Toward fundamental building blocks of supersymmetric dualities

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University of **S**cience and **T**echnology of **C**hina

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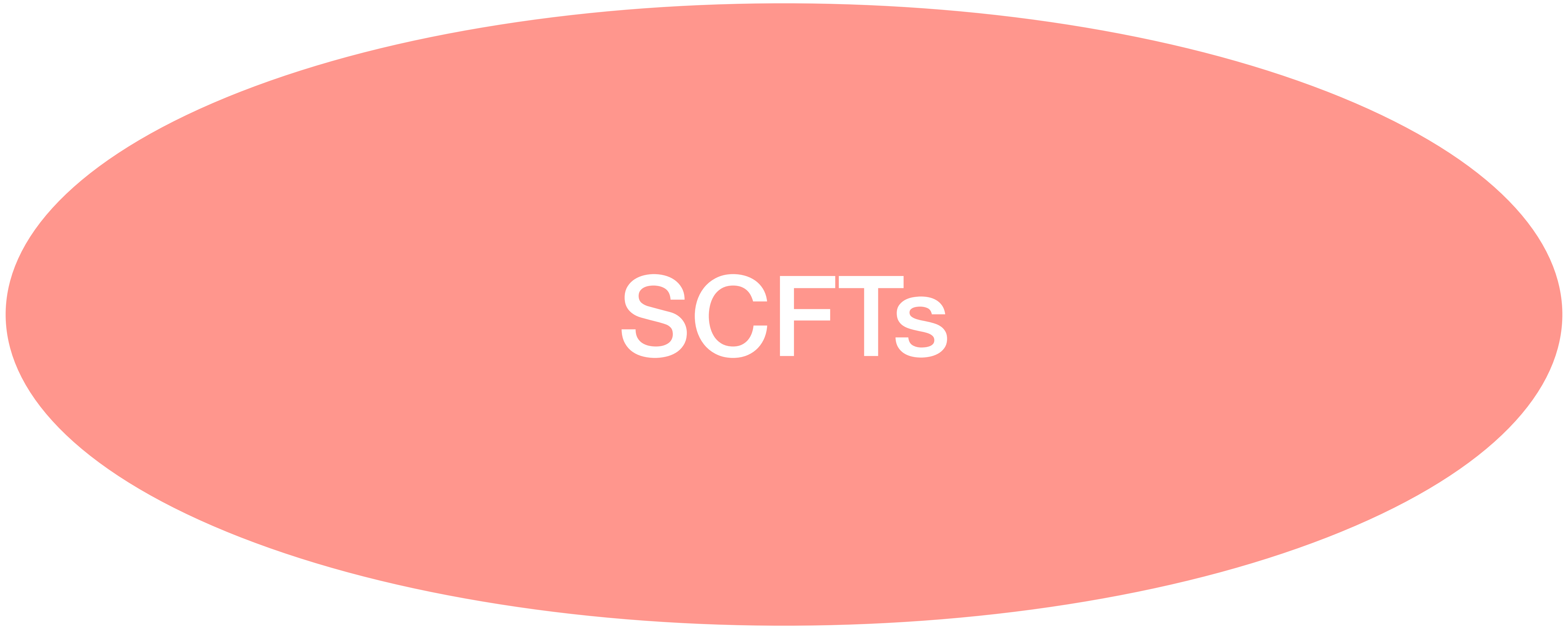
Mainly based on

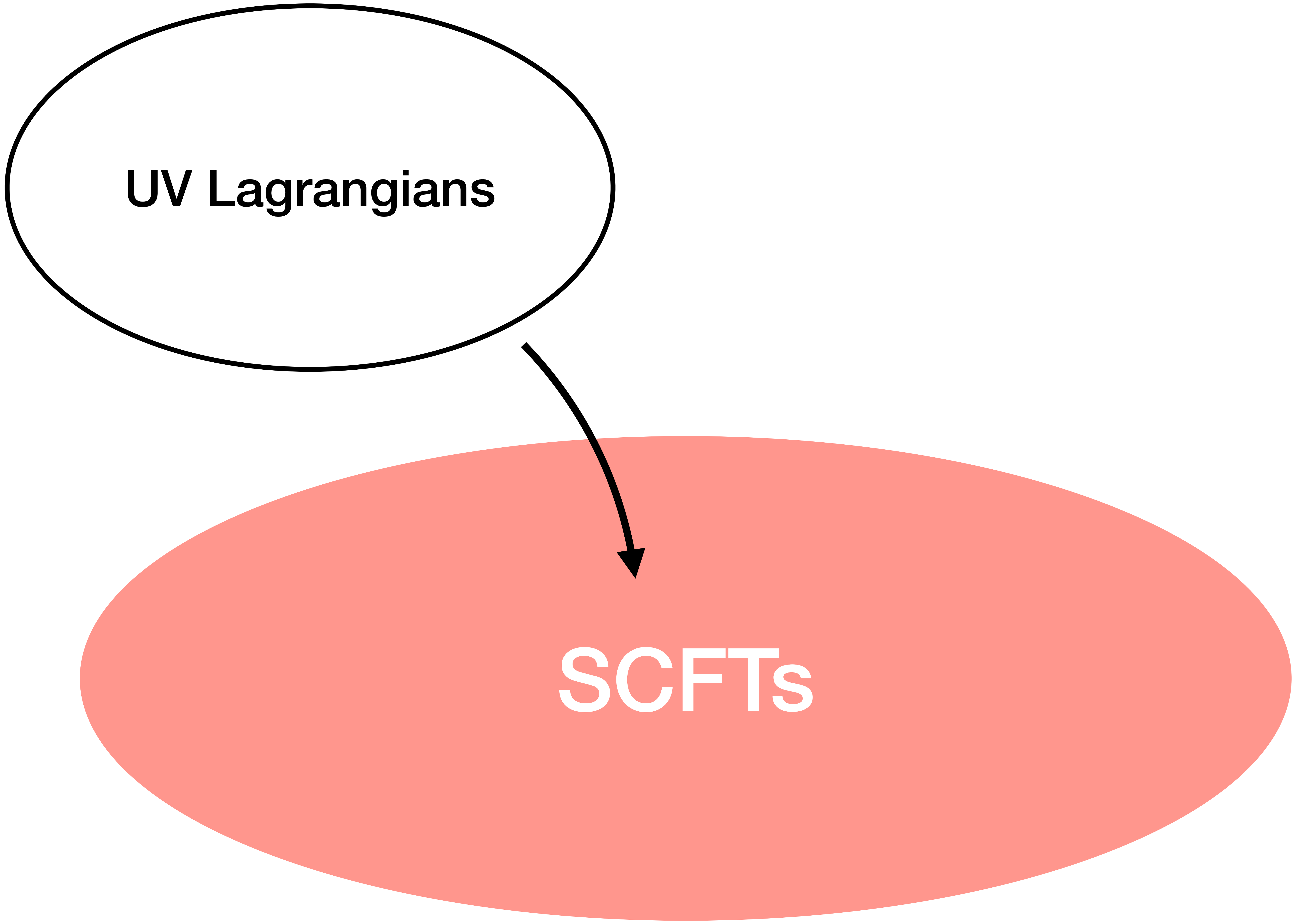
- CH, Sungjoon Kim, “**S-confinement of 3d Argyres-Douglas theories and the Seiberg-like duality with an adjoint,**” to appear,

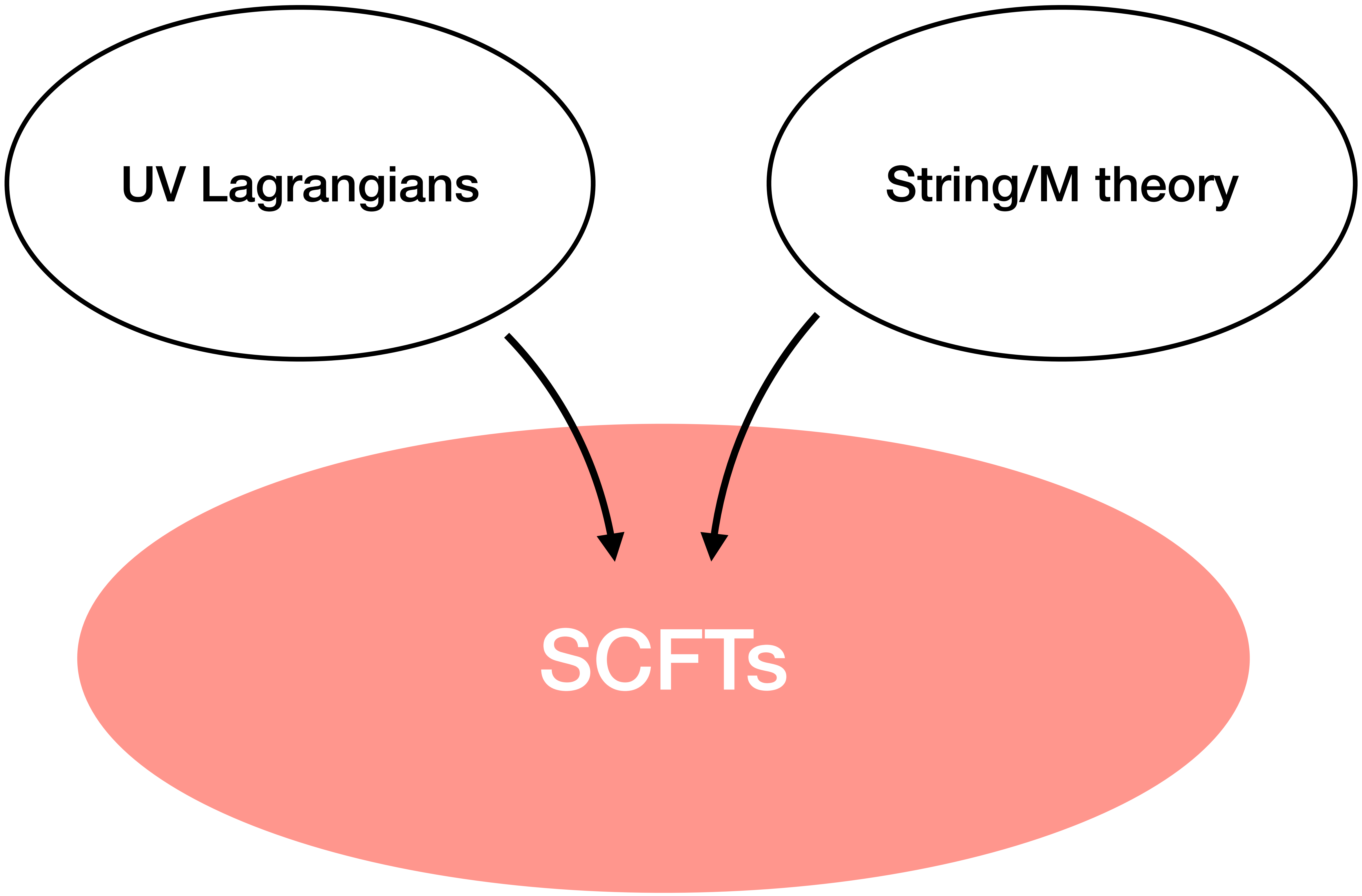
- CH, Sara Pasquetti, Matteo Sacchi, “**Rethinking mirror symmetry as a local duality on fields,**” arXiv:2110.11362,

- *CH, Piljin Yi, Yutaka Yoshida, “**Fundamental Vortices, Wall-Crossing, and Particle-Vortex Duality,**” arXiv:1703.00213.*

- Introduction
- Part I: 3D Reduction of $D_p[SU(N)]$ Argyres-Douglas Theories and S-Confinement
- Part II: Revisit Dualities for Adjoint SQCD
- Conclusion


