identifying Dressed Primordial Black Holes

Based on "Co-Existence test of Primordial black holes and Particle Dark Matter"
HGC, Sunghoon Jung, Philip Lu, and Volodymyr Takhistov, arXiv:2311.17829

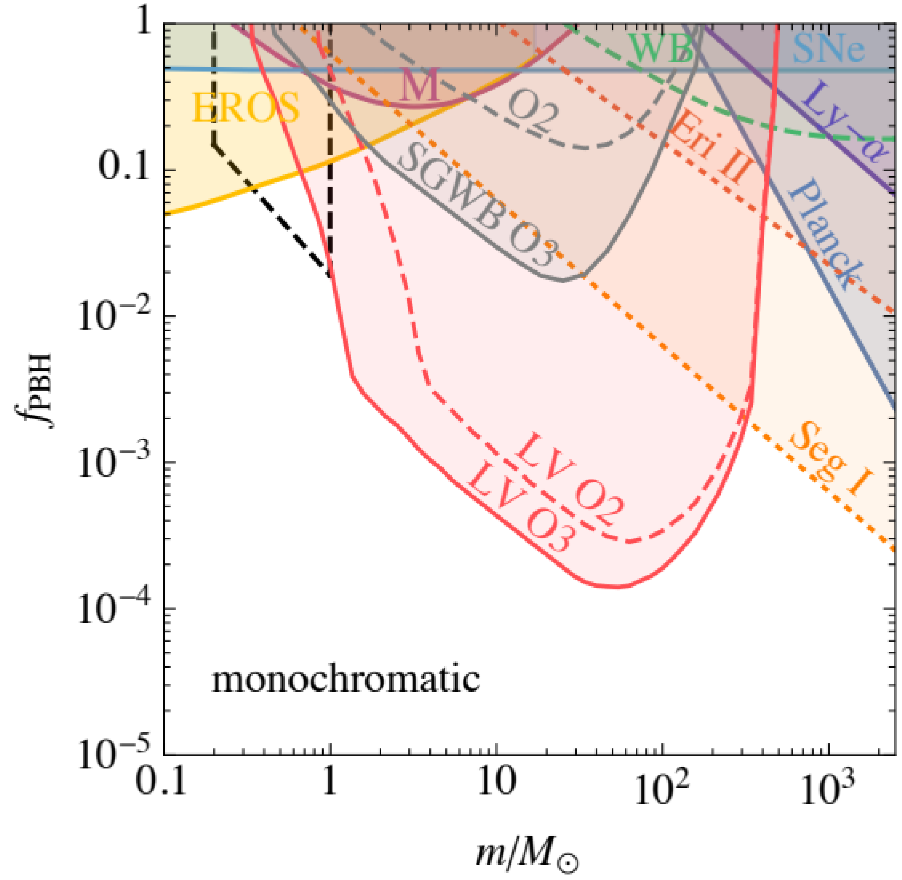
LIGO Black holes

- Origin of the black holes? : Primordial black holes vs Death of stars(astrophysical)
- How can we distinguish their origin?



LIGO Black holes

- PBH is likely a subdominant component of dark matter.
  - from SGWB, BBH distributions

