ABSTRACTS

Function Theory, Operator Theory and Applications

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Contents

Circle companions of Hardy spaces of the unit disk

Raúl E. Curto

We give a complete answer to the following problem:

Find the circle companion of the Hardy space of the unit disk with values in the space of all bounded linear operators between two separable Hilbert spaces.

Classically, the problem asks whether for each function h on the unit *disk*, there exists a "boundary function" bh on the unit *circle* such that the mapping $bh \mapsto h$ is an isometric isomorphism between Hardy spaces of the unit circle and the unit disk with values in some Banach space.

For the case of bounded linear operator-valued functions, we construct a Hardy space of the unit circle such that its elements are SOT measurable, and their norms are integrable: indeed, this new space is isometrically isomorphic to the Hardy space of the unit disk via a "strong Poisson integral."

The talk is based on recent research with In Sung Hwang, Sumin Kim and Woo Young Lee:
Circle companions of Hardy spaces of the unit disk, *J. Funct. Anal.* 285 (2023), no. 12, Paper No. 110159, 23 pp.

Keywords: Operator-valued Hardy spaces, the analytic Radon-Nikodým property, SOT measurable functions, strong Poisson integrals, strong boundary functions, circle companions.

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Subnormality of block Toeplitz operators

Mankunikuzhiyil Abhinand

In this talk, we discuss some sufficient conditions under which a subnormal block Toeplitz operator is either normal or analytic. Let M_n denotes the collection of all $n \times n$ complex matrices. An M_n-valued function D is of the form $D(z) = v \prod_{m=1}^{M} (b_m(z)P_m + (I - P_m))$, where v is an $n \times n$ unitary constant matrix, b_m is a Blaschke factor, of the form $b_m(z) = \frac{z-\alpha_m}{1-\overline{\alpha}_m z}$ ($\alpha_m \in \mathbb{D}$), and P_m is an orthogonal projection in $\mathbb{C}^n$ is called a finite Blaschke-Potapov product. Here, we address the following question posed by R. E. Curto, I. S. Hwang and W. Y. Lee [3]. "When is the subnormal block Toeplitz operator T_Φ either normal or analytic for a matrix-valued function $\Phi = B\Phi^*$, where B is the finite Blaschke-Potapov product ?" This is directly related to Halmos's Problem 5 [4] and Abrahamse's Theorem [2]. This is a joint work with R. E. Curto, I. S. Hwang, W. Y. Lee and T. Prasad [1].

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Sub-Bergman Hilbert spaces on the unit disk

Shuaibing Luo

In this talk, we introduce some basic properties of the sub-Bergman spaces on the unit disk. We investigate when the sub-Bergman space is a complete Nevanlinna-Pick space. We then study the relationship between the range of the defect operators and the symbols being the finite Blaschke products.

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The Fuglede conjecture and the momentum operator on a union of intervals

Dorin Dutkay

The Fuglede conjecture relates Fourier bases on a set to the geometric, tiling properties of that set. However, the conjecture originates in the study of self-adjoint extension of partial differential operators. In this talk we present an overview of the Fuglede conjecture and some connections to the properties of the differential operator, also called momentum operator, on a finite union of intervals.

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Conditional mean embedding and roptimal feature selection via positive definite kernels

Myung-Sin Song

New operator-theoretic approaches to conditional mean embedding (CME). Our present results combine a spectral analysis-based optimization scheme with the use of kernels, stochastic processes, and constructive learning algorithms is being studied. For initially given non-linear data, we consider optimization-based feature selections. This entails the use of convex sets of kernels in a construction of optimal feature selection via regression algorithms from learning models. Thus, with initial inputs of training data (for a suitable learning algorithm), each choice of a kernel K in turn yields a variety of Hilbert spaces and realizations of features. A novel aspect of our work is the inclusion of a secondary optimization process over a specified convex set of positive definite kernels, resulting in the determination of "optimal" feature representations.

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A classification of invariant subspaces of $L^2(\mathbb{T}^n)$ and $L^2(\mathbb{R}^n)$

Caixing Gu*, Dong-O Kang and Michio Seto

In this paper we describe the invariant subspace M for the tuple of multiplciation opeators $T_z = (T_{z_1}, \cdots, T_{z_n})$ on $L^2(\mathbb{T}^n)$ under the condition that the restricted tuple $T_z|M = (T_{z_1}|M, \cdots, T_{z_n}|M)$ is doubly commuting. This exends the result of Ghatage and Mandrekar, and Nakazi (for $n = 2$), and Seto (for $n = 3$). We formulate our invariant subspace theorem using intersecting families of subsets of $\{1, 2, \cdots, n\}$ which are classical and important in finite extremal set theory.