SCFTs


UV Lagrangians
SCFTs


UV Lagrangians
String/M theory
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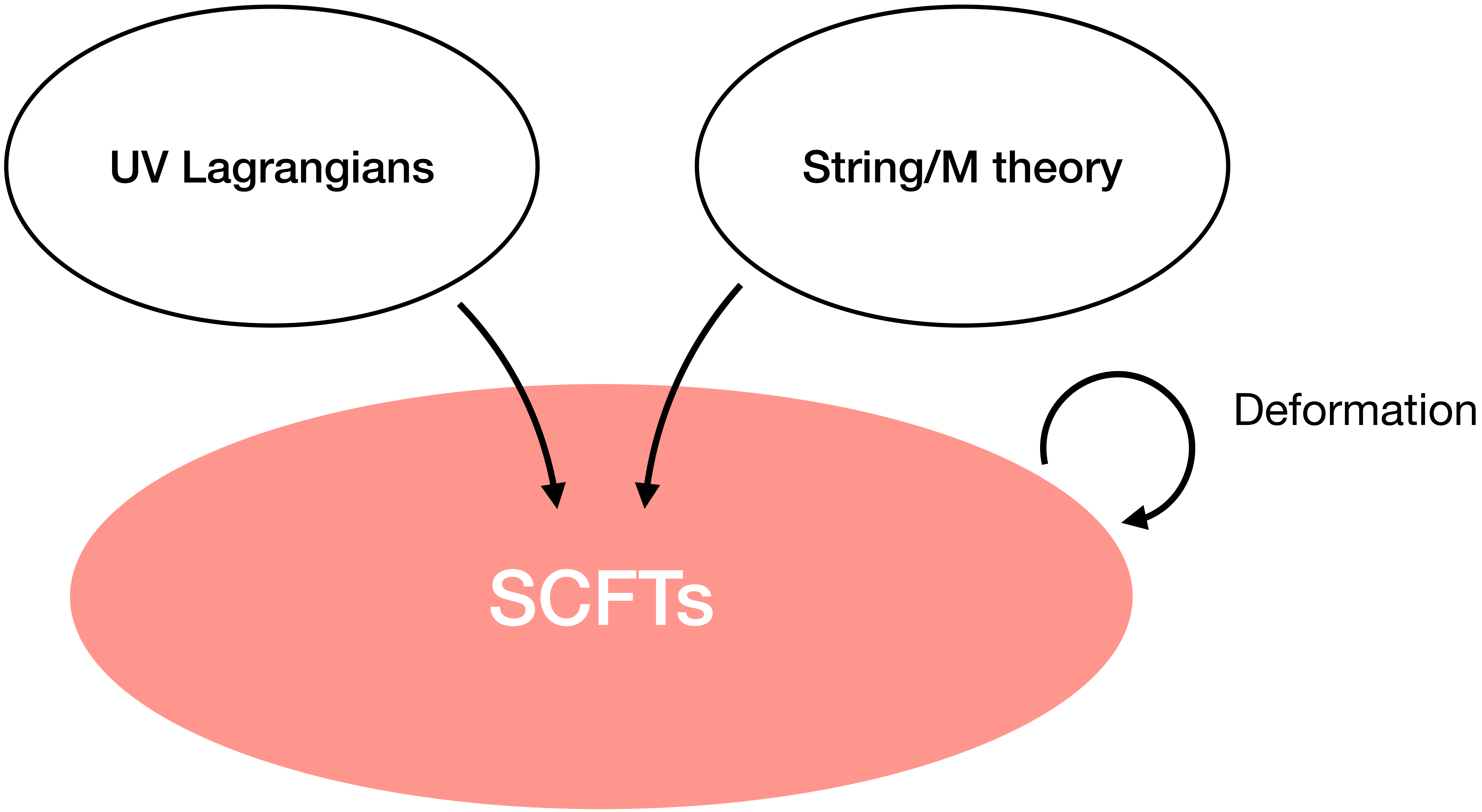
UV Lagrangians
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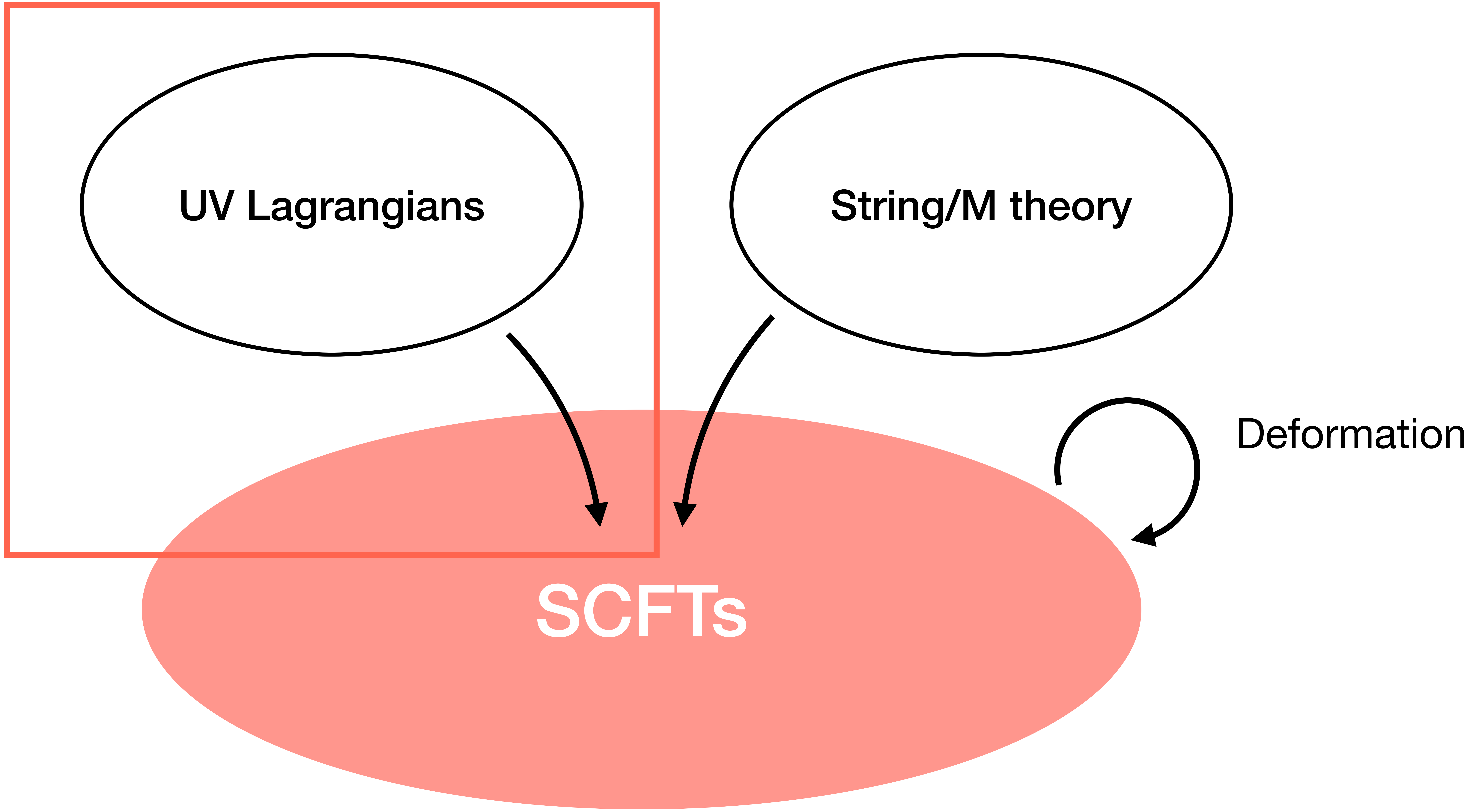
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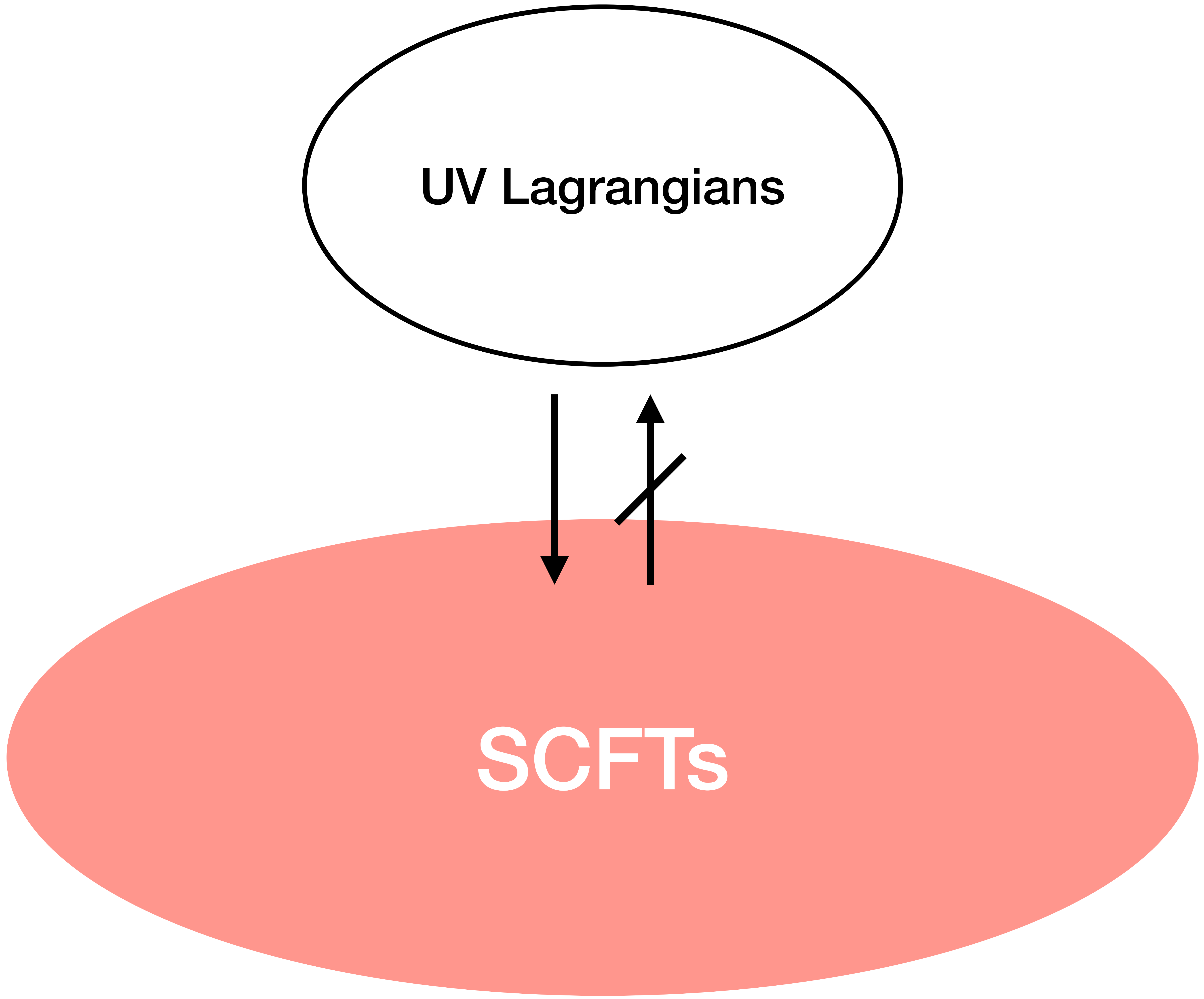
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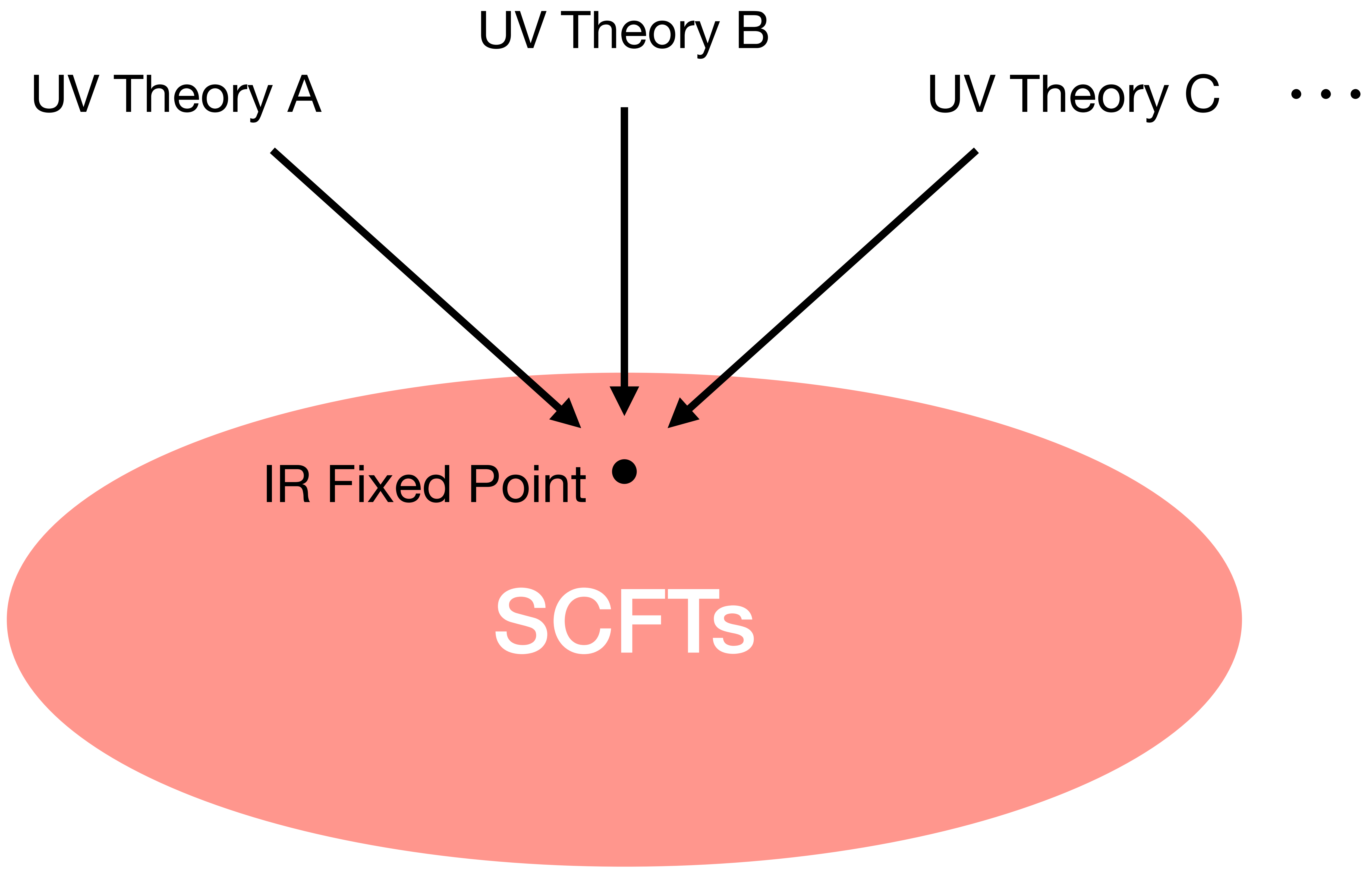
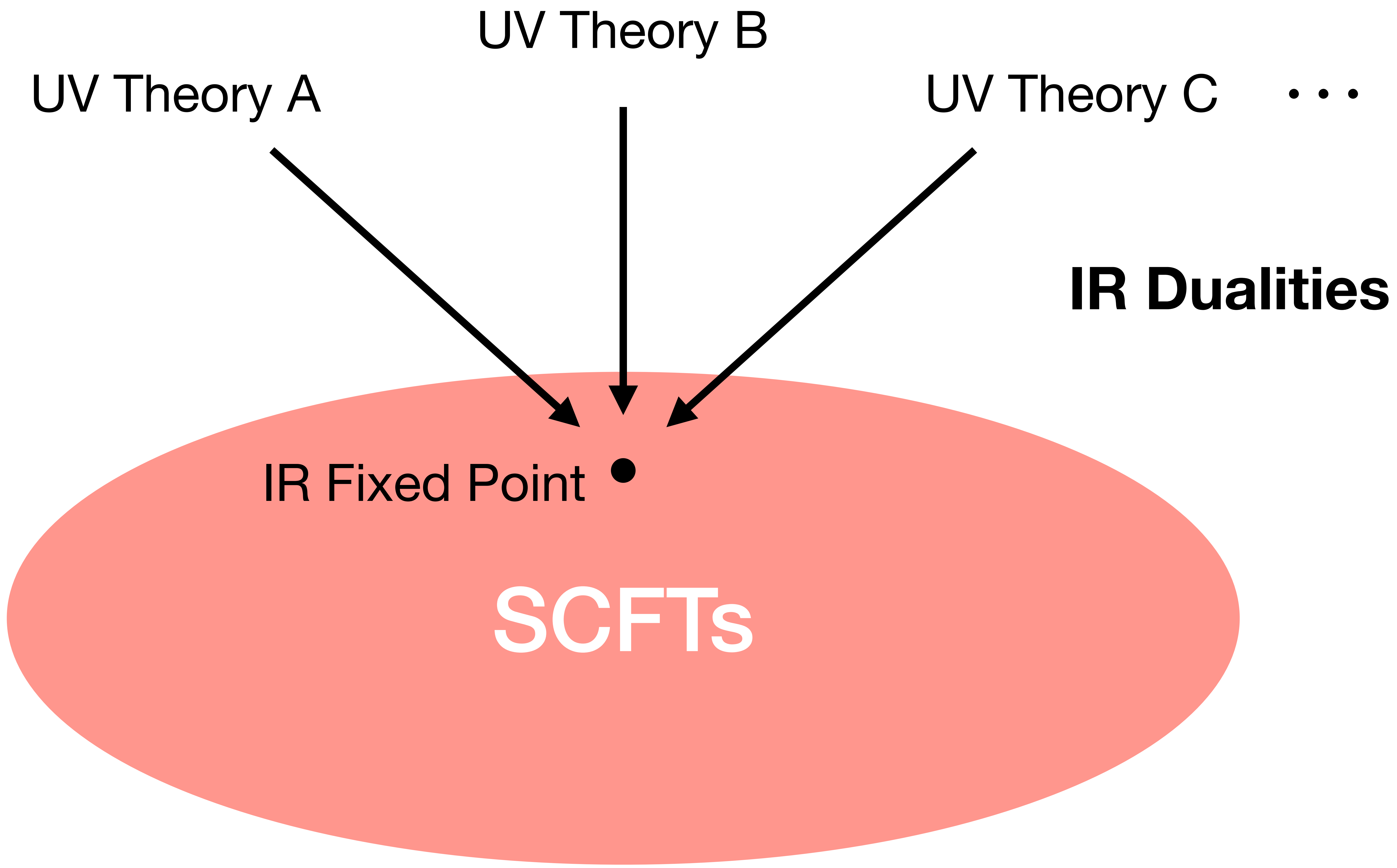
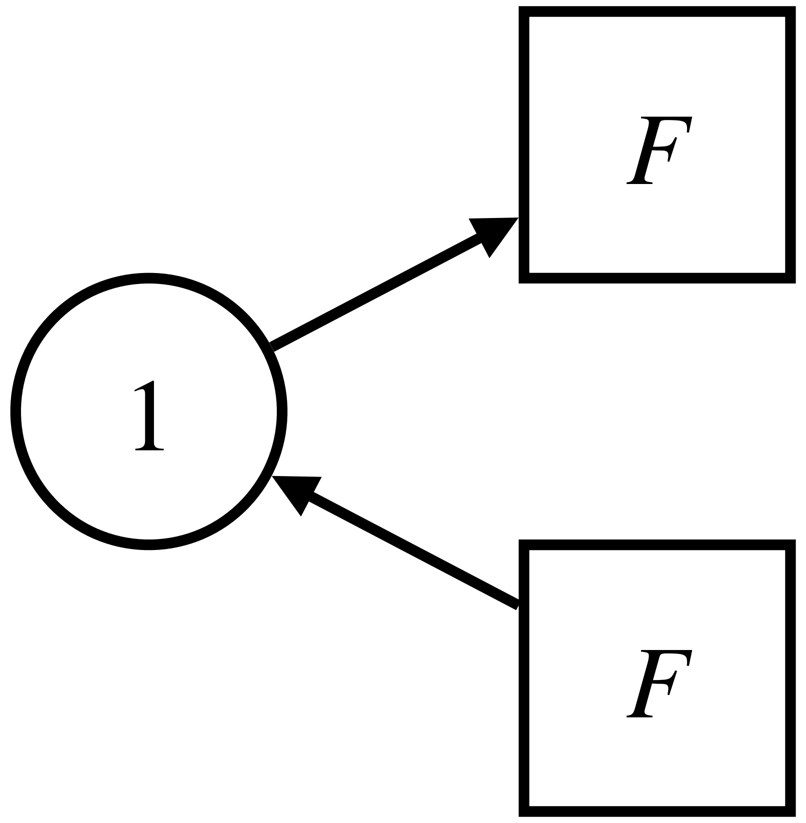
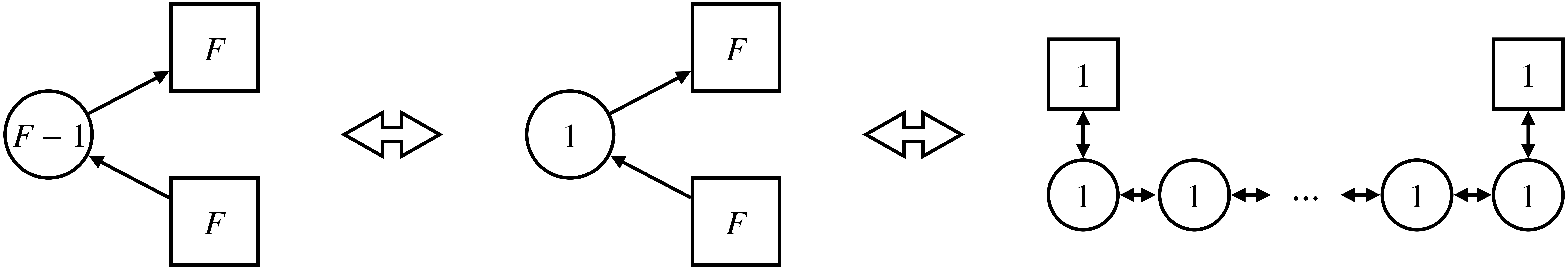

UV Theory B
UV Theory A
UV Theory C
• • •
IR Fixed Point
SCFTs


UV Theory B
UV Theory A
UV Theory C
. . .
IR Dualities
IR Fixed Point
SCFTs

Example

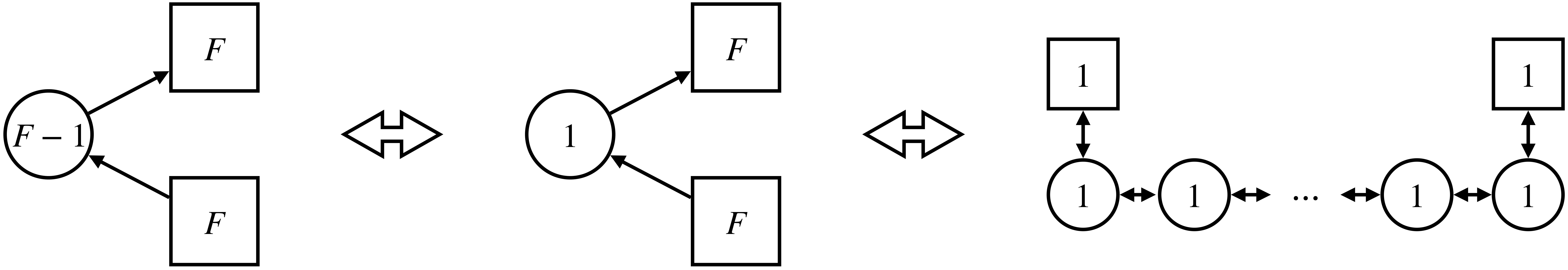


3d $U(1)$ gauge theory with F flavors



Aharony duality
(Seiberg duality in 4d)

3d mirror symmetry



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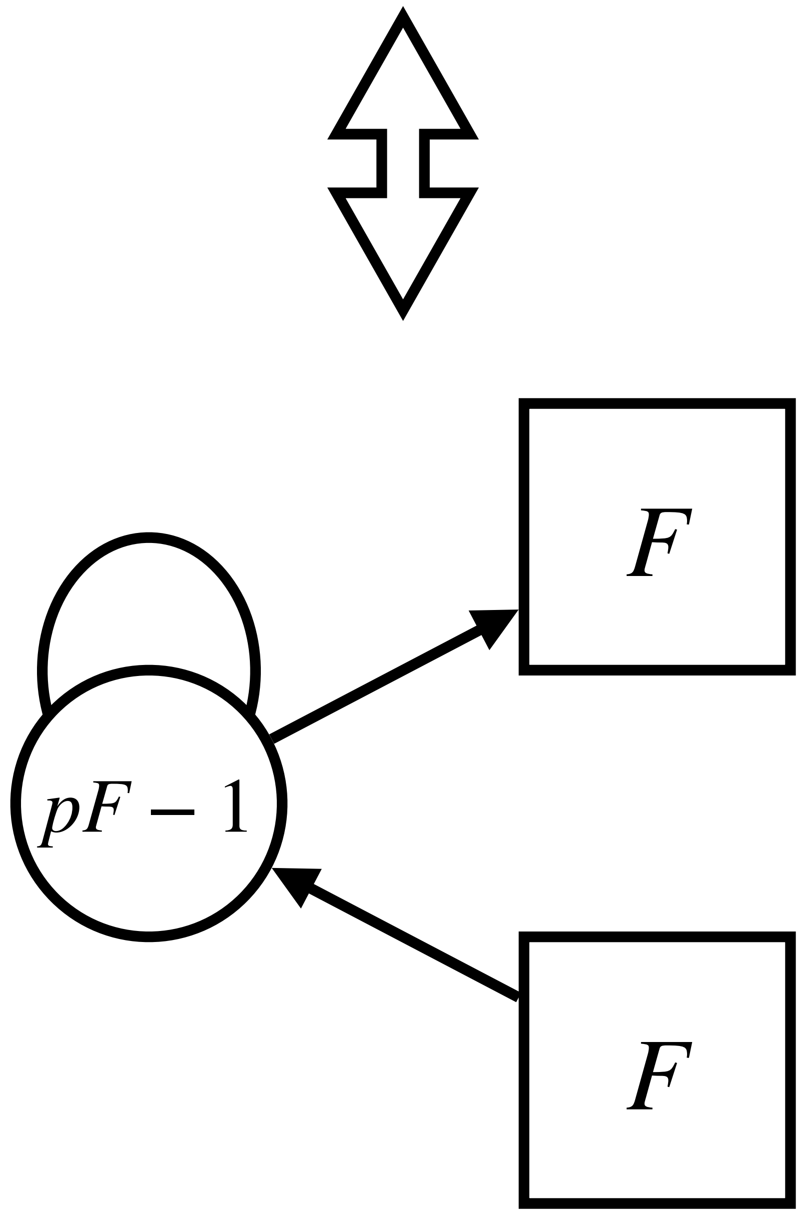

Kim-Park duality
(Kutasov-Schwimmer duality in 4d)

And more…

- (Too) many UV descriptions
- Any organizing principle?
- Can we find the building block allowing the systematic construction of these dualities?