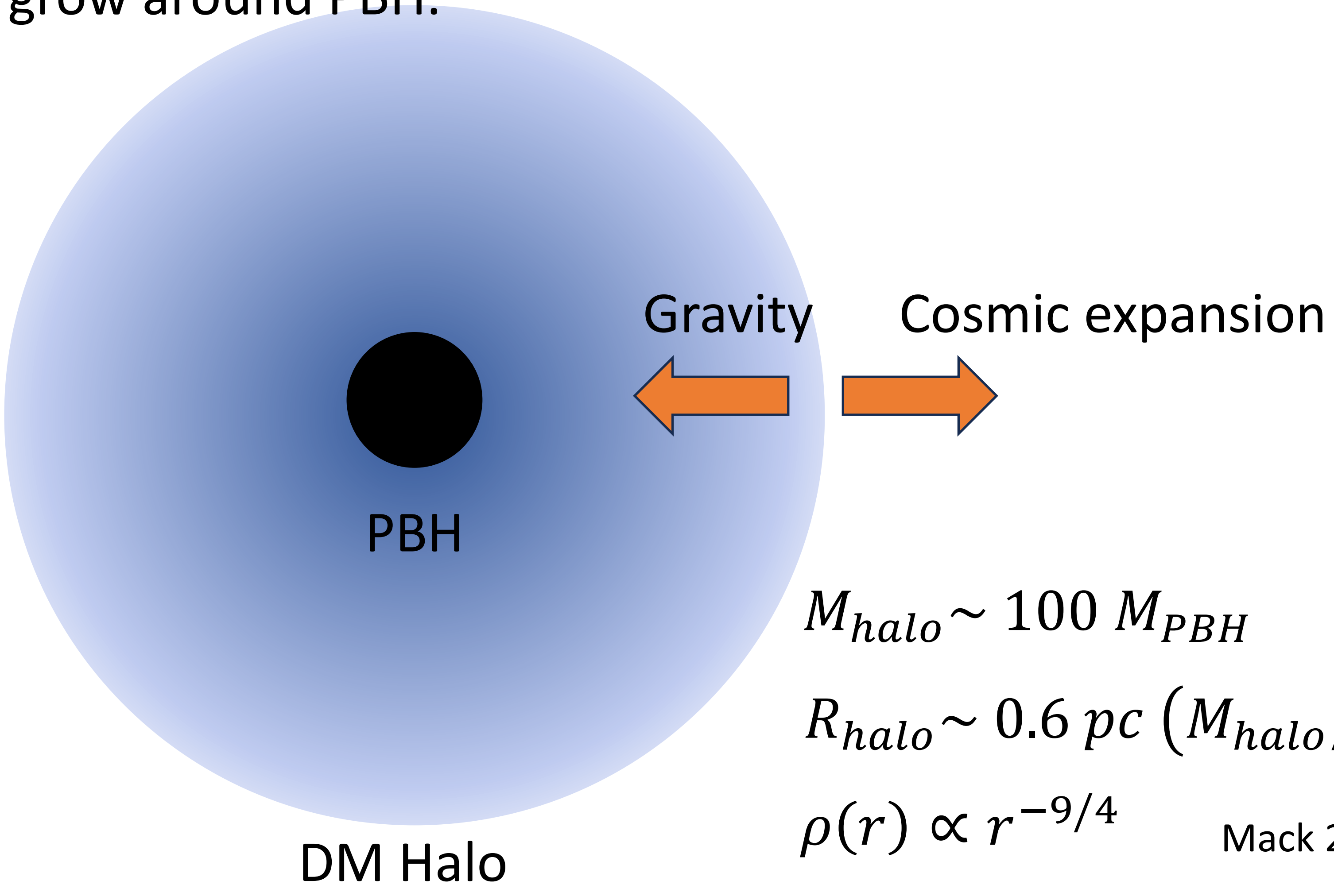


$$f_{PBH} = \frac{\Omega_{PBH}}{\Omega_{DM}} < 10^{-3}$$

Hütsi 2020

Dressed Primordial Black Hole

Before large scale structure formation (z > 30),
Dark matter halo can grow around PBH.



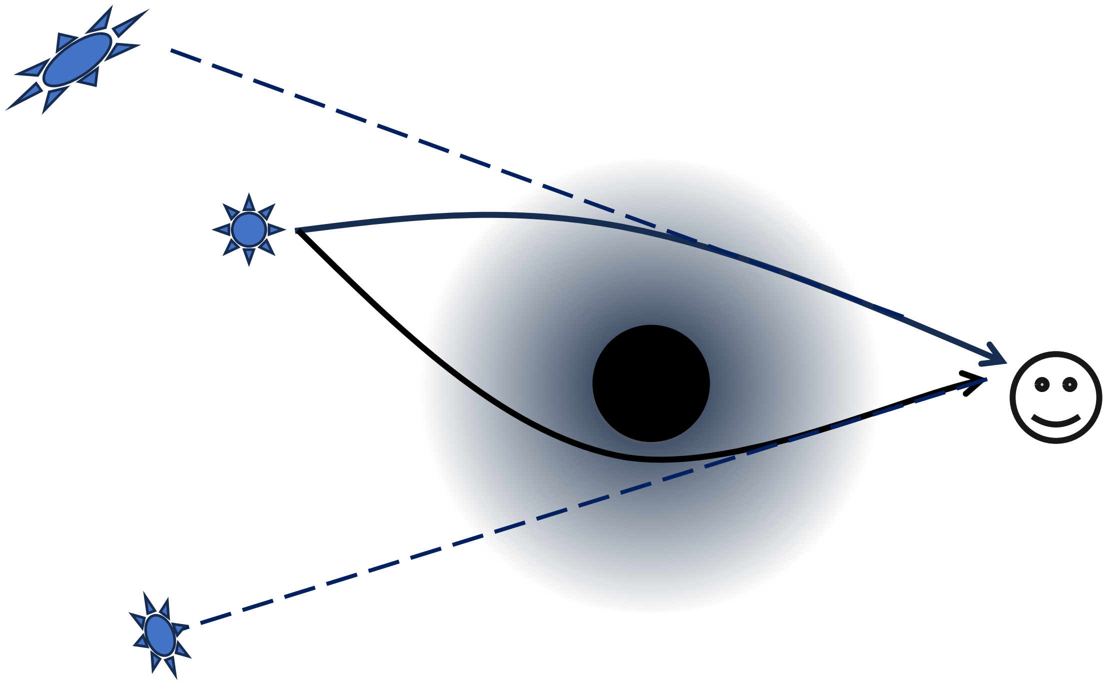
$M_{halo} \sim 100\ M_{PBH}$

$R_{halo} \sim 0.6\, pc\, \left(M_{halo}/M_{\odot}\right)^{1/3}$

$\rho(r) \propto r^{-9/4}$ Mack 2006

Lensing of Dressed PBH

- Produces two images (same as bare PBH)
- Larger magnifications
- Better detection opportunity through microlensing.
  - FRB lensing (Oguri 2022), lensing survey (Cai 2022), etc.



Lensing of Dressed PBH

- However, distinguishing dressed PBH and bare PBH is not yet established.
- For arbitrary two lensing images,

$$F(f) = |\mu_1|^{1/2} + |\mu_2|^{1/2} e^{2\pi i f \Delta t - i\pi/2}$$
$$\propto 1 + \mu_r^{1/2} e^{2\pi i f \Delta t - i\pi/2}$$