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Square root problems of bounded operators

Jasang Yoon

In the talk we consider the following Square Root Problem for measures: Given a positive probability Borel measure μ (supported on an interval $[a, b] \subseteq \mathbb{R}^+$), does there exist a positive Borel measure ν such that $\mu = \nu * \nu$ holds? (Here $*$ denotes the multiplicative convolution, properly defined on $\mathbb{R}^+$.) This problem is closely connected to the subnormality of the Aluthge transform of a unilateral weighted shift. We next consider and study the joint subnormality of the spherical and toral Aluthge transforms of a 2-variable weighted shift from the previous sentence. This is a joint work with R. E. Curto and J. Kim.

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The Gohberg-Sigal theory and eigenvalues of the Laplacian

Hyun Kwon

We estimate the eigenvalues of the Laplacian using single and double layer potentials, geometric basis elements, and the Gohberg-Sigal Theory that gives generalizations of Rouche's and the Residue Theorems. This talk is based on joint work with M. Beceanu, J. Hong, and M. Lim.

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Lifting B-subnormal operators

Il Bong Jung

We study B-operators (abbreviation for Brownian-type operators), which are upper triangular 2×2 block matrix operators with entries satisfying some algebraic constraints. We establish a lifting theorem stating that any B-subnormal operator, i.e., B-operator with subnormal $(2, 2)$-entry, lifts to a B-normal operator, i.e., a B-operator with normal $(2, 2)$ entry, where lifting is understood in the sense of extending entries of the block matrices representing the operators in question. The spectral inclusion and the filling in holes theorems are obtained for such operators.

(This is a joint work with S. Chavan, Z. Jablonski and J. Stochel)

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Fock space approach to the theory of kernel functions

Michio Seto

In machine learning, the theory of reproducing kernel Hilbert spaces has a great application known as kernel method. In this talk, motivated by some problem in machine learning, we give a new approach to the theory of positive definite kernels. Our approach is based on the structure of Fock spaces and an elementary fact in the Hardy space operator theory. As results, a new example of a strictly positive definite kernel and a new proof of universal approximation theorem for Gaussian type kernels are given.

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Linear operators that preserve perimeters of completely positive Boolean matrices

Seok-Zun Song

A factor rank-1 Boolean matrix B can be factored as $\mathbf{c}\mathbf{d}^t$ for column vectors $\mathbf{c}$ and $\mathbf{d}$ of some orders. We define the perimeter of $B = \mathbf{c}\mathbf{d}^t$ as the number of nonzero entries in both $\mathbf{c}$ and $\mathbf{d}$. Every Boolean matrix is a sum of factor rank-1 Boolean matrices. The perimeter of a Boolean matrix C is the minimum sum of the perimeters of the factor rank-1 Boolean matrices whose sum is C.

In this talk, we have some properties of the perimeter of completely positive Boolean matrices and get the characterizations of the linear operators that preserve the perimeters of completely positive Boolean matrices.

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m-symmetric operators with elementary operator entries