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- Can we find the building block allowing the systematic construction of these dualities?

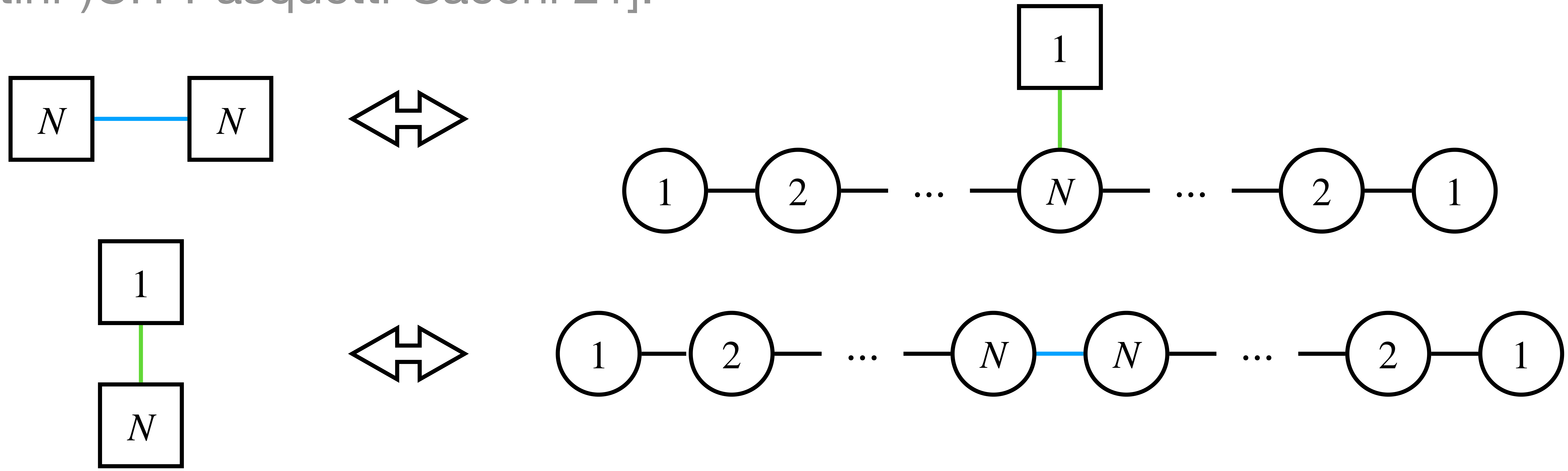
The fundamental mechanism of the dualities is universal.

F
F − 1
F
Aharony duality
(Seiberg duality in 4d)
F
1
F
1
1
1
1
1
…
1
1
3d mirror symmetry
(At least partially,) YES!
F
pF − 1
F
Kim-Park duality
(Kutasov-Schwimmer duality in 4d)
And more…

Building Blocks of 3D Mirror Symmetry

- Typical examples of 3d mirror symmetry are $\mathcal{N}=4$ quiver gauge theories consisting of **bifundamental** and **fundamental** hypers.

- If we know how to dualize bifundamental and fundamental hypers, we can reconstruct the mirror of quiver gauge theories consisting of them [(Bottini-)CH-Pasquetti-Sacchi 21].



- The 3d mirror symmetry of most quiver gauge theories (**good linear** quiver [CH-Pasquetti-Sacchi 21], **bad linear** quiver [Giacomelli-CH-Marino-Pasquetti-Sacchi 22, 23], **circular** quiver [work in progress]) can be constructed by gauging those relations between the bifundamenatal & fundamental hypers.

- ***More surprisingly, the dualization of those hypers to each other can be derived from the Aharony duality!***


F
F − 1
F
Aharony duality
(Seiberg duality in 4d)
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F
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3d mirror symmetry
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pF − 1
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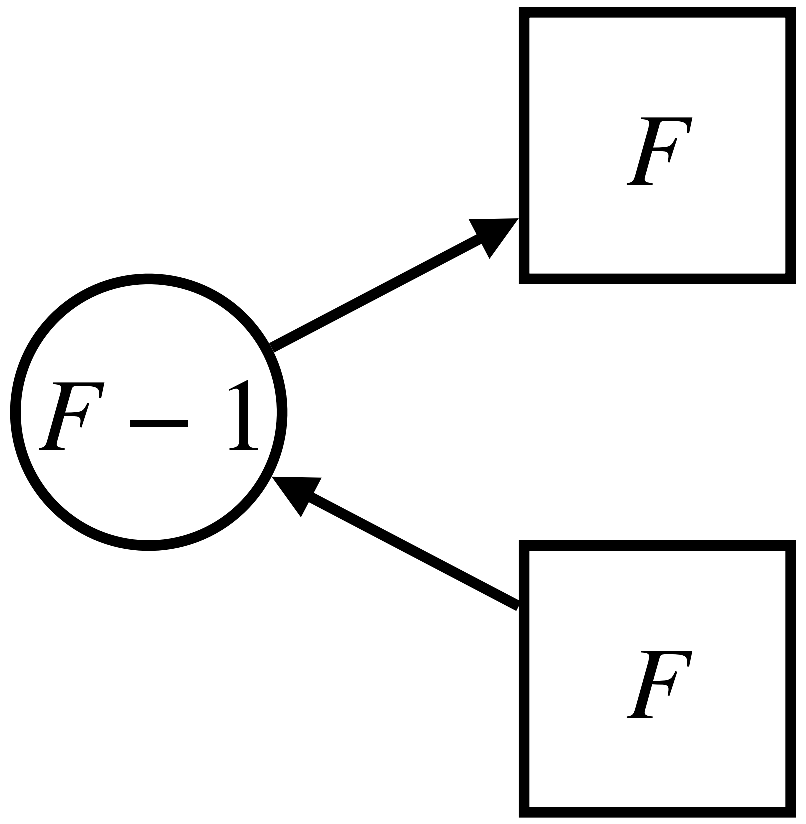

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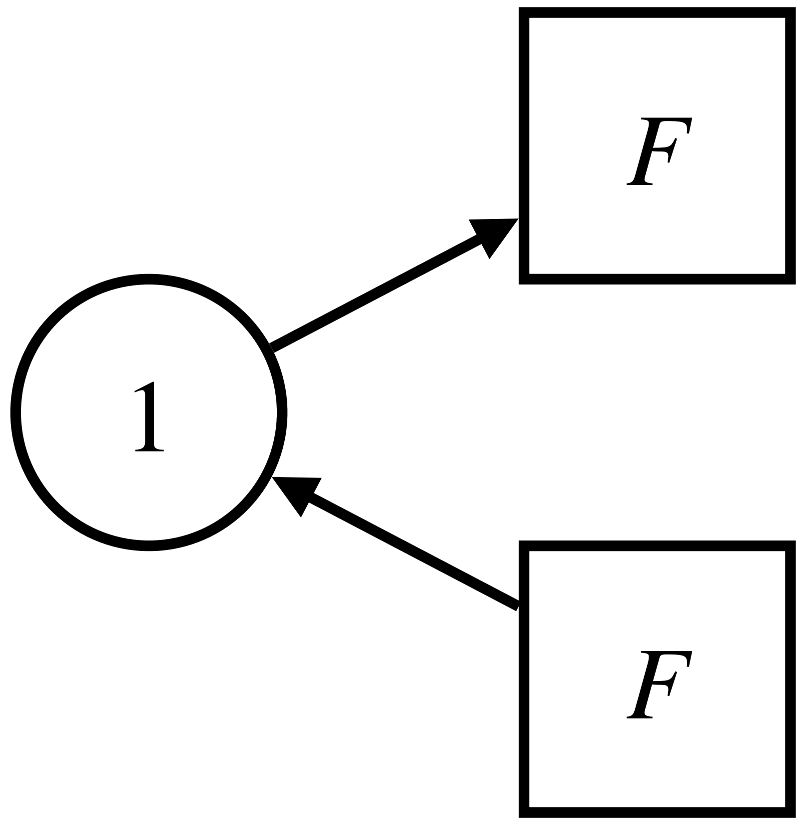
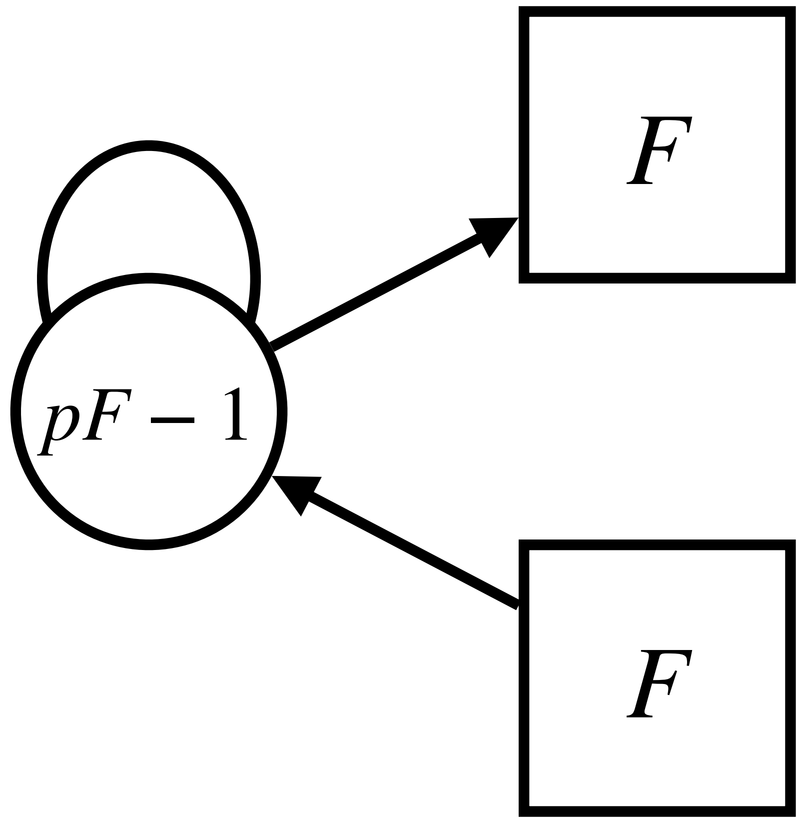
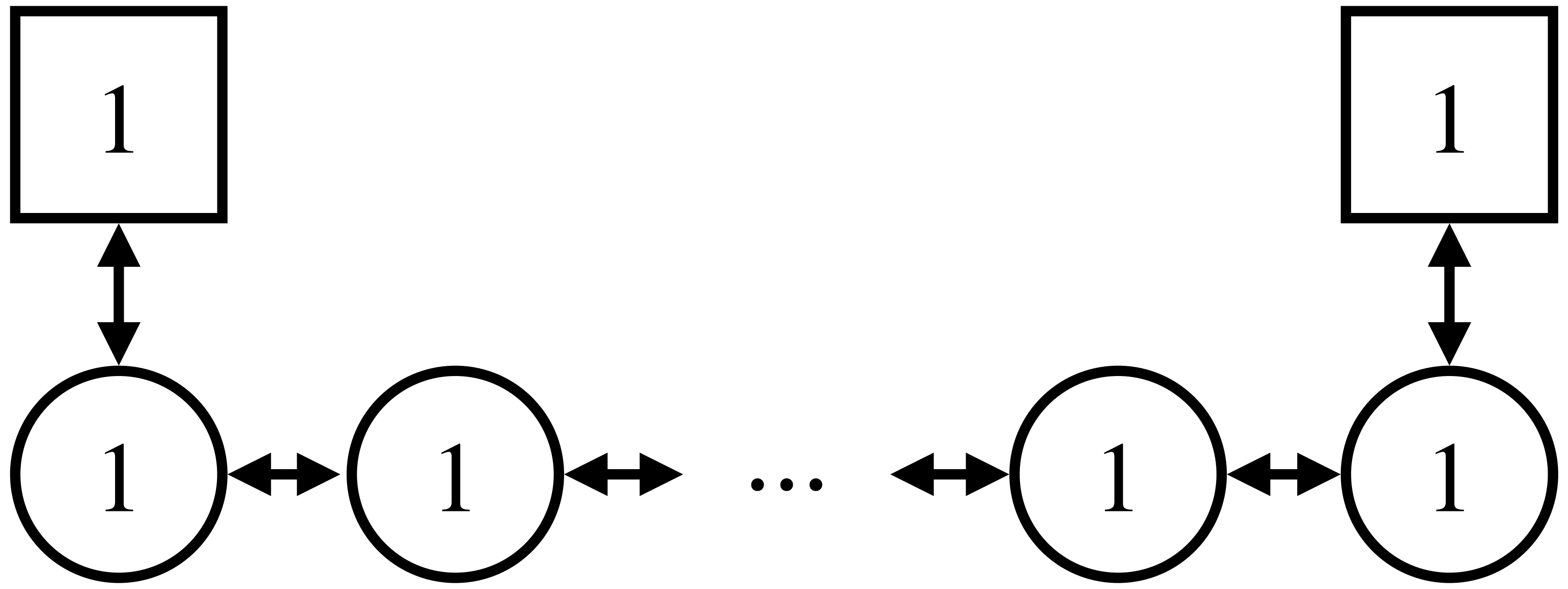




Today's topic



3d mirror symmetry

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And more…

Part I: 3D Reduction of $D_p[SU(N)]$ Argyres-Douglas Theories and S-Confinement

4D $D_p^b(G)$ $\mathcal{N}=2$ Superconformal Field Theories

- An infinite family of 4d $\mathcal{N}=2$ SCFTs with an ADE global symmetry can be engineered by compactifying type IIB string theory on threefold hypersurface singularities in $\mathbb{C}^3 \times \mathbb{C}^*$ [Giacomelli 17]:

$$F(x_1, x_2, x_3, z; G, b, p) = 0$$

⬇

$$D_p^b(G)$$

$$\begin{aligned} G = SU(N)\,, \quad b = N \ &: \qquad F = x_1^2 + x_2^2 + x_3^N + z^p\,, \\ b = N-1 \ &: \qquad F = x_1^2 + x_2^2 + x_3^N + x_3 z^p \end{aligned}$$

4D $D_p^b(G)$ $\mathcal{N}=2$ Superconformal Field Theories

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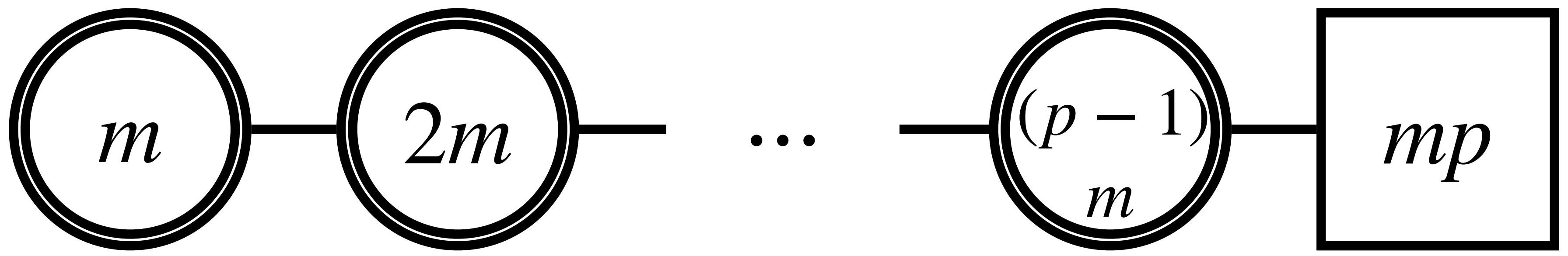
$$G = SU(N)\,, \quad \begin{array}{ll} b = N : & F = x_1^2 + x_2^2 + x_3^N + z^p\,, \\ b = N-1 : & F = x_1^2 + x_2^2 + x_3^N + x_3 z^p \end{array} \quad \equiv D_p[SU(N)]$$

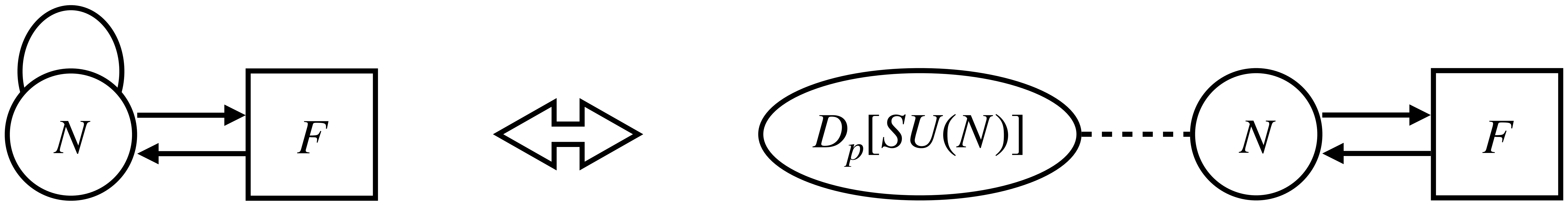
- The $D_p[SU(N)]$ theories allow Lagrangian dual descriptions when $N = mp$ [Cecotti-Del Zotto-Giacomelli 13].