intrinsic luminosity is unknown $\mu_r = |\mu_2/\mu_1|$

From the point mass solutions,

$$y_s = \sqrt{\mu_r^{1/2} + \mu_r^{-1/2} - 2}$$

Impact parameter of point mass lens

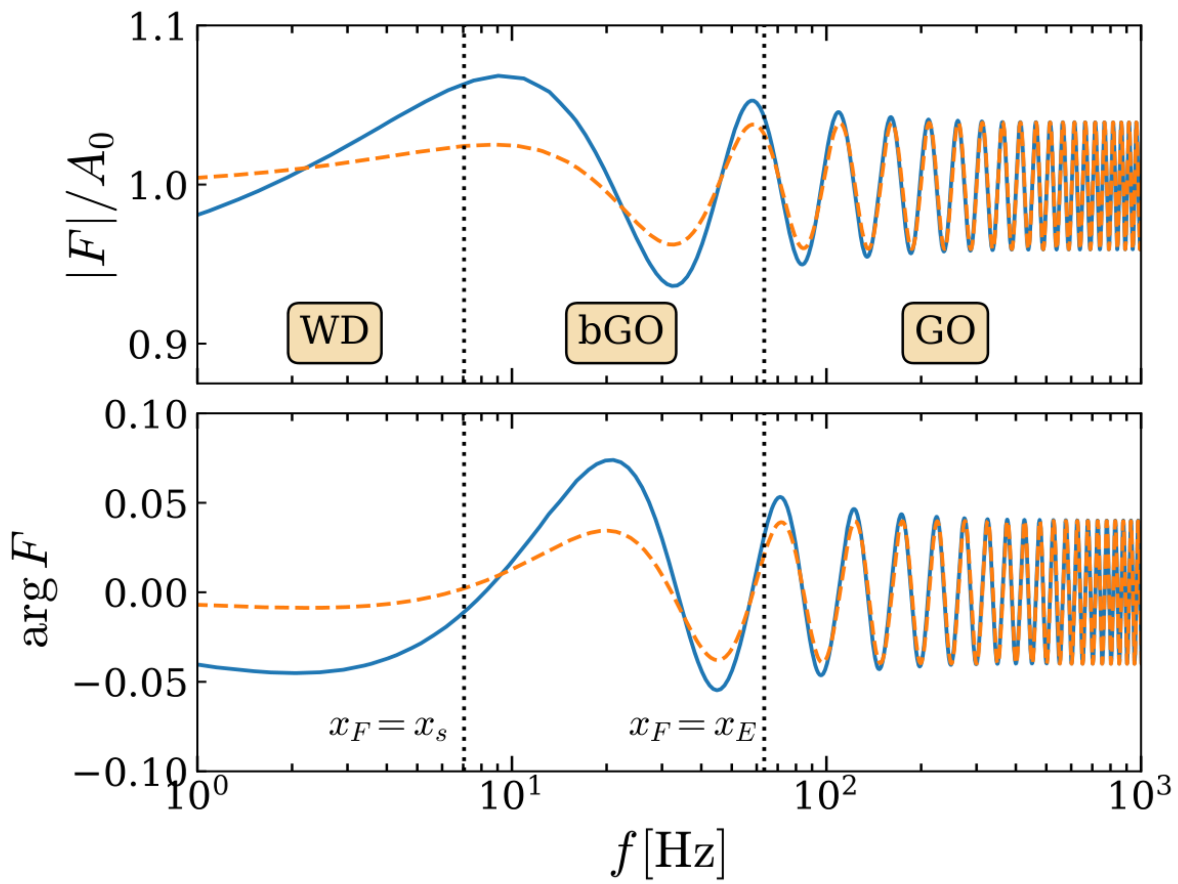
$$M_l = \frac{\Delta t}{2\left(\sqrt{\mu_r + \mu_r^{-1} - 2} - \ln \mu_r\right)}$$

Redshifted mass of point mass lens

If the number of lensing observables is two, it always can be interpreted as a point mass lensing

Diffractive lensing of Dressed PBH

The Key is diffractive lensing (= Halo profile) !