B.P. Duggal

Given Banach space operators A, B, let $\delta_{A,B}$ denote the generalised derivation $\delta(X) = (L_A - R_B)(X) = AX - XB$ and $\triangle_{A,B}$ the length two elementary operator $\triangle_{A,B}(X) = (I - L_A R_B)(X) = X - AXB$. This note considers the structure of m-symmetric operators $\delta^m_{\triangle_{A_1,B_1},\triangle_{A_2,B_2}}(I) = (L_{\triangle_{A_1,B_1}} - R_{\triangle_{A_2,B_2}})^m(I) = 0$ to prove that there exist scalars $\lambda_i \in \sigma_a(B_1)$, $1 \leq i \leq 2$, such that $\delta^m_{\lambda_1 A_1, \lambda_2 A_2}(I) = 0$. Translated to Hilbert space operators A and B this implies that if $\delta^m_{\triangle_{A^*,B^*},\triangle_{A,B}}(I) = 0$, then there exists $\overline{\lambda} \in \sigma_a(B^*)$ such that $\delta^m_{(\lambda A)^*,\lambda A}(I) = 0 = \delta^m_{\overline{\lambda} B,\lambda B^*}(I)$. The operator $\delta^m_{\triangle_{A^*,B^*},\triangle_{A,B}}$ is compact if and only if (i) there exists a real number α and finite sequnces (i) $\{a_j\}_{j=1}^n \subseteq \sigma(A)$, $\{b_j\}_{j=1}^n \subseteq \sigma(B)$ such that $a_j b_j = 1 - \alpha$, $1 \leq j \leq n$; (ii) decompositions $\oplus_{j=1}^n \mathcal{H}_j$ and $\oplus_{j=1}^n \mathtt{H}_J$ of $\mathcal{H}$ such that $\oplus_{j=1}^n (A - a_j I)|_{H_j}$ and $\oplus_{j=1}^n (B - b_j I)|_{\mathtt{H}_j}$ are nilpotent. If $\delta^m_{\triangle_{A^*,B^*},\triangle_{A,B}}(I) = 0$ implies $\delta_{\triangle_{A^*,B^*},\triangle_{A,B}}(I) = 0$, then A and B satisfy a (Putnam-Fuglede type) commutativity theorem; conversely, a sufficient condition for $\delta^m_{\triangle_{A^*,B^*},\triangle_{A,B}}(I) = 0$ to imply $\delta_{\triangle_{A^*,B^*},\triangle_{A,B}}(I) = 0$ is that λA and $\overline{\lambda} B$ satisfy the commutativity property for scalars $\lambda \in \sigma_a(B^*)$. An analogous result is seen to hold for the operators $\triangle^m_{\delta_{A^*,B^*},\delta_{A,B}}$ and $\triangle^m_{\delta_{A^*,B^*},\delta_{A,B}}(I)$.

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On the mixed identity for normed modules

Robin E. Harte* and Dragan S. Djordjević

The mixed identity for normed modules shows that the real and the complex duals of a complex Banach space coincide, offers a partial extension of the von Neumann double commutant theorem to reflexive Banach spaces, explains the Arens multiplication on the second dual of a Banach algebra, and has a high old time playing around among the integration spaces

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Multivariable operator theory and analysis of fractals

Palle Jorgensen

We show that Multivariable operator theory and recursive iteration schemes (usually referred as "the" Kaczmarz algorithms), yield important applications in infinite-dimensional, and non-commutative, settings central to spectral theory of operators in Hilbert space, to optimization, to large sparse systems, to iterated function systems (IFS), and to fractal harmonic analysis. We present a new recursive iteration scheme involving as input a prescribed sequence of selfadjoint projections. Applications include random Kaczmarz recursions, their limits, and their error-estimates.

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Conjugations commuting with unitary operators

Marek Ptak

Conjugations– antilinear, isometric, involutions – commuting with a given unitary operator are investigated. The condition when such conjugation exists is given. Connections of such conjugations with natural decompositions of the unitary operator are presented. Using the spectral theorem we characterize conjugations with those properties completely. The examples of specific unitary operators are pointed out.

Joint work with J. Mashreghi, W. Ross.

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On numerical ranges

Muneo Chō

Let $\mathcal{H}$ be a complex Hilbert space and T be an operator on $\mathcal{H}$. T is said to be *antilinear* if $T(\alpha x + \beta y) = \overline{\alpha}Tx + \overline{\beta}Ty$ for all $\alpha, \beta \in \mathbb{C}, x, y \in \mathcal{H}$. Then the numerical range $W(T)$ of T is defined by $W(T) := \{\langle Tx, x\rangle : \|x\| = 1\}$ (just same). Also. let $\mathcal{X}$ be a complex Banach space, T be an antilinear operator on $\mathcal{X}$. Then the numerical range $V(T)$ of T is defined by $V(T) := \{f(Tx) : (x, f) \in \Pi\}$ (also just same).

I'll introduce properties of those numerical ranges of antilinear operators on Hilbert spces and Banach spaces. And I'll set problems.

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Some results on geometrically regular weighted shifts

George R. Exner

In sectors nicely arranged in the unit square of the parameter space for (N, D), we show that these geometrically regular weighted shifts (GRWS) have representatives in well-studied classes: moment infinitely divisible, subnormal, k– but not $(k+1)$–hyponormal, or completely hyperexpansive. Various tools are used to obtain the results, in particular interpolation of the weight or moment sequence by a variety of well-known functions (such as Bernstein functions) or the presence of a Berger measure. The GRWS also provide subshifts of the Bergman shift with geometric, not linear, spacing in the weights which are moment infinitely divisible.