- On the other hand, the $D_p[SU(N)]$ theories are non-Lagrangian when $\gcd(p, N) = 1$.

The Maruyoshi-Nardoni-Song Duality

- Recently, an interesting 4d $\mathcal{N}=1$ duality involving $D_p[SU(N)]$ has been proposed for $p<N$ satisfying $\gcd(p,N)=1$ [Maruyoshi-Nardoni-Song 23]:



$W = \mathrm{tr}\, X^{p+1}$ $\qquad\qquad$ $W = u_0$

- Deconfinement of the adjoint into a $D_p[SU(N)]$ tail

- Passing many nontrivial tests

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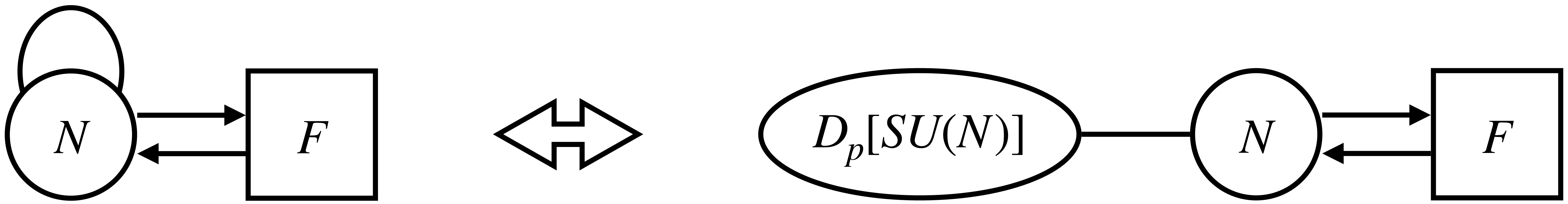

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- Deconfinement of the adjoint into a $D_p[SU(N)]$ tail

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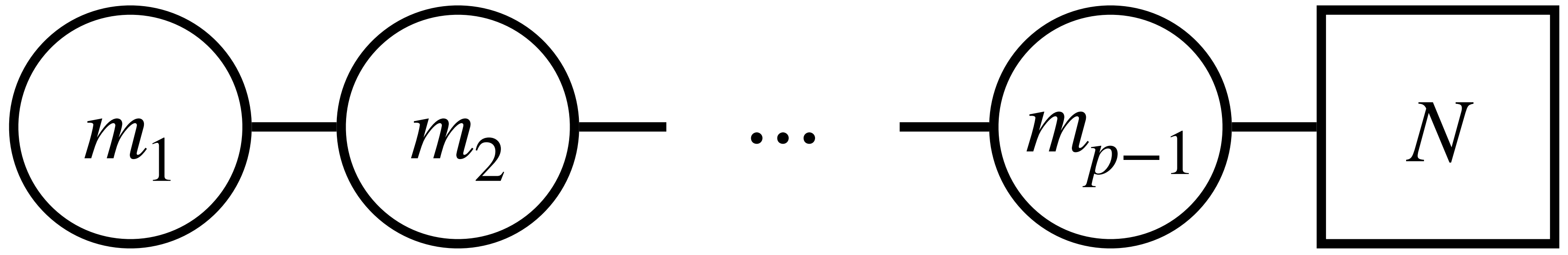
3d version?

3D Reduction of $D_p[SU(N)]$ Theories

- Interestingly, the 3d reduction of 4d $D_p^b[SU(N)]$ theories always has UV *Lagrangian* descriptions [Giacomeeli-Mekareeya-Sacchi 21]; e.g., if $b = N$ and $\gcd(p, N) = 1$,



$m_j = \lfloor jN/p \rfloor\,, \qquad j = 1,\ldots,p-1$

$$W = \sum_{i=1}^{p-1} \mathrm{Tr}_i\, \Phi^{(i)} Q_i \tilde{Q}_i + \sum_{i=1}^{p-2} \mathrm{Tr}_{i+1}\, \Phi^{(i+1)} \tilde{Q}_i Q_i$$

- **Confining deformation?**

S-Confinement of 3D $\mathbb{D}_p[SU(N)]$

- Let's assume some simplifying conditions.
- 3d $\mathbb{D}_p[SU(N)]$ theories are either good or ugly in Gaiotto-Witten's sense.
- Focus on the good case, where each node satisfies

$$m_{j-1} + m_{j+1} - 2m_j \geq 0$$

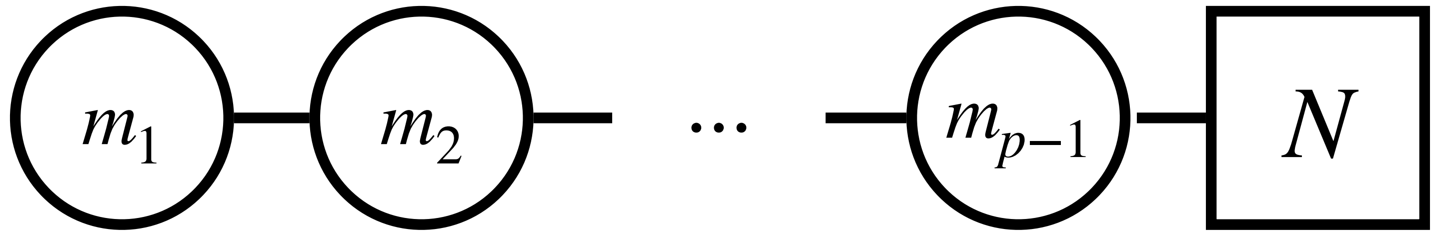
$$\Updownarrow$$

$$N = \pm 1 \mod p$$

- Also assume $p < N$, simplifying the formulas.

- **Proposal:** The 3d $\mathbb{D}_p[SU(N)]$ theory with deformation ΔW is confined.

$D_p[SU(N)]$:



$$m_j = \lfloor jN/p \rfloor\,, \qquad j = 1,\ldots,p-1$$

$\mathbb{D}_p[SU(N)]$ with

$$\Delta W = \eta \sum_{i=1}^{p-1} \mathrm{Tr}\,\Phi^{(i)} + \sum_{i=1}^{p-1} \hat{v}^{(i),+} + \hat{v}^{(1,p-1),-}$$

$$\hat{v}^{(i),\pm} = \left(0^{i-1}\,,\, \pm 1\,, 0^{p-i-1}\right)$$

$$\hat{v}^{(1,p-1),\pm} = (\pm 1\,,\ \ldots\ ,\ \pm 1)$$

$$\Longleftrightarrow$$

A matrix-valued chiral field X with

$$W = \mathrm{Tr}\,X^{p+1}$$

Evidence I

- Superconformal index

$$I \;=\; \mathrm{tr}\,(-1)^F x^{R+2j}$$

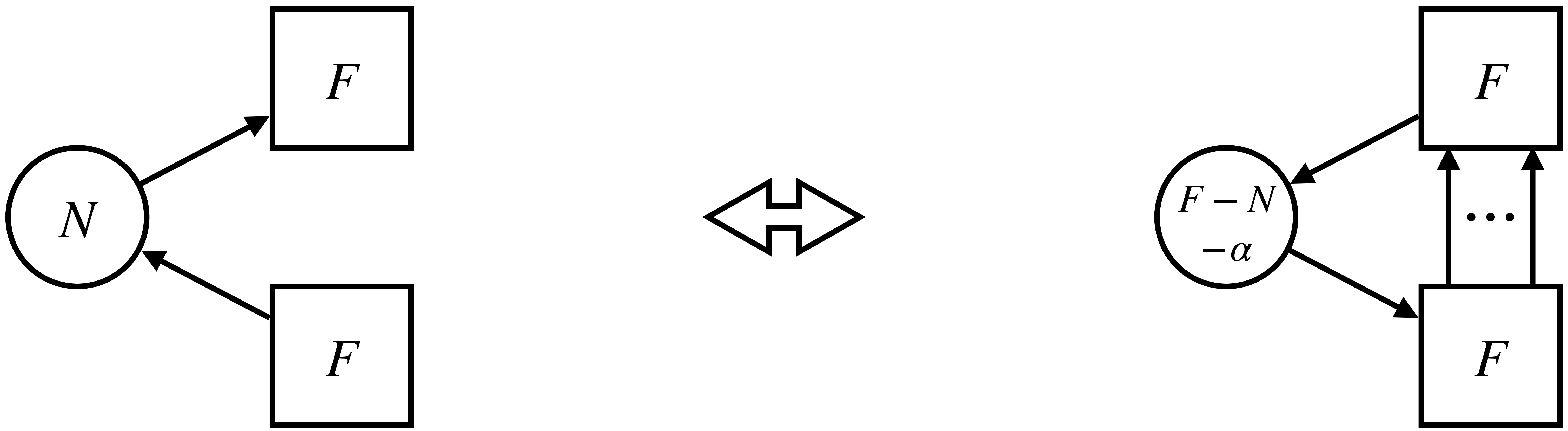
$$\Downarrow$$

$$I_{\mathbb{D}_p[SU(N)]+\Delta W} \quad = \quad PE\left[\frac{N^2\left(x^{\frac{2}{p+1}} - x^{\frac{2p}{p+1}}\right)}{1-x^2}\right] \quad = \quad I_{WZ}$$

- Precisely matching the spectrum of BPS states! (Tested for some N & p)

Evidence II

- More powerfully, one can prove the confinement only assuming the **Aharony-BBP** dualities [Aharony 97, Benini-Benvenuti-Pasquetti 17]:



$$W_A = \begin{cases} \hat{V}^+ + \hat{V}^- \\ \hat{V}^+ \\ 0 \end{cases} \qquad W_B = \begin{cases} \hat{v}^+ + \hat{v}^- + M\tilde{q}q \\ \hat{v}^+ + V^-\hat{v}^- + M\tilde{q}q \\ V^+\hat{v}^+ + V^-\hat{v}^- + M\tilde{q}q \end{cases} \quad \alpha = \begin{cases} 2 \\ 1 \\ 0 \end{cases}$$

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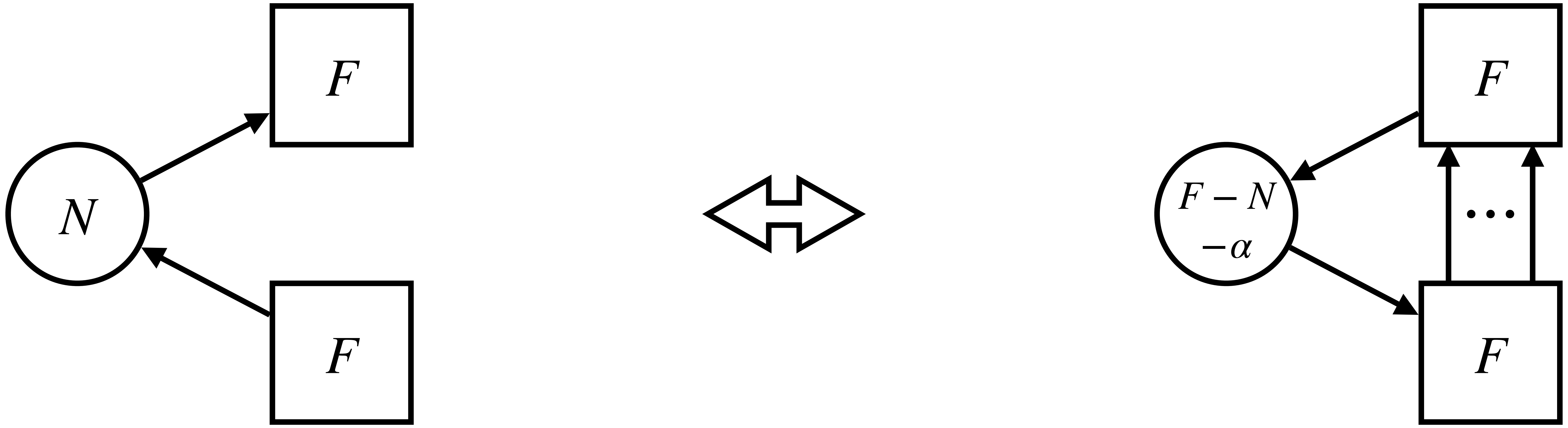

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Aharony (bottom row: $W_A = 0$, $W_B = V^+\hat{v}^+ + V^-\hat{v}^- + M\tilde{q}q$, $\alpha = 0$)

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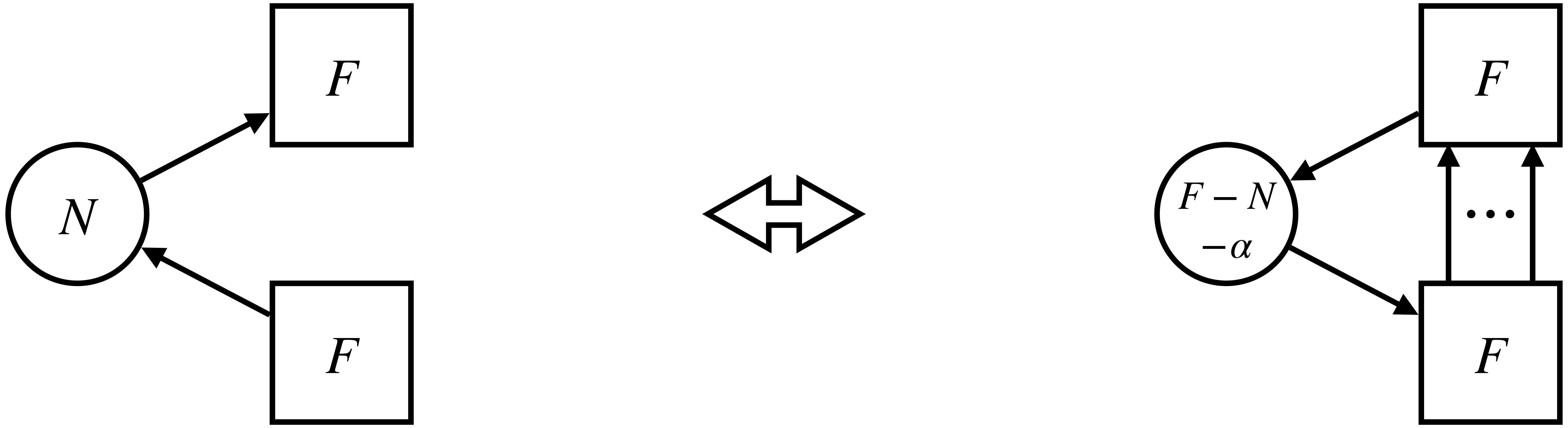

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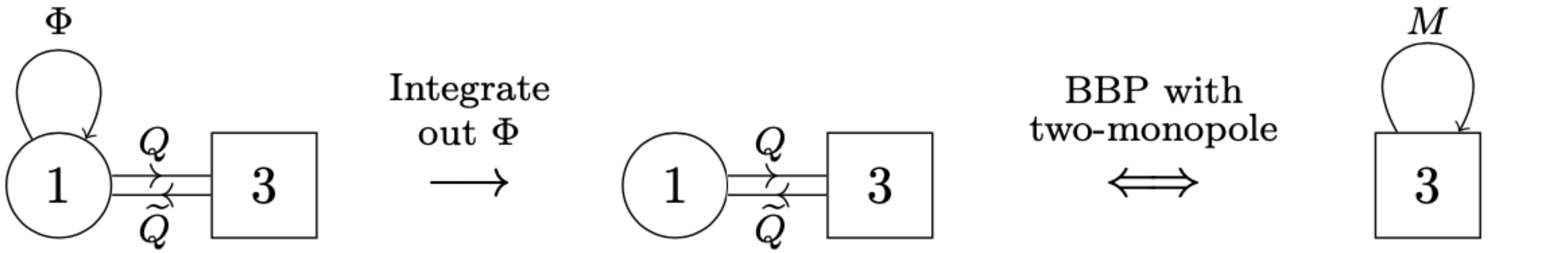
Mass def.