Solid : Dressed PBH
Dashed : Bare PBH

WD : Weak Diffraction
bGO : beyond Geometric Optics

Diffractive lensing of Dressed PBH

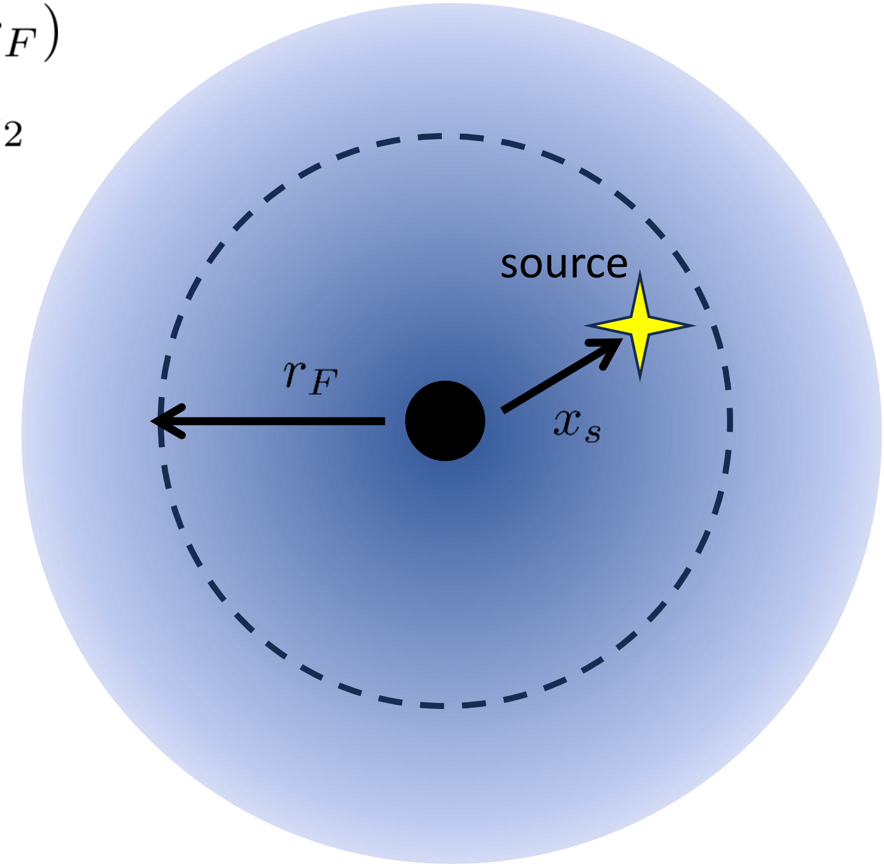
- Weak diffraction : $r_F > x_s$

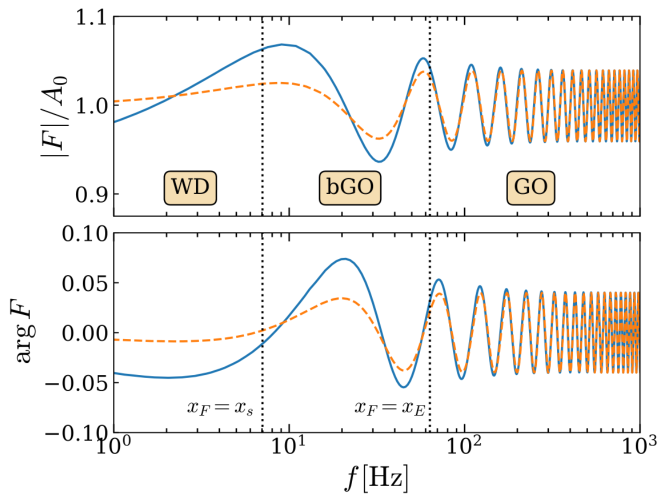
$$F(f) \simeq 1 + \overline{\kappa}(e^{i\frac{\pi}{4}} r_F)$$

$$\overline{\kappa}(e^{i\pi/4} r_F) \propto r_F^{-p} \propto f^{p/2}$$

$p = \frac{5}{4}$ Dressed PBH

$p = 2$ Bare PBH





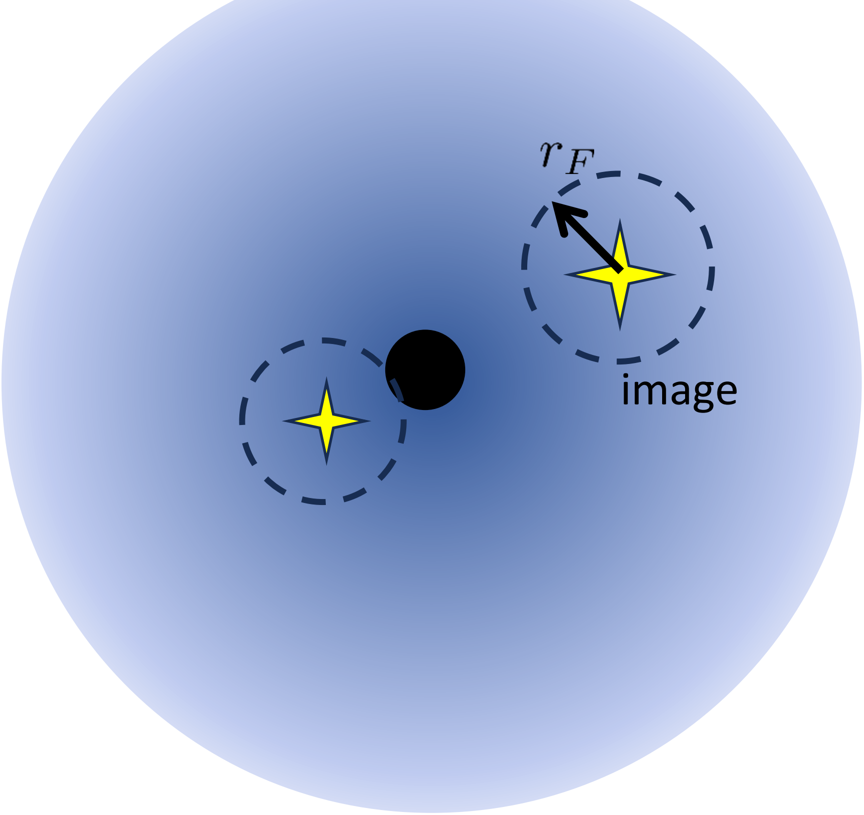
Diffractive lensing of Dressed PBH

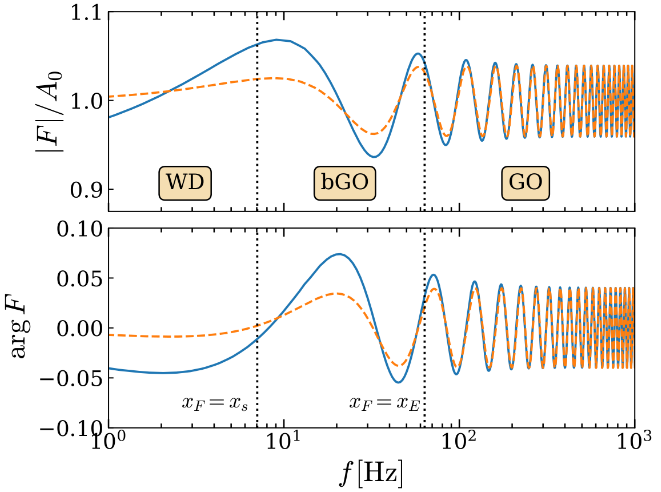
- Beyond Geometric optics : $r_F < x_s$

$$F(f) \simeq \sum_j |\mu_j|^{1/2} e^{2\pi i f T_j} (1 + \overline{\Delta\kappa}_j(e^{i\frac{\pi}{4}} r_F))$$ Diffractive lensing corrections to images

