The 31st KIAS Combinatorics Workshop

Jeju, Korea
May 30 – June 1, 2024

Information

Title: The 31st KIAS Combinatorics Workshop

Date: May 30 – June 1, 2024

Venue: Utop Ubless Hotel, Jeju, Seoul

Web: http://events.kias.re.kr/h/combinatorics/ or kcws.combinatorics.kr

Invited Speakers

Zichao Dong (IBS ECOPRO)

Taehyun Eom (KAIST)

Tony Huynh (Sapienza Università di Roma)

Seonghyuk Im (KAIST)

Dongyeap Kang (IBS ECOPRO)

Ander Lamaison (IBS ECOPRO)

Eunjeong Lee (Chungbuk National University)

Seunghun Lee (KAIST)

Semin Oh (Kyungpook National University)

Seungsang Oh (Korea University)

Organizers

Jaehoon Kim (KAIST)

Jang Soo Kim (Sungkyunkwan University)

Jeong Han Kim (KIAS)

Seog-Jin Kim (Konkuk University)

Young Soo Kwon (Yeungnam University)

Sang June Lee (Kyung Hee University)

Jongyook Park (Kyungpook National University)

Seunghyun Seo (Kangwon National University)

<Time Table>

1st Day: May 30 (Thursday)		
14:30 ~ 15:30	Registration and Opening	
15:30 ~ 18:00	Session (A)	
Chair: Young Soo Kwon		
15:30 ~ 16:10	Tony Huynh	Sharing tea on a graph
16:10 ~ 16:30	Coffee Break	
16:30 ~ 17:10	Taehyun Eom	How computer proves
17:20 ~ 18:00	Zichao Dong	Saturation around the Happy Ending
18:30 ~	Dinner	

2nd Day: May 31 (Friday)		
~ 09:20	Breakfast	
09:20 ~ 10:50	Session (B)	
Chair: Sang June Lee		
09:20 ~ 10:00	Dongyeap Kang	Solution to a problem of Erdos on the chromatic index of hypergraphs with bounded codegree
10:10 ~ 10:50	Ander Lamaison	Uniform Turán density beyond 3-graphs
10:50 ~ 11:10	Coffee Break	
11:10 ~ 12:40	Session (C)	
Chair: Seunghyun Seo		
11:10 ~ 11:50	Eunjeong Lee	Orientations on Dynkin diagrams and topology on flag varieties
12:00 ~ 12:40	Seungsang Oh	Toroidal perfect matchings in the torus grid graph
12:40 ~ 14:30	Lunch	
14:30 ~ 17:30	Problem session and Free discussion	
18:30 ~	Banquet	

3rd Day: June 1 (Saturday)		
~ 09:30	Breakfast	
09:30 ~ 12:00	Session (D)	
Chair: Seog-Jin Kim		
09:30 ~ 10:10	Seonghyuk Im	Dirac's theorem for linear hypergraphs
10:20 ~ 11:00	Semin Oh	On Maximal Fixing Automorphisms of Graphs
11:00 ~ 11:20	Coffee Break	
11:20 ~ 12:00	Seunghun Lee	On conflict-free colorings of cyclic polytopes and the girth conjecture for graphs
12:00 ~	Lunch	

Speaker: Tony Huynh

Affiliation: Sapienza Università di Roma

Title: Sharing tea on a graph

Abstract

Consider the following procedure on a graph G. Initially, there is 1 unit of tea at a fixed vertex r of G and all other vertices have no tea. At any time in the procedure, we can choose an edge uv of G and equalize the amount of tea between u and v. We prove that for every vertex x of G, the amount of tea at x is always at most $1/(d+1)$, where d is the distance from x to r. This bound is best possible and answers a question of Nina Gantert. This problem is motivated by the analysis of consensus formation in the Deffuant model for social interaction, which I will also discuss. This is joint work with J. Pascal Gollin, Kevin Hendey, Hao Huang, Bojan Mohar, Sang-il Oum, Neil N.Y. Yang, Wei-Hsuan Yu, and Xuding Zhu.