Derivation Using the BBP Dualities

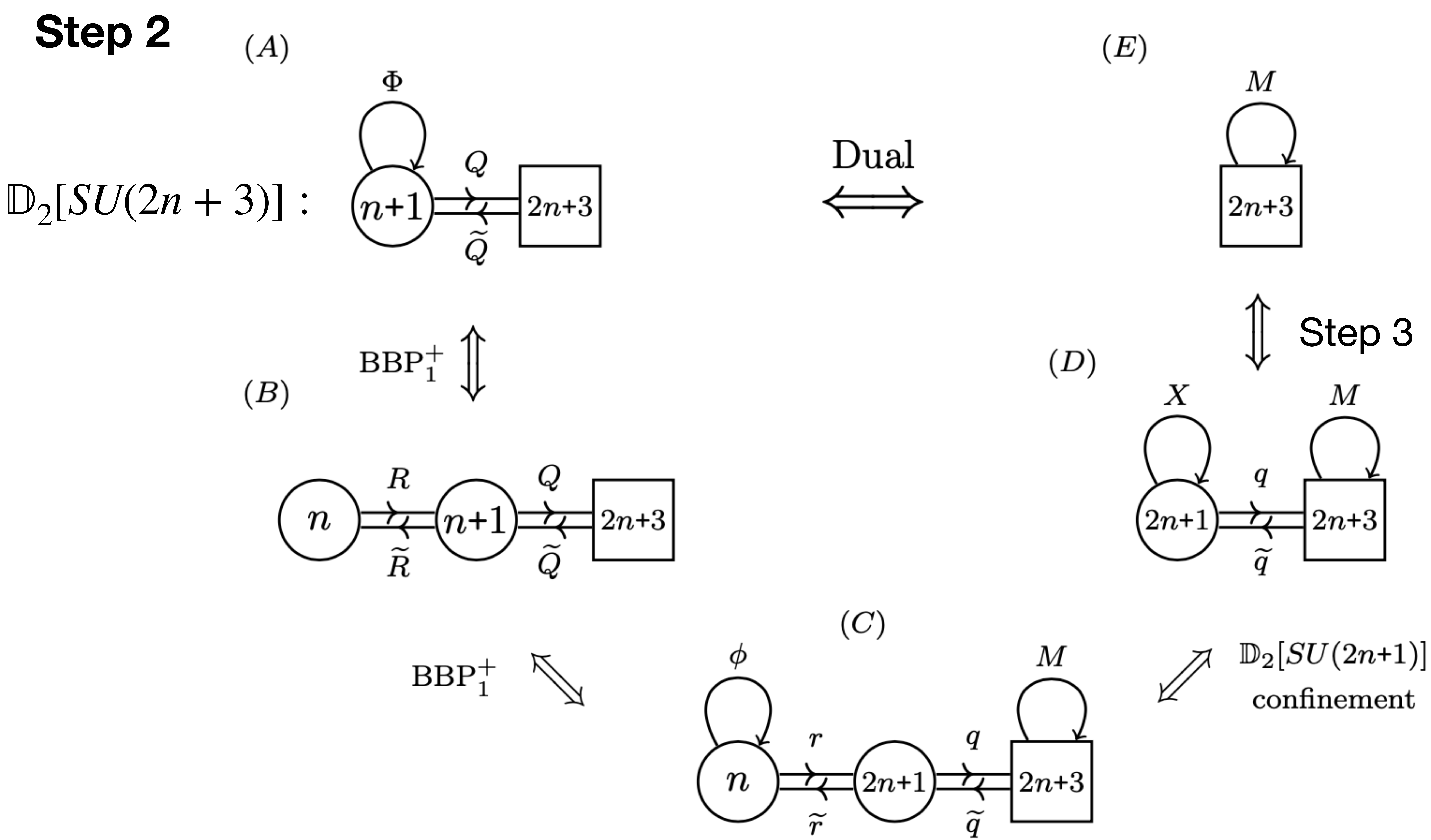
- Let's consider the $p = 2$ case. (Assume the gauge rank N is odd.)

- **Step 1**

$\mathbb{D}_2[SU(3)]$:



$W = \Phi\,\mathrm{Tr}(Q\widetilde{Q}) + \eta\,\Phi + \hat{V}^{+} + \hat{V}^{-}$ $\quad$ $W = \hat{V}^{+} + \hat{V}^{-}$ $\quad$ $W = \mathrm{Tr}\,M^3$


Step 2
(A)
$\mathbb{D}_2[SU(2n+3)]$:
Φ
Q
n+1
2n+3
$\widetilde{Q}$
Dual
(E)
M
2n+3
Step 3
BBP_1^+
(B)
n
R
n+1
Q
2n+3
$\widetilde{R}$
$\widetilde{Q}$
(D)
X
M
q
2n+1
2n+3
$\widetilde{q}$
(C)
BBP_1^+
ϕ
r
n
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q
M
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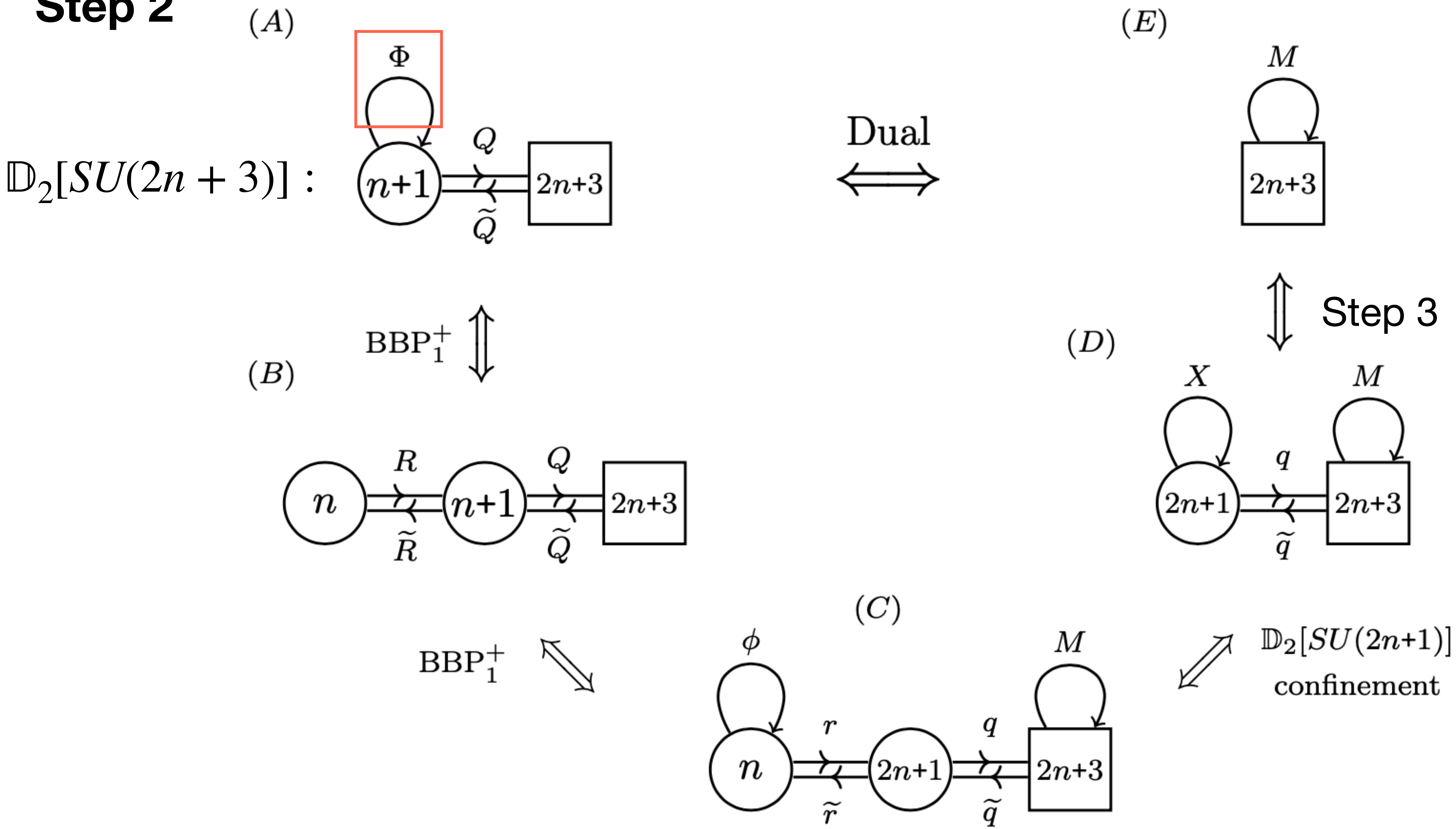
$\widetilde{r}$
$\widetilde{q}$
$\mathbb{D}_2[SU(2n+1)]$
confinement

Step 2



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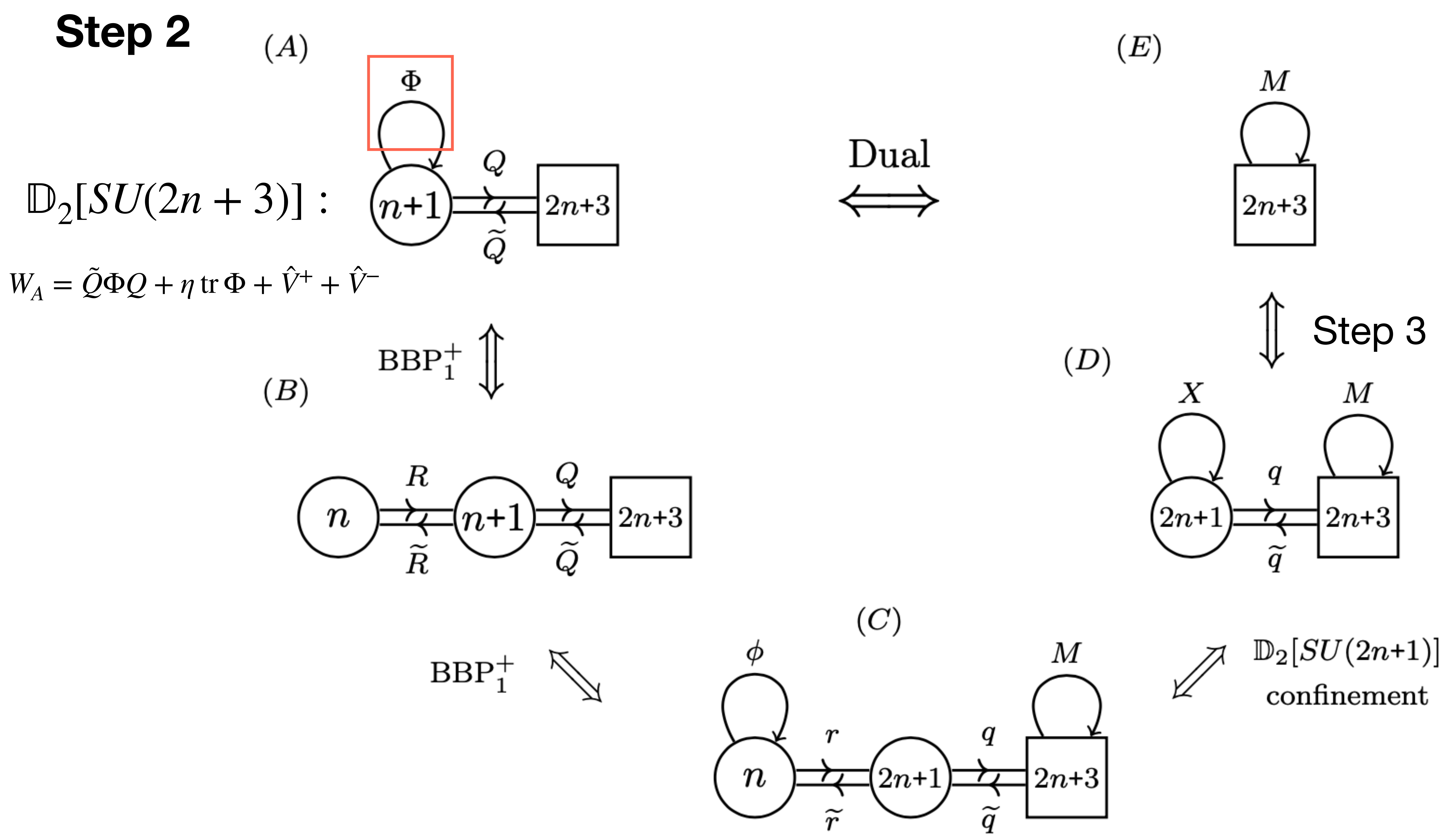

Step 2
(A)
Φ
$\mathbb{D}_2[SU(2n+3)]$:
n+1
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$W_A = \tilde{Q}\Phi Q + \eta \, \mathrm{tr}\, \Phi + \hat{V}^+ + \hat{V}^-$
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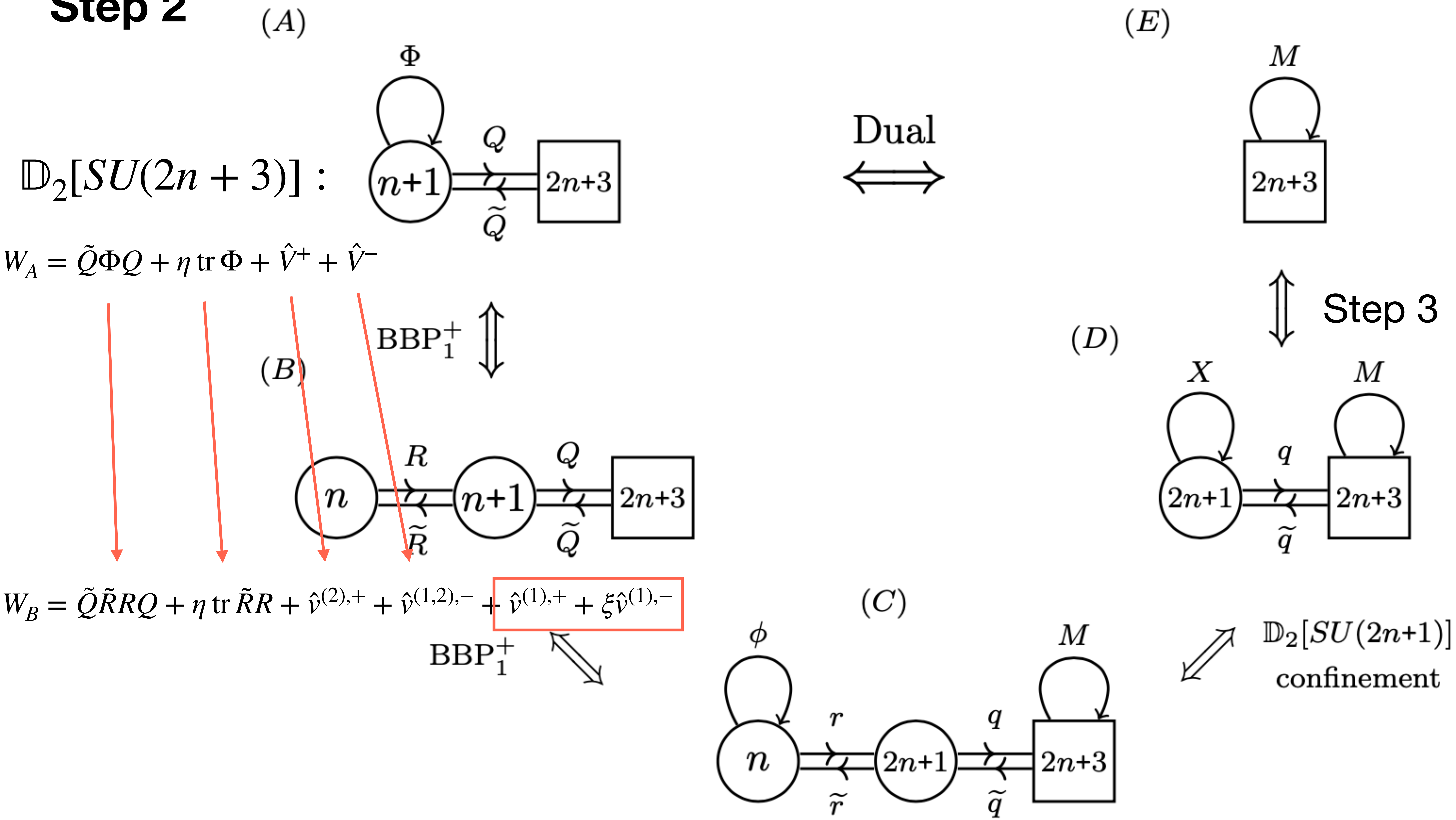
confinement

Step 2



(A)
Φ
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$\mathbb{D}_2[SU(2n+3)]$:
n+1
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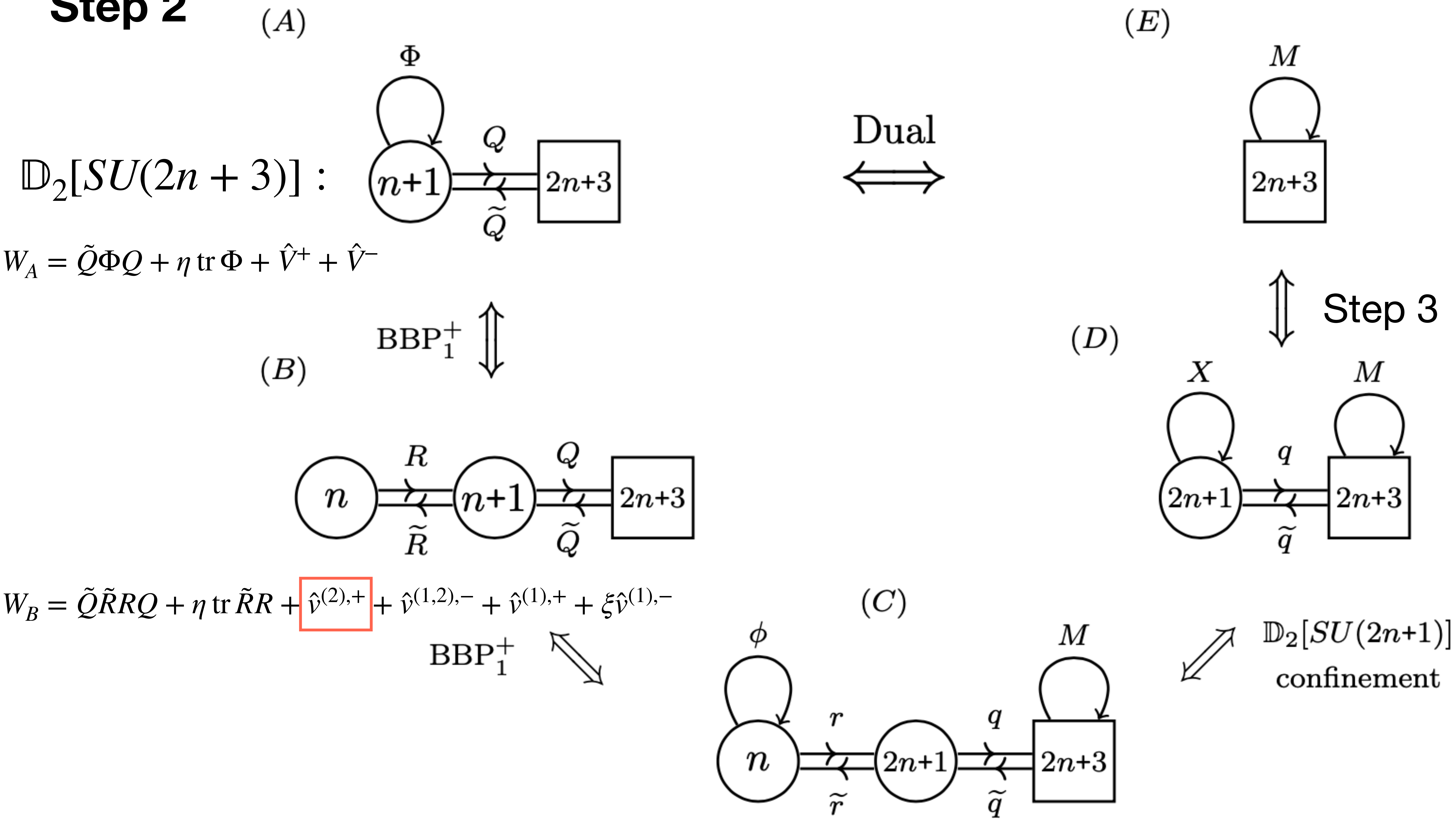