$$\overline{\Delta\kappa}(e^{i\pi/4} r_F) \simeq r_F^2 \propto \frac{1}{f}$$

$\overline{\Delta\kappa} = 0$ Bare PBH

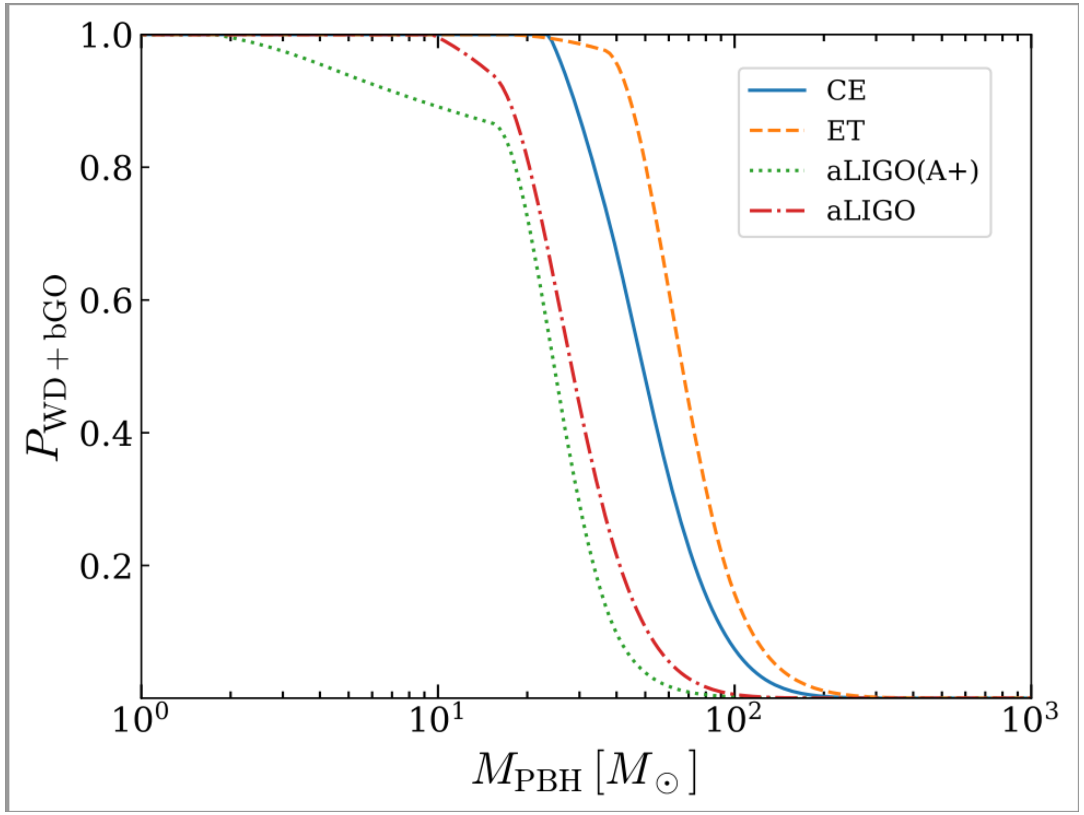




Diffractive lensing probability

Assuming lensing detection, we compute the probability of

$$r_E < \max r_F \text{ \& SNR>10 in WD+bGO}$$

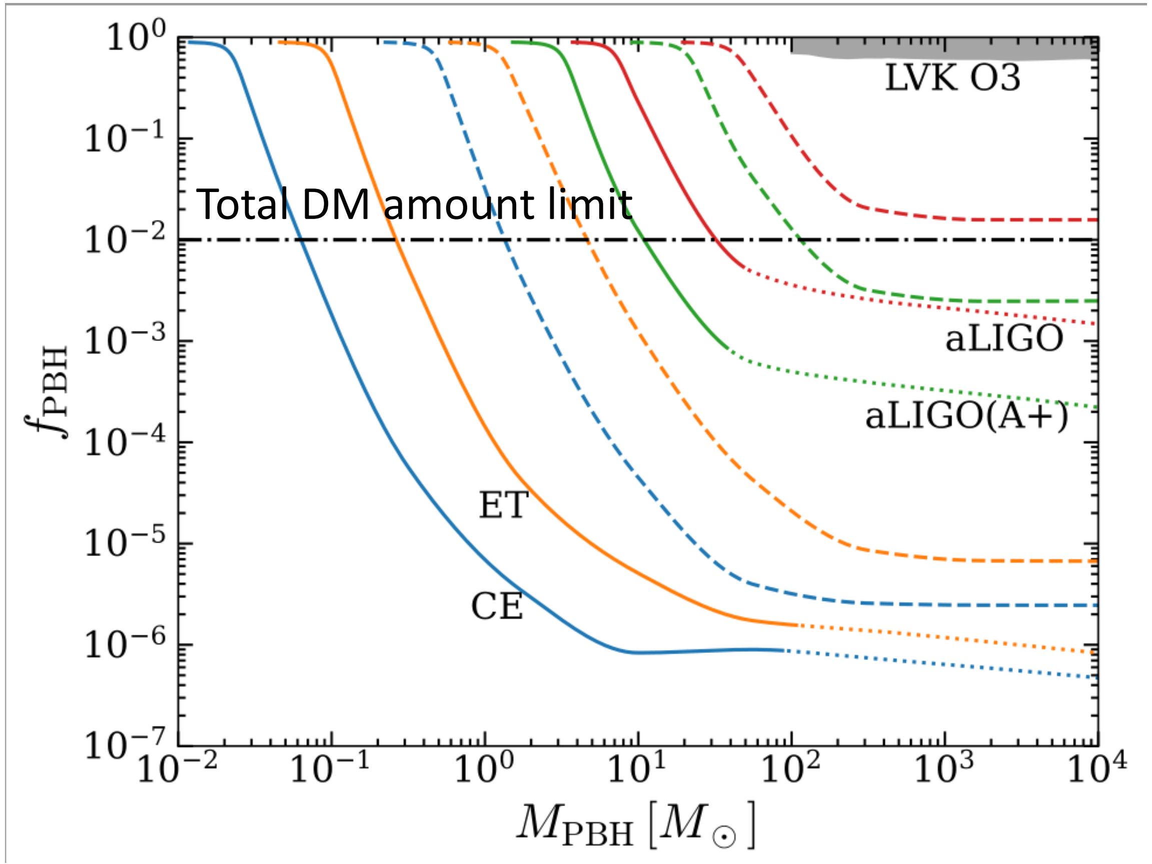


$10^2 M_\odot > M_{PBH}$ dressed PBH can be identified!