This new family of weighted shifts provides a useful addition to the library of shifts with which to explore new definitions and properties.

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Some quotients for geometrically regular weighted shifts

Chafiq Benhida

In joint work with Raúl E. Curto (University of Iowa, USA) and George R. Exner (Bucknell University, USA) we consider geometrically regular weighted shifts (GRWS), namely those with weights $\alpha_n = \sqrt{\frac{p^n+N}{p^n+D}}$, where $p > 1$ and N and D are parameters so that $(N, D) \in (-1, 1) \times (-1, 1)$. We study the zone of pairs (M, P) for which the weight $\frac{\alpha(N,D)}{\alpha(M,P)}$ gives rise to a moment infinitely divisible ($\mathcal{MID}$) or a subnormal weighted shift, and deduce immediately the analogous results for product weights $\alpha(N, D)\alpha(M, P)$ instead of quotients.

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Koopman operators with intrinsic observables in rigged reproducing kernel Hilbert spaces

Isao Ishikawa

The Koopman operator is the equivalent notion of the composition operator, that is defined as the pull-back of a map onto a function space. The composition operator has been studied for a long time, and their properties have been researched in various function spaces. In recent years, the composition operator, as the Koopman operator, has been actively researched for analyzing time-series data generated from dynamical systems in the context of engineering and physics. In this talk, we explain mathematical results and motivation of Koopman operators on reproducing kernel Hilbert spaces within the context of data analysis. Additionally, I introduce an algorithm for estimating composition operators from finite data, using these results.

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Asymptotics of block Toeplitz determinants with piecewise continuous symbols

Torsten Ehrhardt

Under some assumptions, we determine the asymptotics (as $n \to infty$) of the determinants of block Toeplitz matrices $T_n(c) = [c_{i-j}]_{i,j=1}^n$ where c is a piecewise continuous $N \times N$-matrix valued function defined on the unit circle with Fourier coefficients $c_k \in \mathbb{C}^{N\times N}$ and the block size N being fixed. This joint work with E.L. Basor and J. Virtanen generalizes the the following "classical results": For smooth symbols (satisfying some regularity conditions) the Toeplitz determinant asymptotics is given by the Strong Szegö Limit theorem (1952) in the scalar case $N = 1$, and by the Szegö-Widom Limit Theorem (1976) in the block case $N > 1$. The corresponding the asymptotics is given by $\det T_n(c) \sim G^n E$ with certain constants $G, E \neq 0$. On the other hand, scalar $(N = 1)$ piecewise continuous symbols are as special case of so-called Fisher-Hartwig symbols, where the asymptotic is either of the form $\det T(c) \sim G^n n^\Omega E$ or maybe be even more complicated. Thus the result we obtain generalized the Fisher-Hartig asymptotics to the block case. Whereas we prove that the asymptotics is similar to the scalar case, the proof is more complicated as certain obstacles are encountered on the block case. The proof is heavily based on operator theory and uses so-called localization theorems, i.e., the asymptotics for symbols with multiple discontinuities is reduced particular symbols with a singles discontinuity. Furthermore, we only able to treat the cases with small jump parameters. Interestingly as a side-result we obtain also formula for the Fredholm index of a block Toeplitz operator $T(c)$ with piecewise continuous symbol, which complements previously known formulas.

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Log-majorization by grand Furuta inequality

Masatoshi Fujii

This talk is based on a joint work with Y. Seo.

The grand Furuta inequality has been established as an interpolation between Furuta inequality and Ando-Hiai inequality:

(GFI) *If* $A \geq B > 0$ *and* $t \in [0,1]$*, then*

$$[A^{\frac{r}{2}}(A^{-\frac{t}{2}}B^pA^{-\frac{t}{2}})^sA^{\frac{r}{2}}]^{\frac{1}{q}} \leq A^{\frac{(p-t)s+r}{q}}$$

holds for $r \geq t$*,* $p \geq 0$*,* $q \geq 1$ *and* $s \geq 1$ *with* $(1-t+r)q \geq (p-t)s+r$*.*

As an application, we have the following log-majorization:

Theorem 1. Suppose that $0 \leq q \leq p$, $0 \leq t \leq 1$, $r \geq t$ and $pt = qr$. Then

$$A^{\frac{r-1}{2}}(A^{-r} \natural_{\frac{\beta}{p+r}} B^p)A^{\frac{r-1}{2}} \prec_{(log)} A^{\frac{t-1}{2}}(A^{-t} \natural_{\frac{\beta}{q+t}} B^q)A^{\frac{t-1}{2}}$$

holds for $\beta \geq q+t$, where $A \natural_\alpha B = A^{\frac{1}{2}}(A^{-\frac{1}{2}}BA^{-\frac{1}{2}})^\alpha A^{\frac{1}{2}}$ for $A, B > 0$.