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BBP$_1^+$
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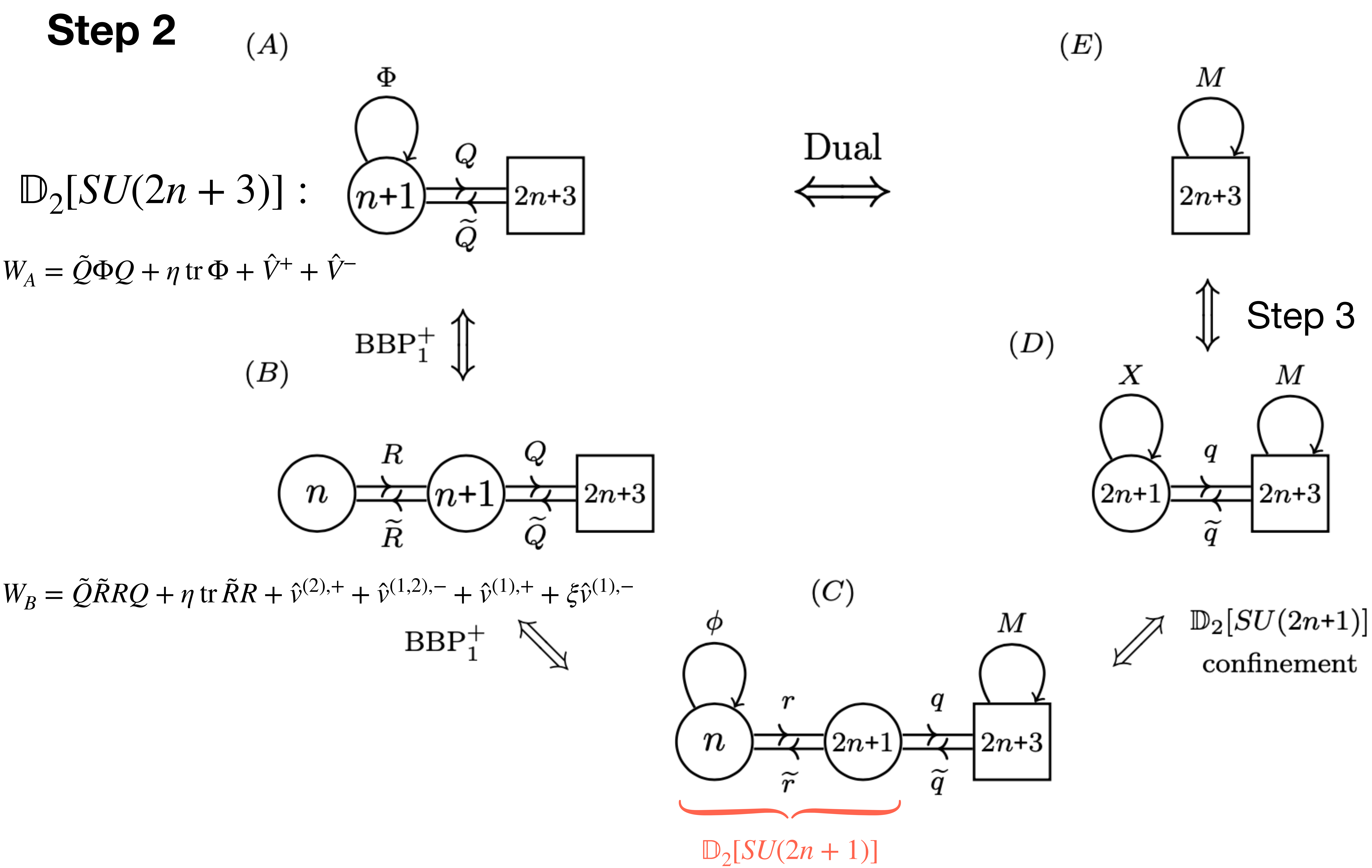
confinement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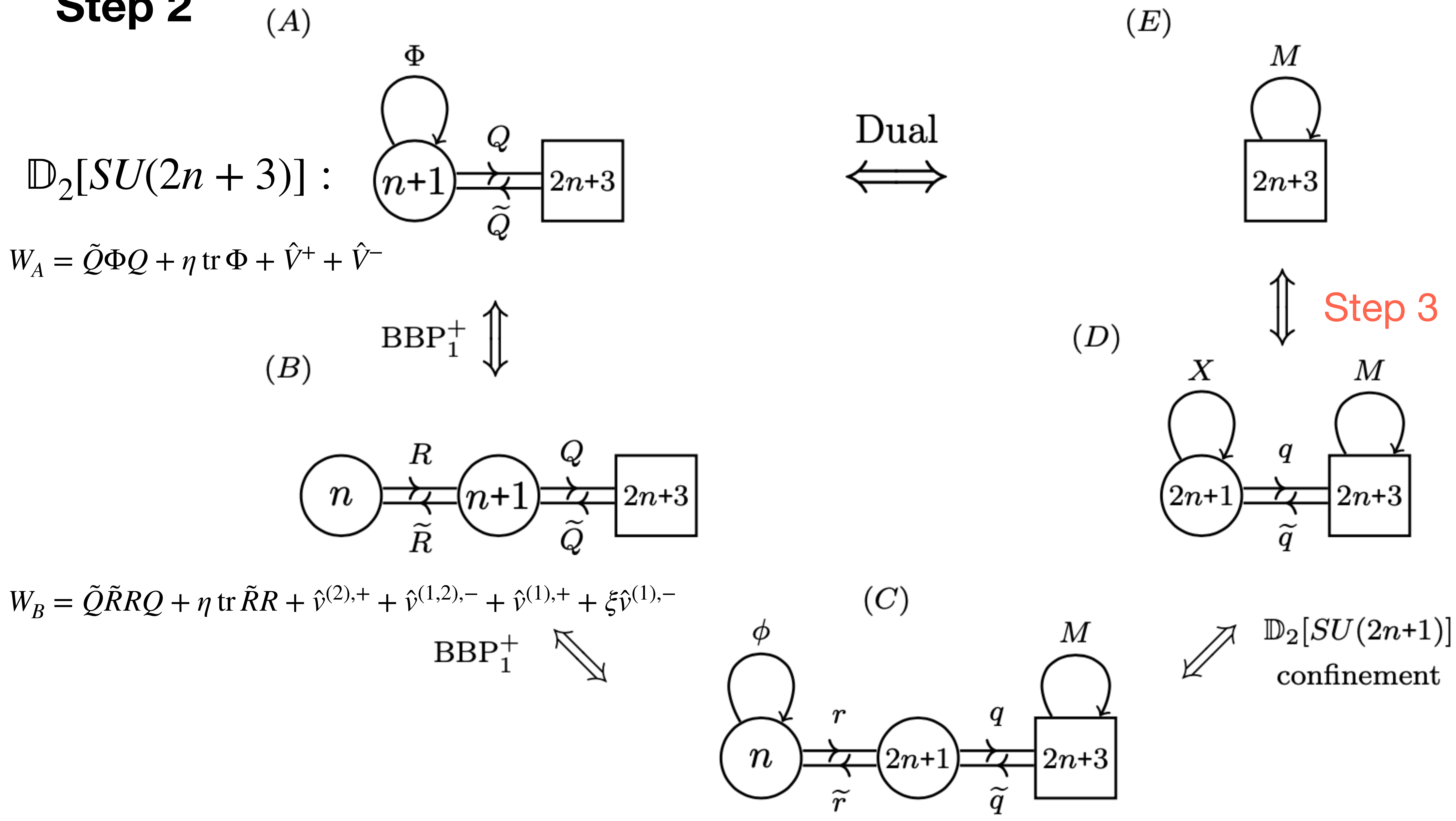
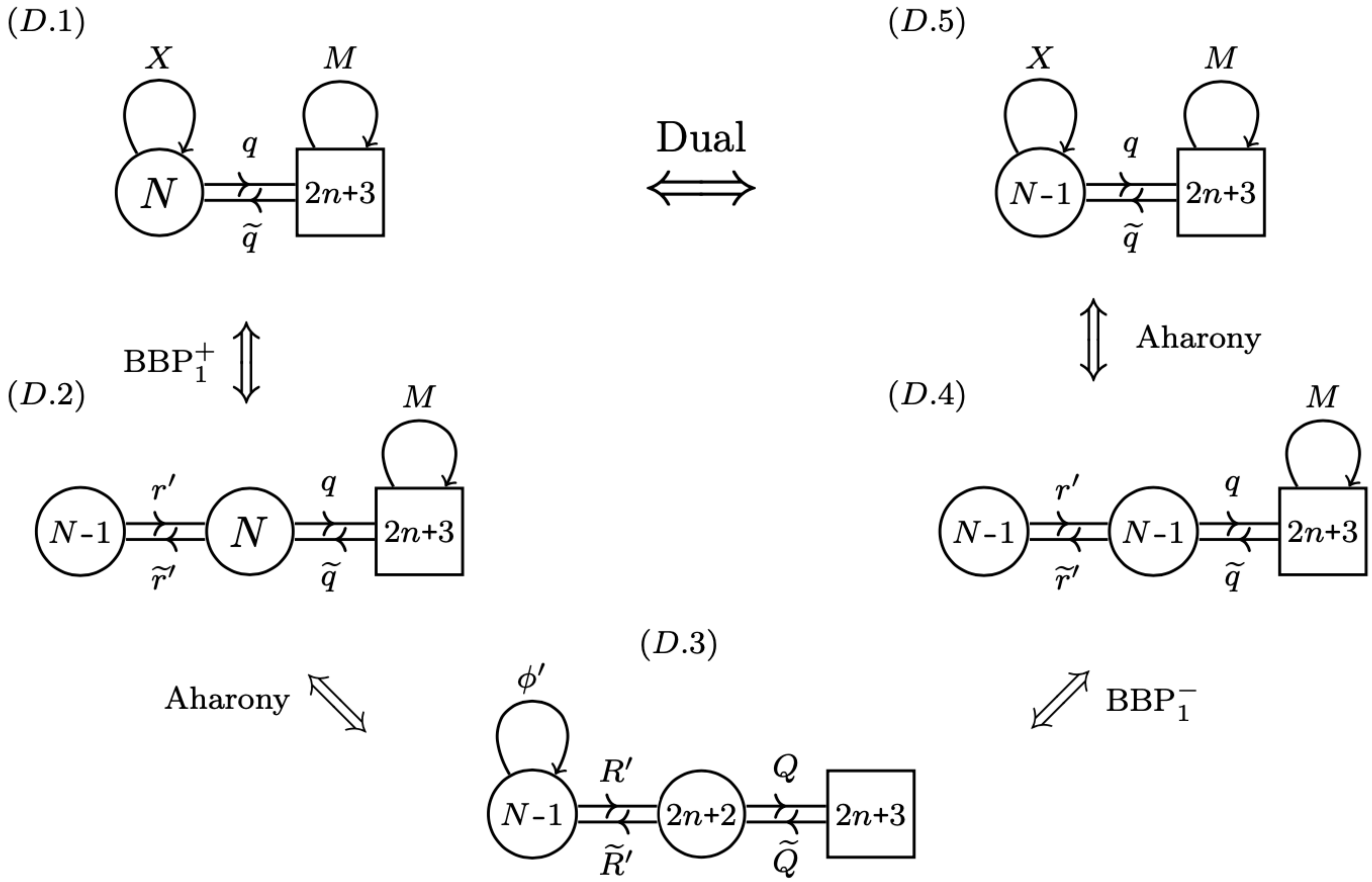
R
$\widetilde{R}$
n+1
Q
$\widetilde{Q}$
2n+3
$W_B = \tilde{Q}\tilde{R}RQ + \eta \operatorname{tr}\tilde{R}R + \hat{v}^{(2),+} + \hat{v}^{(1,2),-} + \hat{v}^{(1),+} + \xi\hat{v}^{(1),-}$
(D)
X
M
2n+1
q
$\widetilde{q}$
2n+3
BBP_1^{+}
(C)
ϕ
n
r
$\widetilde{r}$
2n+1
q
$\widetilde{q}$
M
2n+3
$\mathbb{D}_2[SU(2n+1)]$
confinement
$\mathbb{D}_2[SU(2n+1)]$

Step 2


(A)
$\mathbb{D}_2[SU(2n+3)]$:
Φ
Q
n+1
2n+3
$\tilde{Q}$
$W_A = \tilde{Q}\Phi Q + \eta \, \mathrm{tr}\, \Phi + \hat{V}^{+} + \hat{V}^{-}$
Dual
(E)
M
2n+3
Step 3
BBP_1^+
(B)
n
R
n+1
Q
2n+3
$\tilde{R}$
$\tilde{Q}$
$W_B = \tilde{Q}\tilde{R}RQ + \eta \, \mathrm{tr}\, \tilde{R}R + \hat{v}^{(2),+} + \hat{v}^{(1,2),-} + \hat{v}^{(1),+} + \xi\hat{v}^{(1),-}$
(D)
X
M
2n+1
q
2n+3
$\tilde{q}$
BBP_1^+
(C)
ϕ
n
r
2n+1
q
M
2n+3
$\tilde{r}$
$\tilde{q}$
$\mathbb{D}_2[SU(2n+1)]$
confinement

Step 3



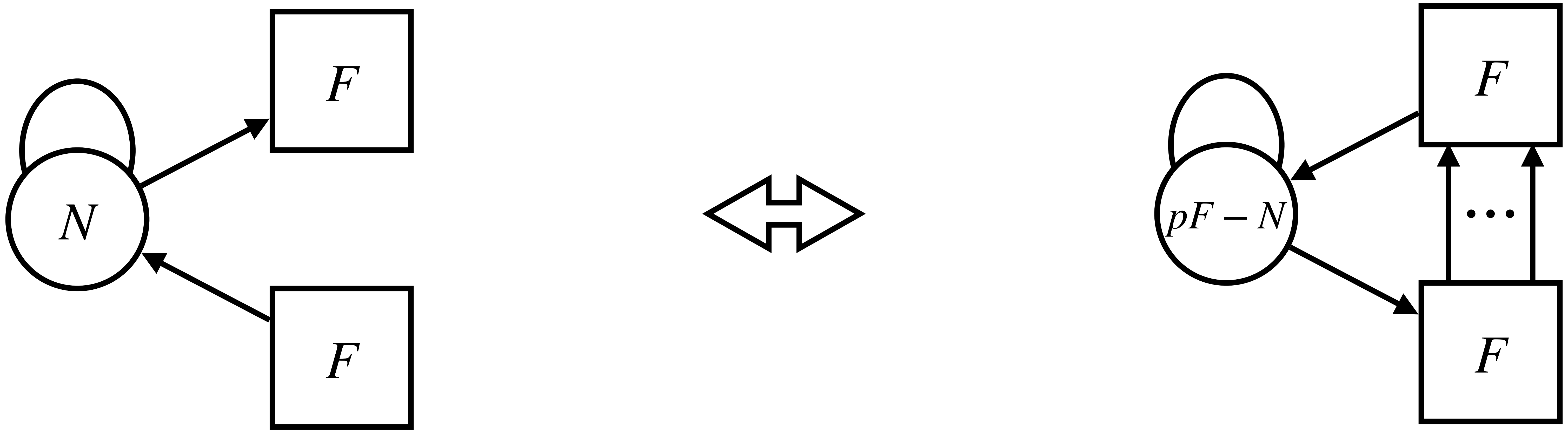
(D.1)
X
M
q
N
2n+3
q̃
Dual
(D.5)
X
M
q
N-1
2n+3
q̃
BBP$_1^+$
Aharony
(D.2)
M
N-1
r′
N
q
2n+3
r̃′
q̃
(D.4)
M
N-1
r′
N-1
q
2n+3
r̃′
q̃
(D.3)
Aharony
BBP$_1^-$
ϕ′
R′
Q
N-1
2n+2
2n+3
R̃′
Q̃

Part II: Revisit Dualities for Adjoint SQCDs

Dualities for 3D Adjoint SQCDs

- A variety of Seiberg-like dualities for adjoint SQCDs have been studied.