Diffractive lensing probability

Overall detection & identification prospects(5 years)

Diffractive lensing will reveal dressed PBHs in $10^{-1}M_{\odot} < M_{PBH} < 10^{2}M_{\odot}$



Solid : Dressed PBH
Dotted : Indistinguishable

Dashed : Bare PBH

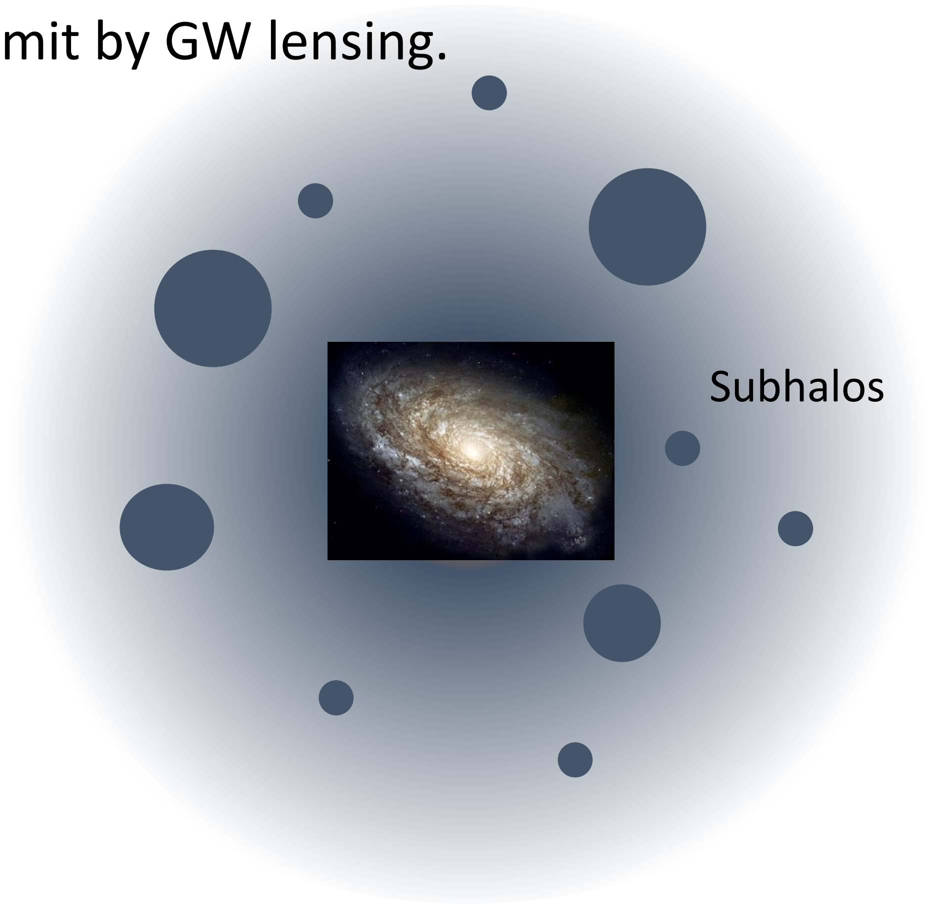
Probing small dark matter halos with weak diffractive lensing

Based on "Small-scale shear: peeling off diffuse subhalos with gravitational waves"

Han Gil Choi, Chanung Park and Sunghoon Jung, Phys. Rev. D **104**, 063001 (2021)