Speaker: Taehyun Eom

Affiliation: KAIST

Title: How computer proves

Abstract

Even before computers became electronic, computers were with combinatorics. The first computer program is known as the program to calculate Bernoulli numbers. The first computer-assisted theorem that caused a lot of controversy was the four color theorem. Moreover, the four color theorem is now fully computer-verified by Coq, a proof assistant language. Nowadays, with progress in AI, it is not amazing that Stack Exchange made a site for proof assistants, which shows a large expectation for the AI do mathematics. In this talk, I will introduce the Curry-Howard correspondence between proofs and programming with some simulated results and what it tells to combinatorics. This correspondence is based on the similarity between modus ponens and function application, and is one of the technical bases of current proof assistants.

Speaker: Zichao Dong

Affiliation: IBS ECOPRO

Title: Saturation around the Happy Ending

Abstract

We consider saturation problems related to the celebrated Erdős–Szekeres convex polygon problem. For each $n \geq 7$, we construct a planar point set of size $(7/8) \cdot 2^{n-2}$ which is saturated for convex n-gons. That is, the set contains no n points in convex position while the addition of any new point creates such a configuration. This demonstrates that the saturation number is smaller than the Ramsey number for the Erdős–Szekeres problem. The proof also shows that the original Erdős–Szekeres construction is indeed saturated. Our construction is based on a similar improvement for the saturation version of the cups-versus-caps theorem. Moreover, we consider the generalization of the cups-versus-caps theorem to monotone paths in ordered hypergraphs. In contrast to the geometric setting, we show that this abstract saturation number is always equal to the corresponding Ramsey number. This is joint work with Gabór Damásdi, Manfred Scheucher, and Ji Zeng.

Speaker: Dongyeap Kang

Affiliation: IBS ECOPRO

Title: Solution to a problem of Erdos on the chromatic index of hypergraphs with bounded codegree

Abstract

In 1977, Erdős asked the following question: for any integers $t, n \in \mathbb{N}$, if $G_1, \ldots, G_n$ are complete graphs such that each G_i has at most n vertices and every pair of them shares at most t vertices, what is the largest possible chromatic number of the union $\bigcup_{i=1}^{n} G_i$? The equivalent dual formulation of this question asks for the largest chromatic index of an n-vertex hypergraph with maximum degree at most n and codegree at most t. For the case $t = 1$, Erdős, Faber, and Lovász famously conjectured that the answer is n, which was recently proved for all sufficiently large n. In this talk, we will present the resolution of the conjecture for large hypergraphs in a strong sense. This is joint work with Tom Kelly, Daniela Kühn, Abhishek Methuku, and Deryk Osthus.

Speaker: Ander Lamaison

Affiliation: IBS ECOPRO

Title: Uniform Turán density beyond 3-graphs

Abstract

The uniform Turán density $\pi_u(F)$ of a hypergraph F, introduced by Erdős and Sós, is the smallest value of d such that any hypergraph H where all linear-sized subsets of vertices of H have density greater than d contains F as a subgraph. Over the past few years the value of $\pi_u(F)$ was determined for several classes of 3-graphs, but no nonzero value of $\pi_u(F)$ has been found for r-graphs with $r > 3$. In this talk we show the existence of r-graphs F with $\pi_u(F) = \binom{r}{2}^{-\binom{r}{2}}$, which we conjecture is minimum possible. Joint work with Frederik Garbe, Daniel Il'kovic, Dan Král' and Filip Kučerák.

Speaker: Eunjeong Lee

Affiliation: Chungbuk National University

Title: Orientations on Dynkin diagrams and topology on flag varieties

Abstract

Let G be a simply laced simple Lie group, T a maximal torus of G, and B a Borel subgroup of G containing T. The homogeneous space G/B is a smooth projective algebraic variety, called a flag variety. The left multiplication of T on G induces that on G/B. The geometric and topological properties of the flag variety G/B and its subvarieties are closely related to combinatorics on the Weyl group of G. In this talk, we study how orientations on Dynkin diagrams determine toric Schubert varieties. This talk is based on joint work with Mikiya Masuda and Seonjeong Park.