- E.g., the Kim-Park duality for 3d U(N) gauge theories with a single adjoint [Kim-Park 13]:



$+ V_i$

$$W = \mathrm{Tr}\, X^{p+1}$$

$$W = \mathrm{Tr}\, x^{p+1} + \sum_{i=0}^{p-1} \left(M_{p-i-1}\, \tilde{q} x^i q + V^+_{p-i-1}\, \hat{v}^+_i + V^-_{p-i-1}\, \hat{v}^-_i \right)$$

Dualities for 3D Adjoint SQCDs

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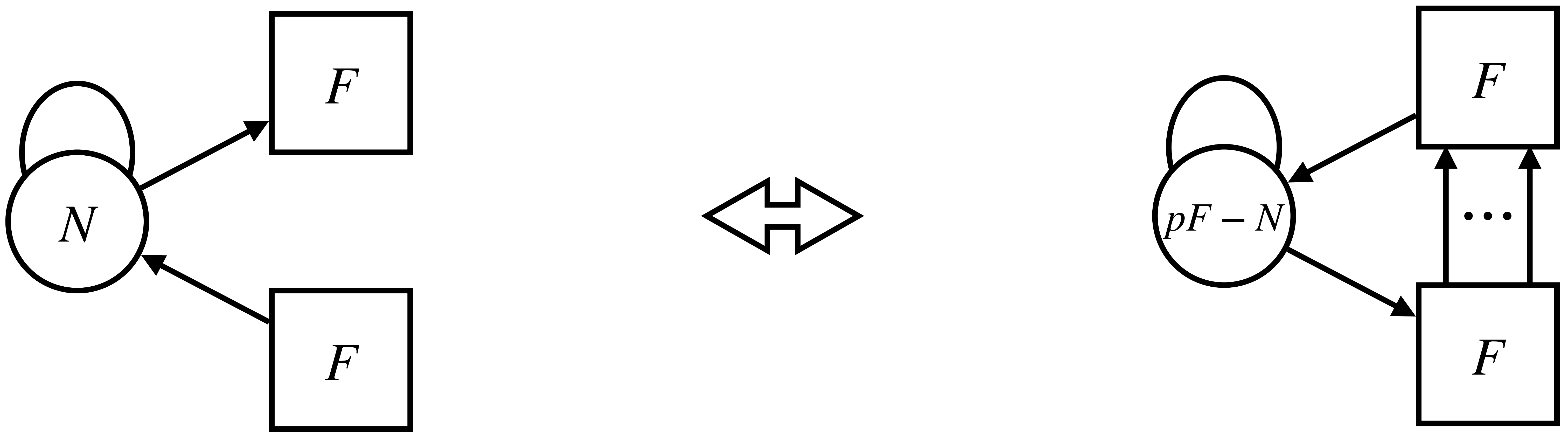


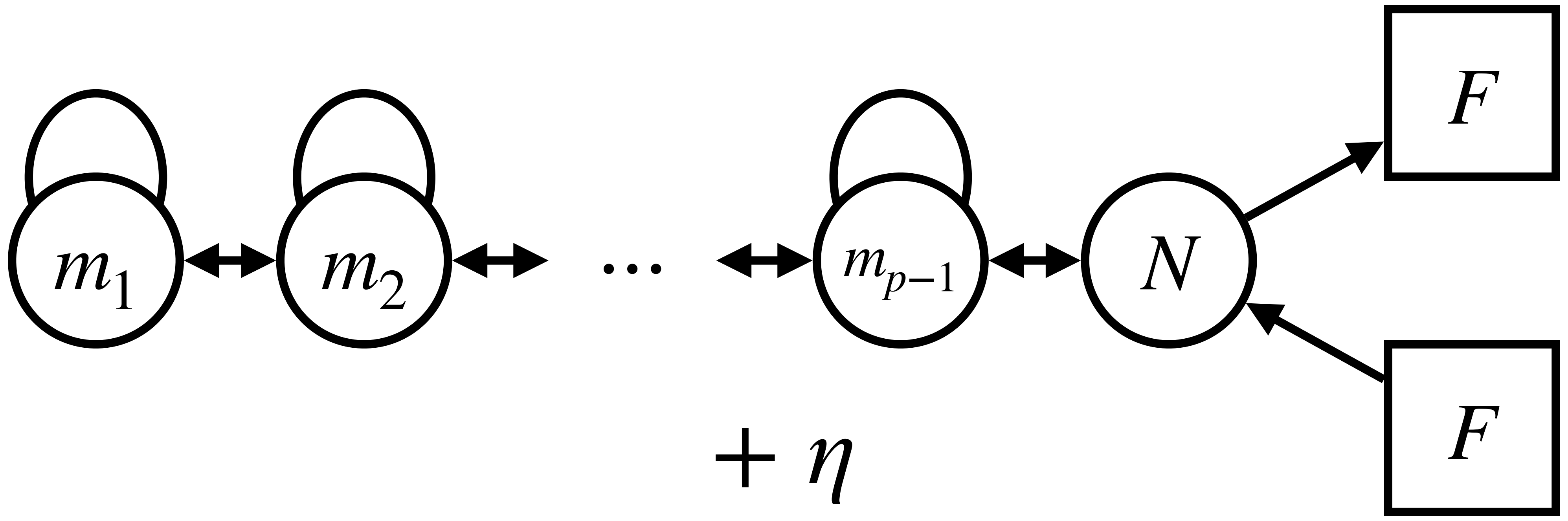
$+ V_i$

$$W = \mathrm{Tr}\, X^{p+1}$$

$$W = \mathrm{Tr}\, x^{p+1} + \sum_{i=0}^{p-1} \left(M_{p-i-1}\, \tilde{q} x^i q + V^{+}_{p-i-1}\, \hat{v}^{+}_i + V^{-}_{p-i-1}\, \hat{v}^{-}_i \right)$$

Allowing the deconfinement of the adjoint into the $\mathbb{D}_p[SU(N)]$ tail

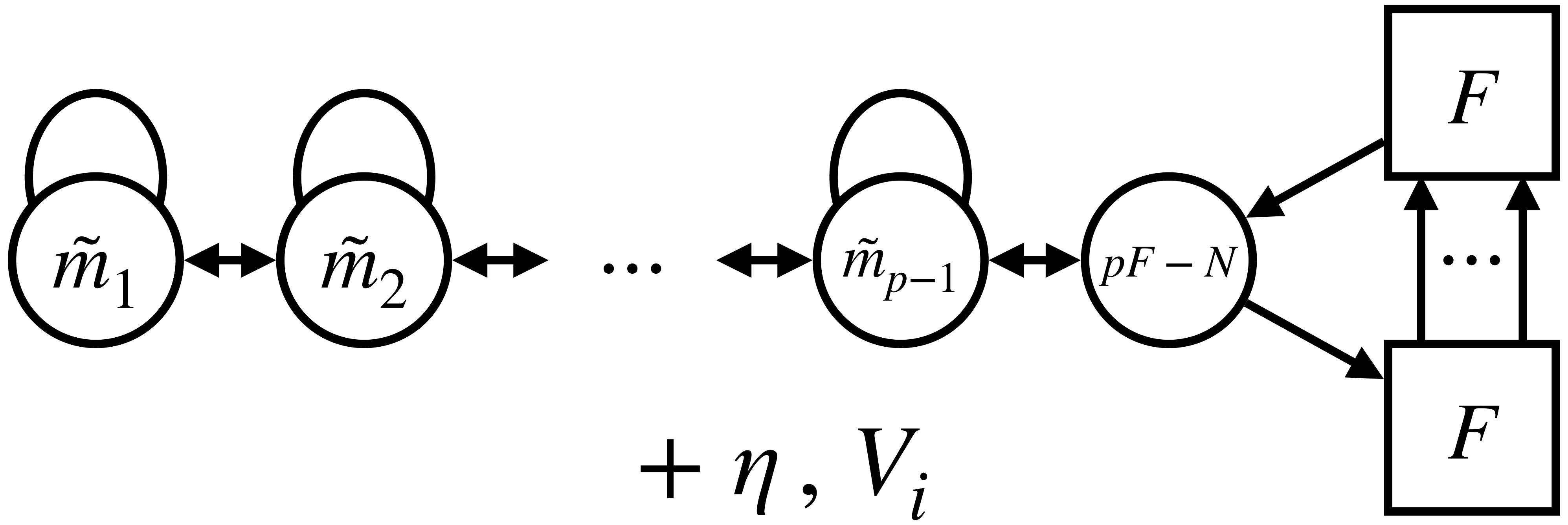
Deconfined Kim-Park Duality



$$W_A = \sum_{i=1}^{p-1} \mathrm{Tr}_i\, \Phi^{(i)} Q_i \tilde{Q}_i + \sum_{i=1}^{p-2} \mathrm{Tr}_{i+1}\, \Phi^{(i+1)} \tilde{Q}_i Q_i + \eta \sum_{i=1}^{p-1} \mathrm{Tr}\, \Phi^{(i)} + \sum_{i=1}^{p-1} \hat{V}^{(i),+} + \hat{V}^{(1,p-1),-}$$

$$m_j = \lfloor jN/p \rfloor\,, \qquad j = 1,\ldots,p-1$$

$\Updownarrow$



$$W_B = \sum_{i=1}^{p-1} \mathrm{Tr}_i\, \Phi^{(i)} Q_i \tilde{Q}_i + \sum_{i=1}^{p-2} \mathrm{Tr}_{i+1}\, \Phi^{(i+1)} \tilde{Q}_i Q_i + \eta \sum_{i=1}^{p-1} \mathrm{Tr}\, \Phi^{(i)} + \sum_{i=1}^{p-1} \hat{V}^{(i),+} + \hat{V}^{(1,p-1),-} + \ldots$$

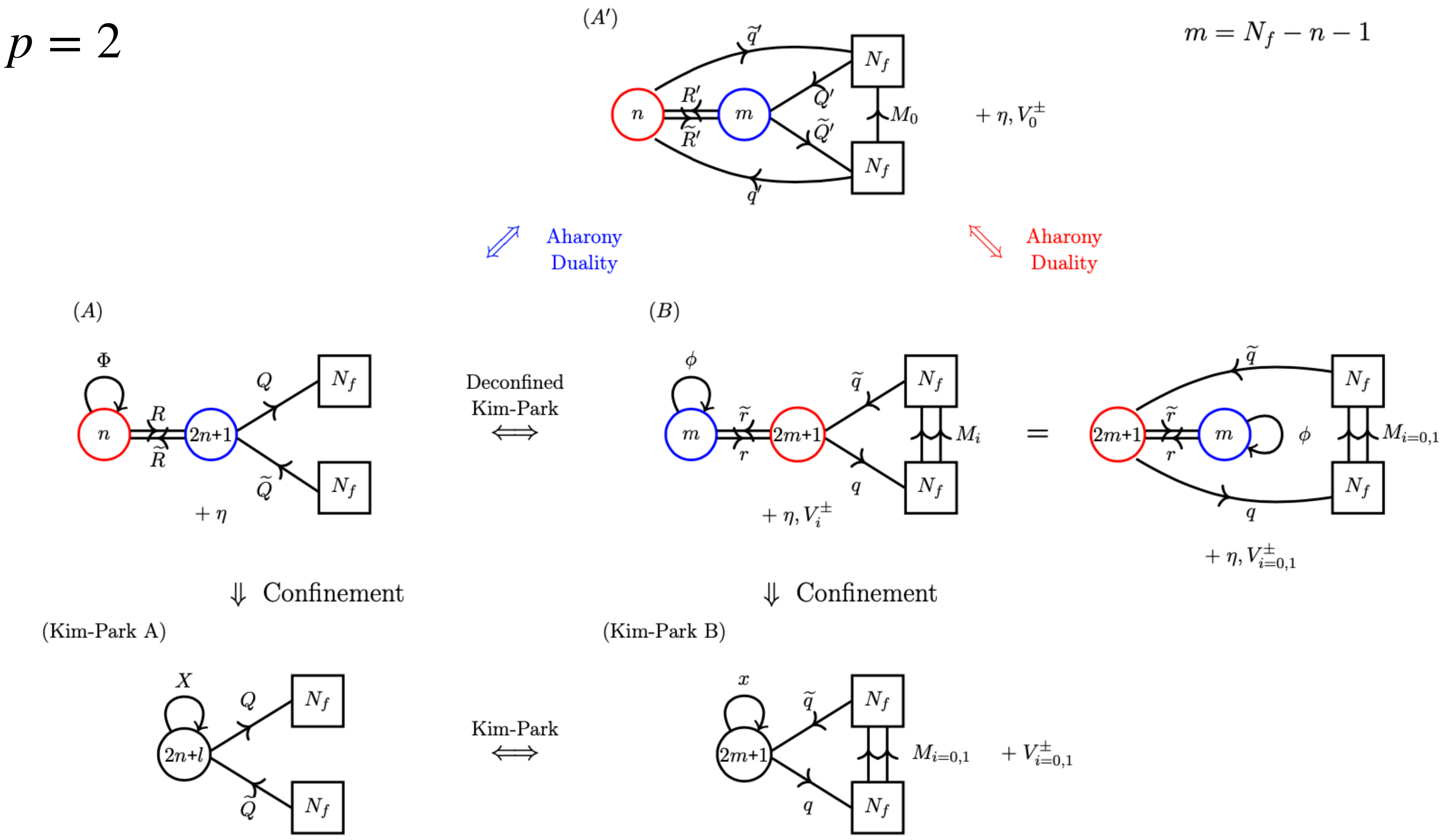
$$\tilde{m}_j = \lfloor j(pF-N)/p \rfloor = jF + m_{p-j} - m_p\,, \qquad j = 1,\ldots,p-1$$

- Matching superconformal indices (tested for some N & p)

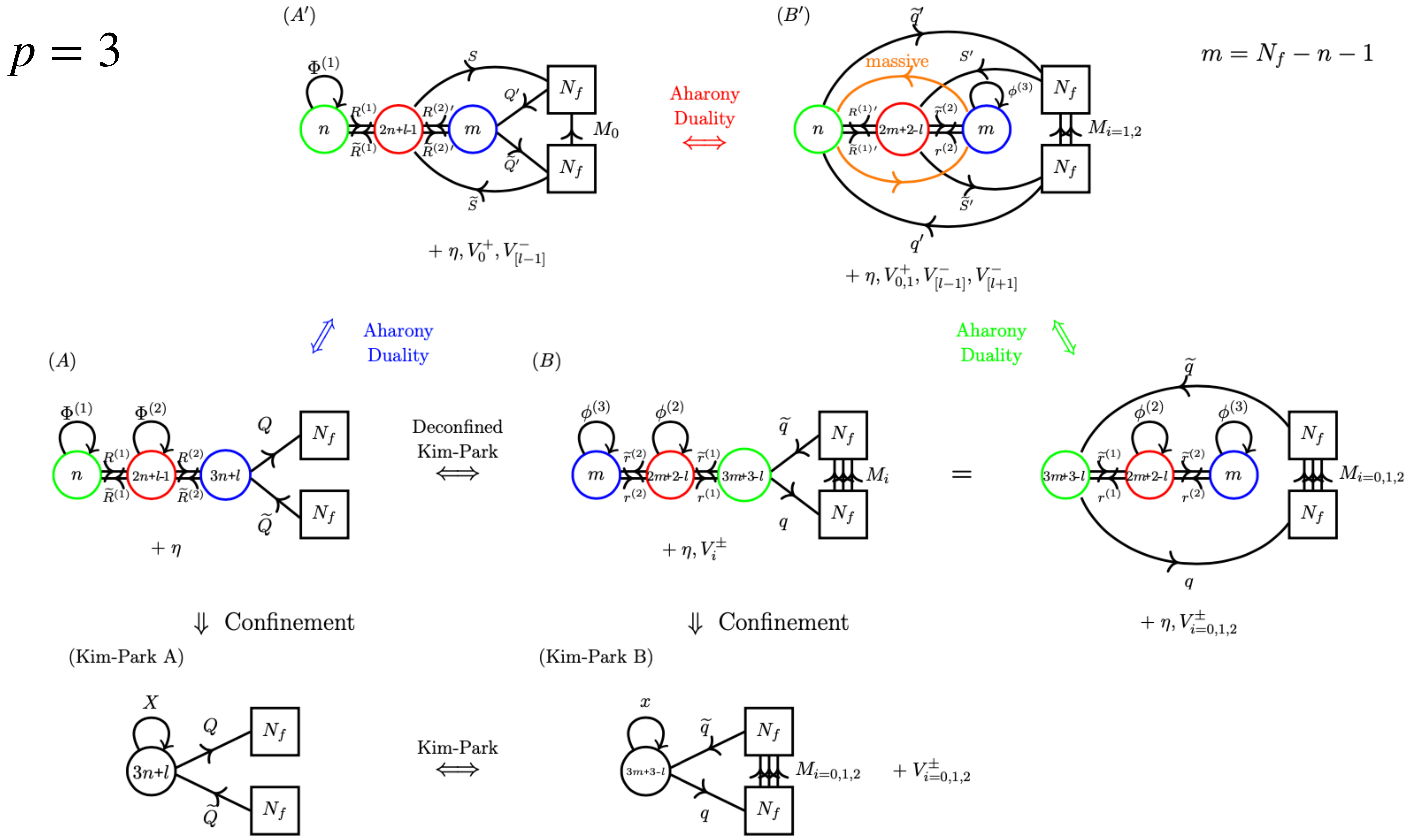
- E.g., the chiral ring generators for $p = 2$:

Kim–Park A	Theory A	Theory A$'$	Theory B	Kim–Park B
$\tilde{Q}Q$	$\tilde{Q}Q$	M_0	M_0	M_0
$\tilde{Q}XQ$	$\tilde{Q}\tilde{R}RQ$	$q'\tilde{q}'$	M_1	M_1
$\mathrm{Tr}\, X$	$\eta \sim \mathrm{Tr}\, \tilde{R}R$	η	$\eta \sim \mathrm{Tr}\, \tilde{r}r$	$\mathrm{Tr}\, x$
$\hat{V}_0^{\pm}$	$\hat{V}^{(2),\pm}$	$V_0^{\pm}$	$V_0^{\pm}$	$V_0^{\pm}$
$\hat{V}_1^{\pm}$	$\hat{V}^{(1,2),\pm}$	$\hat{v}^{(1),\pm}$	$V_1^{\pm}$	$V_1^{\pm}$

- Again, proved only assuming the Aharony duality



$p = 2$
(A')
$m = N_f - n - 1$
$\widetilde{q}'$
R'
$\widetilde{R}'$
Q'
$\widetilde{Q}'$
q'
N_f
M_0
$+\ \eta, V_0^\pm$
Aharony Duality
Aharony Duality
(A)
(B)
Φ
R
$\widetilde{R}$
Q
$\widetilde{Q}$
$2n{+}1$
$+\ \eta$
Deconfined Kim-Park
ϕ
$\widetilde{r}$
r
$\widetilde{q}$
q
$2m{+}1$
M_i
$+\ \eta, V_i^\pm$
$M_{i=0,1}$
$+\ \eta, V_{i=0,1}^\pm$
⇓ Confinement
⇓ Confinement
(Kim-Park A)
(Kim-Park B)
X
$2n{+}l$
Kim-Park
x
$+\ V_{i=0,1}^\pm$


p = 3
m = N_f − n − 1
(A′)
(B′)
Aharony Duality
massive
+ η, V_0^+, V_[l−1]^−
+ η, V_{0,1}^+, V_[l−1]^−, V_[l+1]^−
Aharony Duality
Aharony Duality
(A)
(B)
Deconfined Kim-Park
+ η
+ η, V_i^±
+ η, V_{i=0,1,2}^±
M_{i=1,2}
M_i
M_{i=0,1,2}
⇓ Confinement
⇓ Confinement
(Kim-Park A)
(Kim-Park B)
Kim-Park
+ V_{i=0,1,2}^±

- The Aharony duality, or its monopole deformed cousin, is a **building block** of various supersymmetric 3d dualities, such as 3d $U(N)$ mirror symmetry and the Seiberg-like duality with an adjoint matter.

- Furthermore, those underlying relations between different supersymmetric dualities provide **new proof of various special function identities** through the localization computation of supersymmetric partition functions (Spiridonov, Rains, …)

- E.g., the partition function identities for the Aharony duality on S^3 [van de Bult 08], $S^2 \times S^1$ [CH-Yi-Yoshida 17] have been proven.