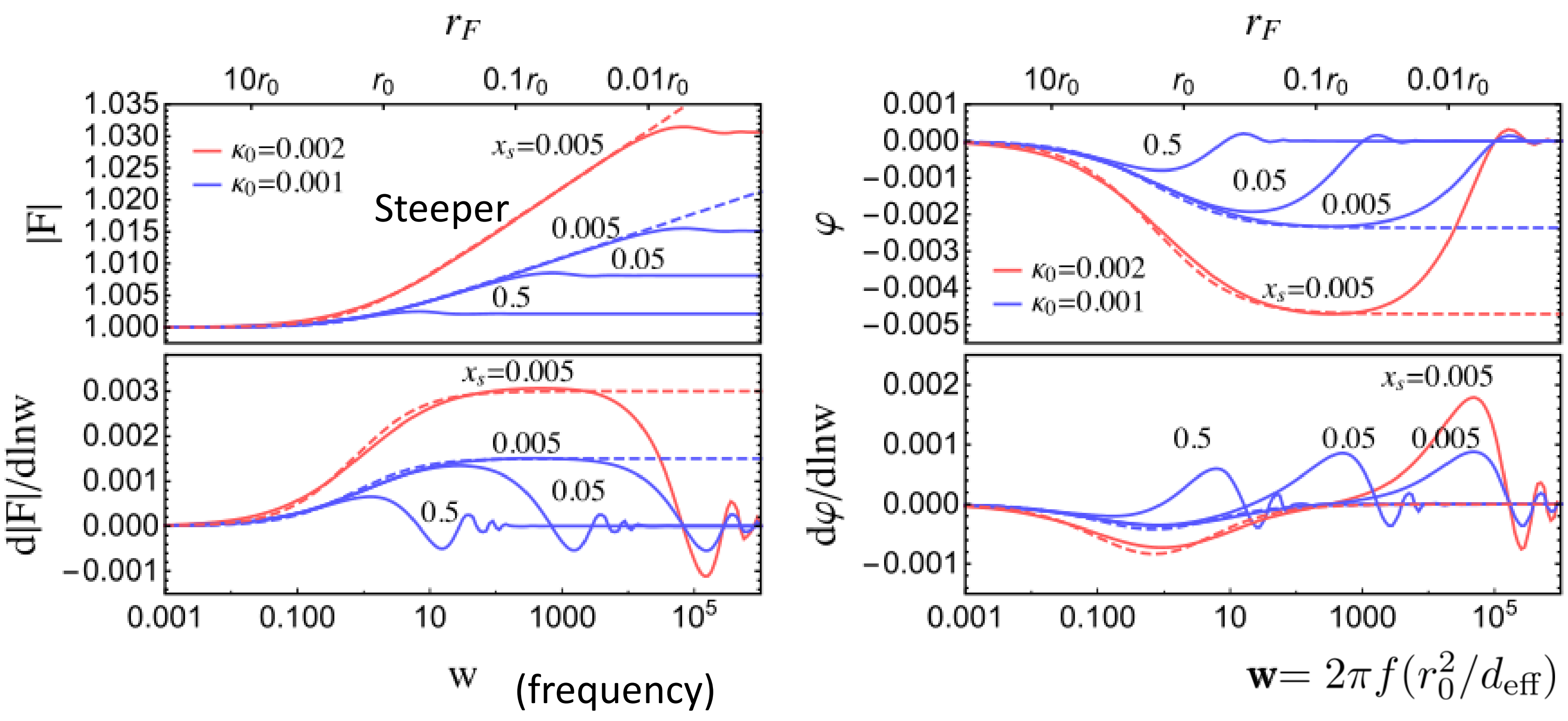
Motivations – Dark matter halo

- Can we detect diffuse lens object like **Dark matter halo by lensing**?
- **Small Dark matter halo can give a hint on dark matter properties.**
- Lensing constraints of small dark matter (sub)halo
  - $\mathrm{M_{sub}} > 10^7 M_\odot$ (Nadler 2021)
- We want to lower the limit by GW lensing.



Weak Diffractive lensing

- F(w) of Navarro-Frenk-White(NFW) profile. Numerical vs Analytic
- Good matches when $r_F > x_s$
- The **slope of F(w) follows the slope** of the DM halo profile.



Detection of Diffractive lensing

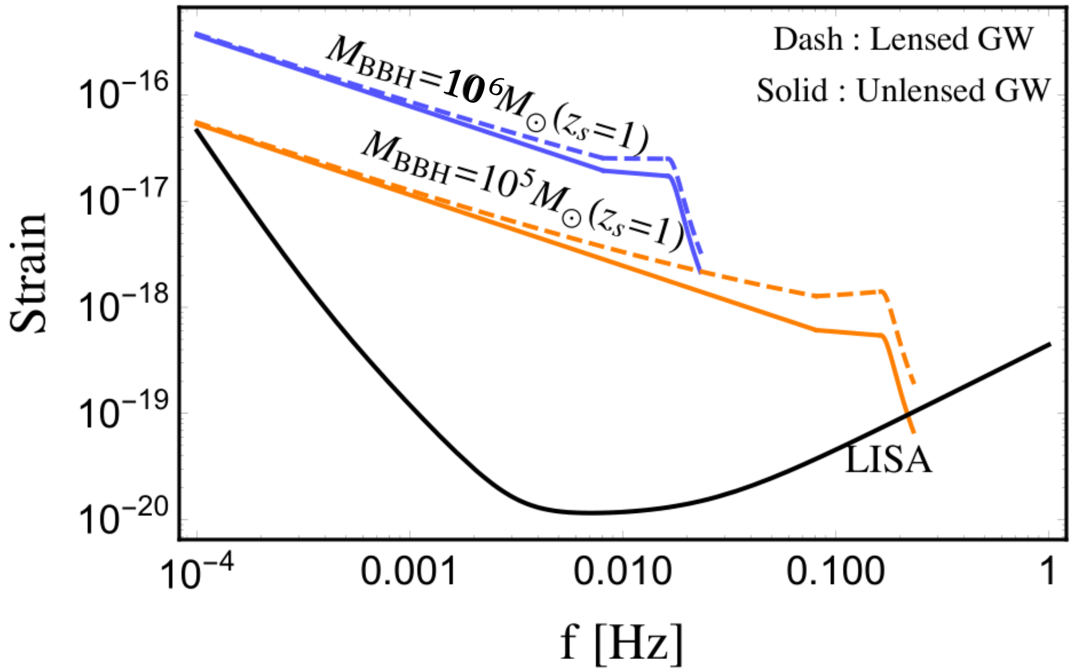
Multiple frequencies replace multiple background sources

Gaussian Noise, Small detector noise, Ignore correlations with source intrinsic parameters

$$\text{log-likelihood} \simeq \min_{A,\phi,t} 2 \sum_{f_j} \frac{|F(f_j) - Ae^{i\phi}e^{2\pi i f_j t}|^2 |h_0(f_j)|^2}{S_n(f_j)} \Delta f$$

Lensing by Singular Isothermal sphere lens ($M = 10^5\ M_{\odot}, z_l = 0.35$)