Speaker: Seungsang Oh

Affiliation: Korea University

Title: Toroidal perfect matchings in the torus grid graph

Abstract

A perfect matching in a graph is a subset of edges where each vertex is incident to exactly one edge from the subset. The enumeration of perfect matchings in a rectangular grid graph on a plane was initially achieved by Kasteleyn as well as Temperley and Fisher in 1961. This study focuses on counting perfect matchings in a grid graph on the torus. We introduce two categories of perfect matchings: conventional ones characterized by fixed positions and toroidal ones up to cyclic rotations on the torus. We provide a recursive matrix relation to obtain the number of perfect matchings characterized by fixed positions and another recursive formula considering rotational characteristics in toroidal perfect matchings in a torus grid graph.

Speaker: Seonghyuk Im

Affiliation: KAIST

Title: Dirac's theorem for linear hypergraphs

Abstract

Dirac's theorem states that any n-vertex graph G with even integer n satisfying $\delta(G) \geq n/2$ contains a perfect matching. We generalize this to k-uniform linear hypergraphs by proving the following. Any n-vertex k-uniform linear hypergraph H with minimum degree at least $\frac{n}{k} + \Omega(1)$ contains a matching that covers at least $(1 - o(1))n$ vertices. This minimum degree condition is asymptotically tight and obtaining perfect matching is impossible with any degree condition. Furthermore, we show that if $\delta(H) \geq (\frac{1}{k} + o(1))n$, then H contains almost spanning linear cycles, almost spanning hypertrees with $o(n)$ leaves, and "long subdivisions" of any $o(\sqrt{n})$-vertex graphs. This is joint work with Hyunwoo Lee.

Speaker: Semin Oh

Affiliation: Kyungpook National University

Title: On Maximal Fixing Automorphisms of Graphs

Abstract

In linear algebra, a reflection is a linear transformation from a vector space to itself with a hyperplane as the set of fixed points. For a linear transformation t, the following two statements are equivalent: (i) t is a reflection; (ii) every linear transformation t' fixing all fixed points of t is equal to t or the identity.

Motivated by (ii), we consider a function representing "reflection" in graphs. For a graph G, we define an automorphism σ of G as a *maximal fixing automorphism* (*MFA*) if every automorphism τ of G fixing all fixed points of σ is equal to σ or the identity. The identity automorphism of G is the trivial MFA. A graph is *MFA-free* if it has no nontrivial MFAs.

A graph is asymmetric if it has no nontrivial automorphisms. A graph is involution-free if it has no involutions, i.e., automorphisms of order 2. In 1988, Nešetřil conjectured that there exists only a finite number of finite minimal asymmetric graphs. In 1992, Nešetřil and Sabidussi conjectured that the set of finite minimal asymmetric undirected graphs and the set of finite minimal involution-free undirected graphs are the same. These two conjectures were confirmed in 2017 by Pascal Schweitzer and Patrick Schweitzer.

In this talk, we investigate the relationship between asymmetric graphs, involution-free graphs, and MFA-free graphs.

Speaker: Seunghun Lee

Affiliation: KAIST

Title: On conflict-free colorings of cyclic polytopes and the girth conjecture for graphs

Abstract

We study conflict-free colorings for hypergraphs derived from the family of facets of d-dimensional cyclic polytopes. For odd dimensions d, the problem is fairly easy. However, for even dimensions d the problem becomes highly non-trivial. We provide sharp asymptotic bounds for the conflict-free chromatic number in all even dimensions $4 \leq d \leq 20$ except for $d = 16$. We also provide non-trivial upper and lower bounds in any even dimension d. We exhibit a strong relation to the famous Erdős girth conjecture in extremal graph theory which might be of independent interest for the study of conflict-free colorings. Improving the upper or lower bounds for general even dimensions d would imply an improved lower or upper bound (respectively) on the Erdős girth conjecture. Finally, we extend our result for dimension 4 showing that the hypergraph whose hyperedges are the union of two discrete intervals from $[n]$ of cardinality at least 3 has conflict-free chromatic number $\Theta(\sqrt{n})$. This is a joint work with Shakhar Smorodinsky.