As a matter of fact, it suffices to show the following operator inequality, which is proved by (GFI).

Theorem 2. Suppose that $0 \leq q \leq p$, $0 \leq t \leq 1$, $r \geq t$ and $pt = qr$. Then

$$(A^{\frac{t}{2}}B^qA^{\frac{t}{2}})^{\frac{\beta}{q+t}} \leq A \ \Rightarrow \ (A^{\frac{r}{2}}B^pA^{\frac{r}{2}})^{\frac{\beta}{p+r}} \leq A$$

holds for $\beta \geq q+t$.

Theorem 1 induces the monotonicity of a relative entropy of Tsallis type discussed by Seo and myself.

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Theory of some classes of operators related to subnormal operators

T. Prasad

In this talk, we discuss some structural and spectral properties of natural extensions of subnormal operators, namely n-subnormal operators ($T \in B(H)$: T^n is subnormal) and sub-n-normal ($T \in B(H)$: T has n-normal extension) operators. We discuss some inclusion relations among the above mentioned classes and other related classes and also characterization of these classes of operators in the view point of some concrete operators. This is joint work with R.E. Curto, which aims to develop a theory for operators that are n-th roots of subnormal operators.

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Solvable discrete quantum groups

Masato Tanaka

In 2009, Etingof–Nikshych–Ostrik formulated nilpotency and solvability for fusion categories. If we define nilpotency and solvability for finite quantum groups via this language, nilpotency does not imply solvability in general. In 2016, Cohen–Westreich proposed definitions of nilpotency and solvability via integrals. Their definitions are satisfactory in that nilpotency implies solvability and the analogue of Burnside's $p^a q^b$ theorem holds. In this talk, we introduce Cohen–Westreich's definitions for solvability and nilpotency and give examples of nilpotent finite quantum groups. We also generalize these definitions to the case of discrete quantum groups and consider examples. A part of this talk is based on the joint work with Gerard Glowacki (Nagoya University) and Masamune Hattori (Nagoya University).

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Inverse Laplace transforms of harmonic and hyper-harmonic sequences in Banach algebras and modules

Bogdan Djordjević

Let M be a right Banach module for the unital complex Banach algebra A, and simultaneously a left Banach module for the unital complex Banach algebra B. In this talk we will discuss the inverse Laplace transforms of the M-valued mappings

$$k \mapsto (a+k)^{-n}c(b+k)^{-m}$$

where k is a positive number, n, m are natural numbers and $a \in A$, $b \in B$ and $c \in M$. These findings are obtained in a joint work with Zora Lj. Golubovic, Faculty of Mathematics University of Belgrade.

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Regularly varying functions

Dragan S. Djordjević

We investigate integrals of algebra-valued functions or algebra-valued measures. Results are applied to algebra-valued orthogonal polynomials and algebra-valued regularly varying functions.

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Paired operators and their kernels

M. Cristina Camara

Paired operators are defined as $S_{A,B} = AP + BQ$ or $\Sigma_{A,B} = PA + QB$, where A, B are operators on a Banach space X and P, Q are complementary projections on X. In this talk we take $X = L^2(T)$, where T denotes the unit circle, P and Q are the orthogonal projections onto the Hardy space H^2 of the unit disk and its orthogonal, respectively, and A, B are multiplication operators on $L^2(T)$. Paired operators are dilations of Toeplitz operators and the kernels of such operators, together with their analytic projections, are generalisations of Toeplitz kernels. It is thus natural to ask whether various properties of Toeplitz operators have analogues for paired operators. In this talk we address this question and present generalisations of some well known properties of Toeplitz operators and their kernels. The results are applied to describing the kernels of finite-rank asymmetric truncated Toeplitz operators. This is based on joint work with Jonathan Partington.

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Invariant and hyperinvariant subspace problem for Hilbert and right Hamilton spaces

Sa Ge Lee

We give a corrected version of earlier works announced in arxiv and the operator theory conference in SNU, 2023.

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