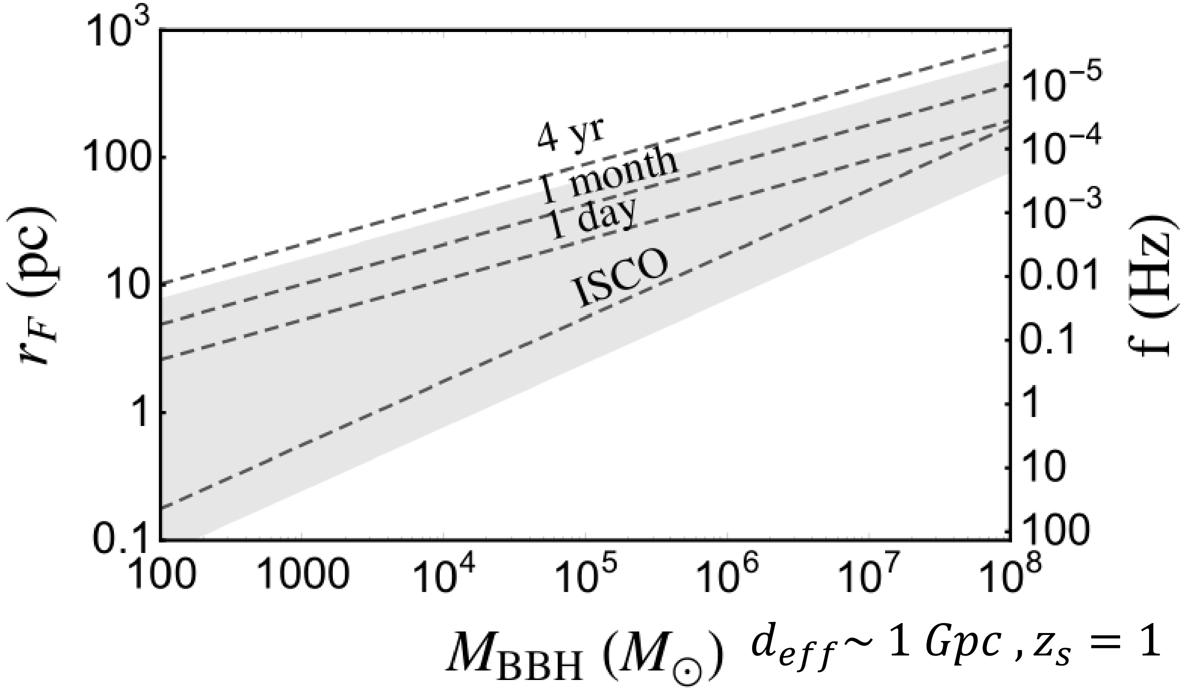
Ex) spectrum change



Detection of Diffractive lensing

- Small DM halo with **10 pc** length scale can be probed by **Massive BBH mergers**

$$r_F \equiv \sqrt{\lambda d_l} \simeq 5.56\mathrm{pc}\sqrt{\left(\frac{d_l}{\mathrm{Gpc}}\right)\left(\frac{0.1\mathrm{Hz}}{f}\right)}$$

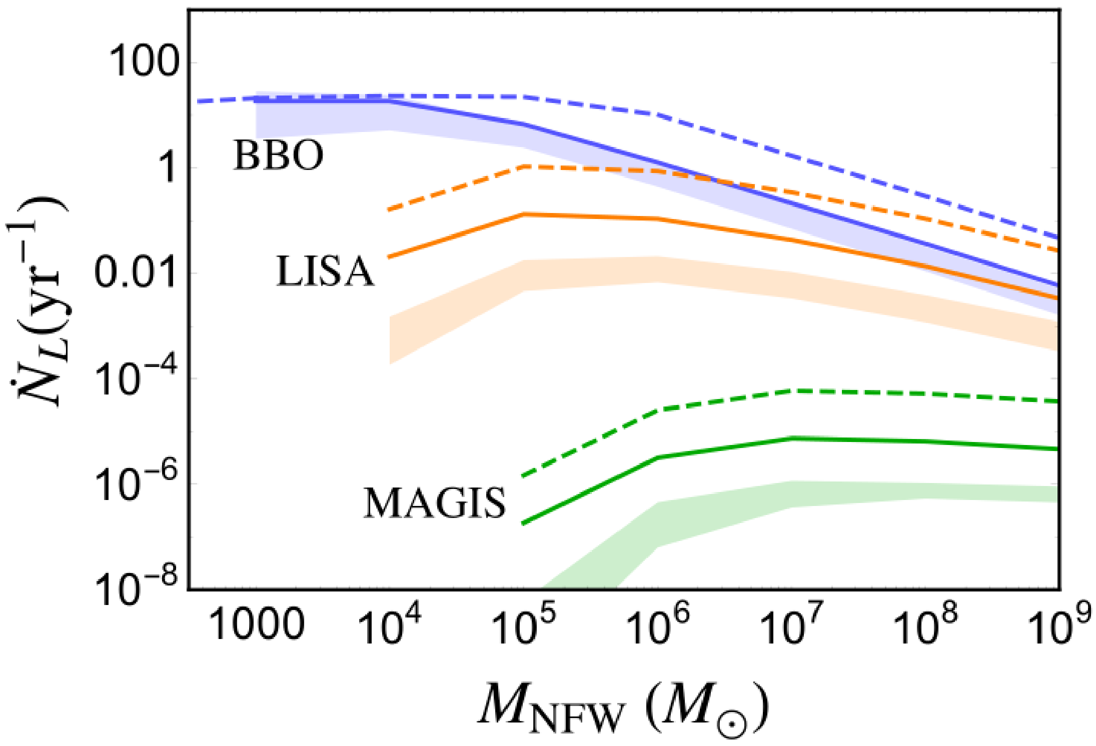


$d_{eff} \sim 1\ Gpc$, $z_s = 1$

Prospects

- Big Bang Observer (BBO) can detect **(CDM)** $\mathbf{10^3 M_\odot}$ **halo** more than 10 per year.
  - The others are less promising due to strong detector noise.

- The prospects highly depends on **massive BBH merger population** and **DM halo population**.



BBH Merger rate
Solid : 0.01 $Gpc^{-3}yr^{-1}$
Shaded : astrophysical (Bonetti 2018)

Dashed : No halo mass distribution

Summary

1. Diffractive lensing is wave optics phenomena which links lensing amplification to lens profile.

2. Dressed PBH and Bare PBH lensing can be distinguished by diffractive lensing.

3. r_F of GW from massive BBHs can be few parsecs. Therefore, light sub halos can be detected through diffractive lensing. Powerful GW can detect few tens of $10^{3\sim4}M_{\odot}$ DM halo per year.

4. Diffractive lensing will be powerful tool for mapping a dark matter mass distribution.