  -> *The identities for the (deconfined) Kim-Park duality are also implied.*

Factorization of the 3d superconformal index

- 3d superconformal index

$$I \;=\; \mathrm{tr}\,(-1)^F x^{R+2j} e^{i\mu Q}$$

SUSY localization [Kim 09, Imamura-Yokoyama 11]

$$I(x;\mu) = \sum_{\mathfrak{m}\in\mathbb{Z}^N/S^N} \frac{1}{W_{\mathfrak{m}}} \oint \frac{d^N a}{(2\pi)^N} Z_{cl}(x;\mu,a;\mathfrak{m})\, Z_{1-loop}(x;\mu,a;\mathfrak{m})$$

$$Z^{chiral}_{1-loop}(x;\mu,a;\mathfrak{m}) = \prod_{\rho} \left(e^{i\rho(a+\mu)}x^{-1}\right)^{-\frac{\rho(m)}{2}} \frac{\left(e^{-i\rho(a+\mu)}\, x^{2-R+\,\rho(m)}\,;x^2\right)}{\left(e^{i\rho(a+\mu)}\, x^{R+\,\rho(m)}\,;x^2\right)}$$

$$\vdots$$

- Factorization [CH-Kim-Park 12] (Holomorhpic blocks, Higgs-branch localization)

$$I(x;\mu) = \sum_{\mathfrak{m}\in\mathbb{Z}^N/S^N} \frac{1}{W_{\mathfrak{m}}} \oint \frac{d^N a}{(2\pi)^N} Z_{cl}(x;\mu,a;\mathfrak{m})\, Z_{1-loop}(x;\mu,a;\mathfrak{m})$$

⬇

$$I = \sum_{\text{Higgs vacua}} Z_{pert}\, Z_{vortex}\, \bar{Z}_{pert}\, \bar{Z}_{vortex}$$

- For the Aharony duality

$$Z_{pert} = \tilde{Z}_{pert}\,\tilde{Z}_M$$

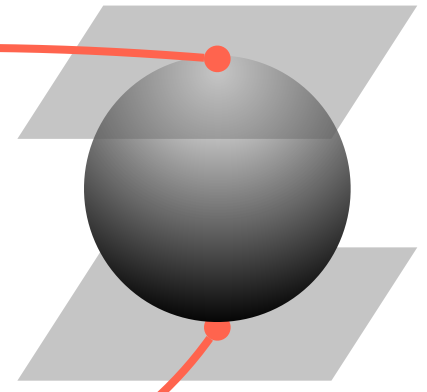
Proved

$$Z_{vortex} = \tilde{Z}_{vortex}\,\tilde{Z}_V$$

Checked by series expansion up to a finite order

- Factorization [CH-Kim-Park 12] (Holomorhpic blocks, Higgs-branch localization)

$$I(x;\mu) = \sum_{\mathfrak{m}\in\mathbb{Z}^N/S^N} \frac{1}{W_{\mathfrak{m}}} \oint \frac{d^N a}{(2\pi)^N} Z_{cl}(x;\mu,a;\mathfrak{m})\, Z_{1-loop}(x;\mu,a;\mathfrak{m})$$



$$I = \sum_{\text{Higgs vacua}} Z_{pert} Z_{vortex} \bar{Z}_{pert} \bar{Z}_{vortex}$$

BPS partition functions on $\mathbf{R}^2 \times S^1$

- For the Aharony duality

$$Z_{pert} = \tilde{Z}_{pert} \tilde{Z}_M$$

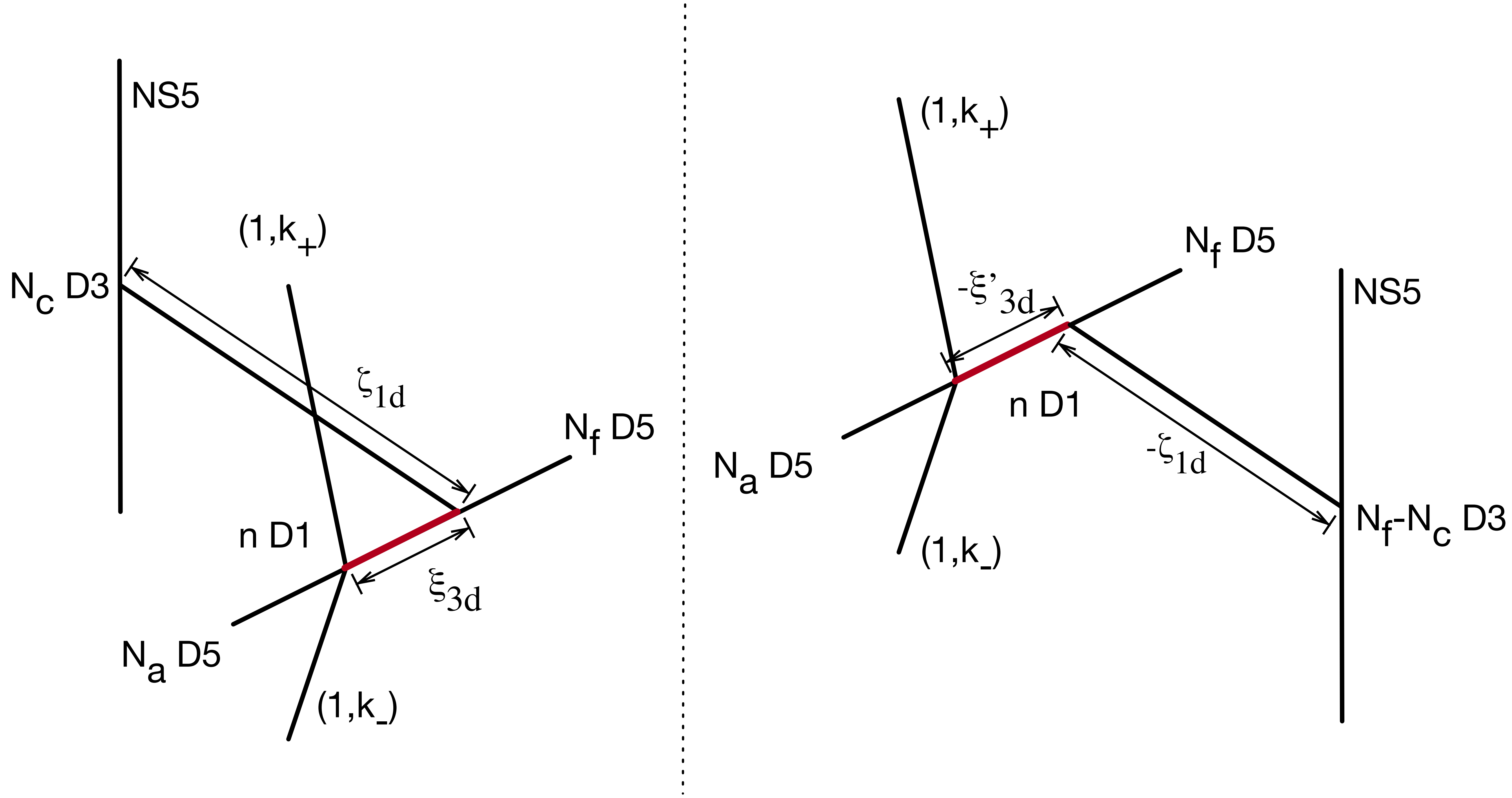
Proved

$$Z_{vortex} = \tilde{Z}_{vortex} \tilde{Z}_V$$

Checked by series expansion up to a finite order

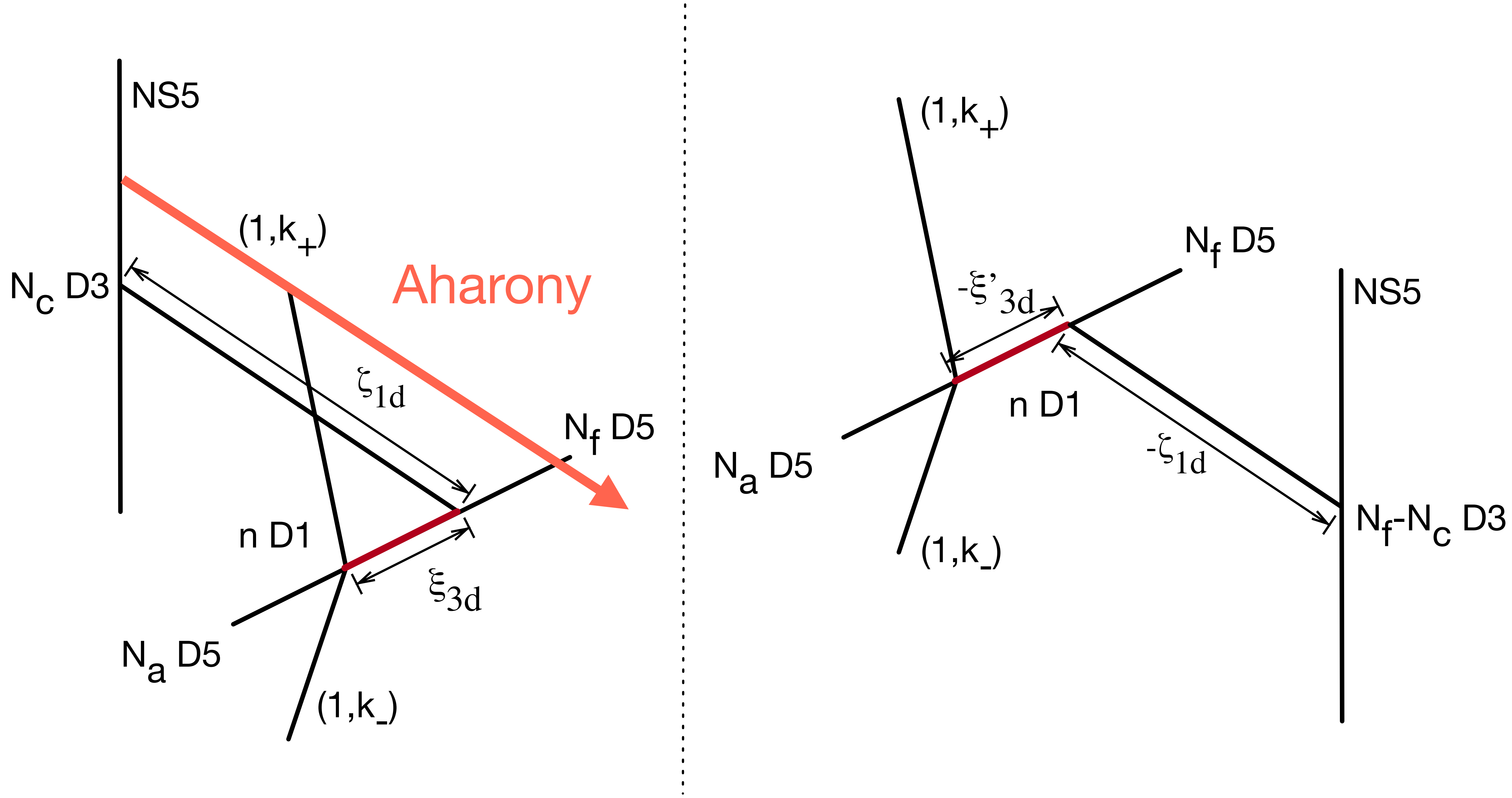
The Aharony Duality and Vortex Wall-Crossing

- Type IIB brane picture



The Aharony Duality and Vortex Wall-Crossing

- Type IIB brane picture



The Aharony Duality and Vortex Wall-Crossing

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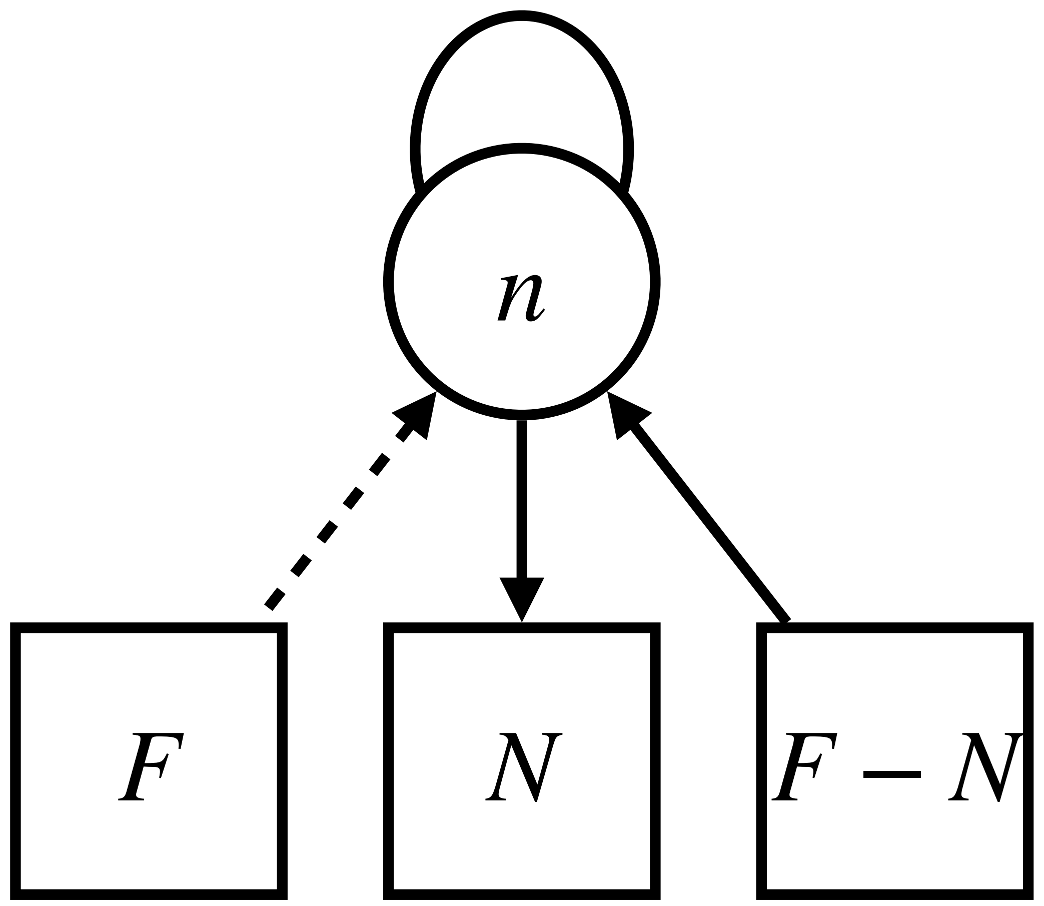

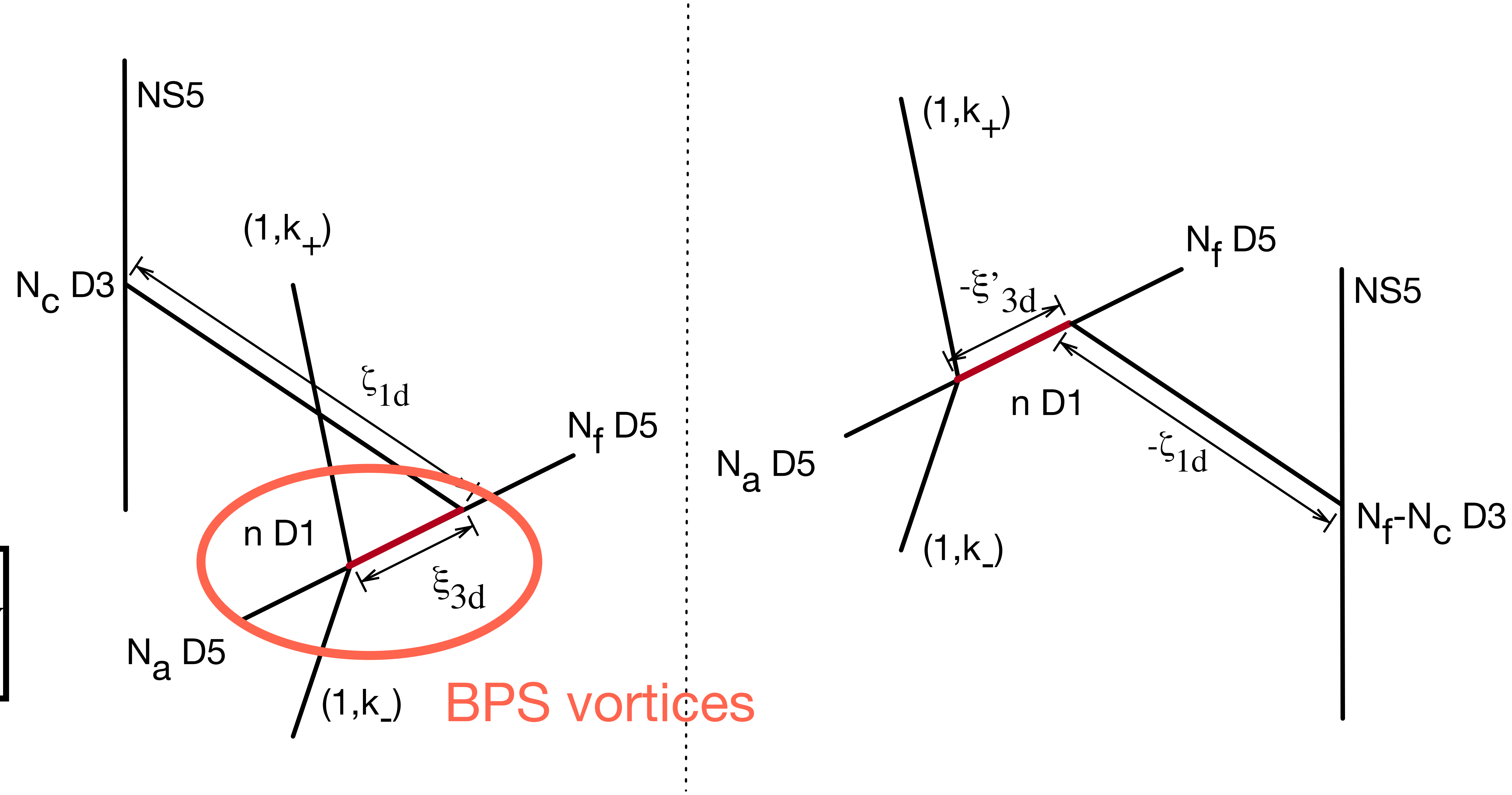

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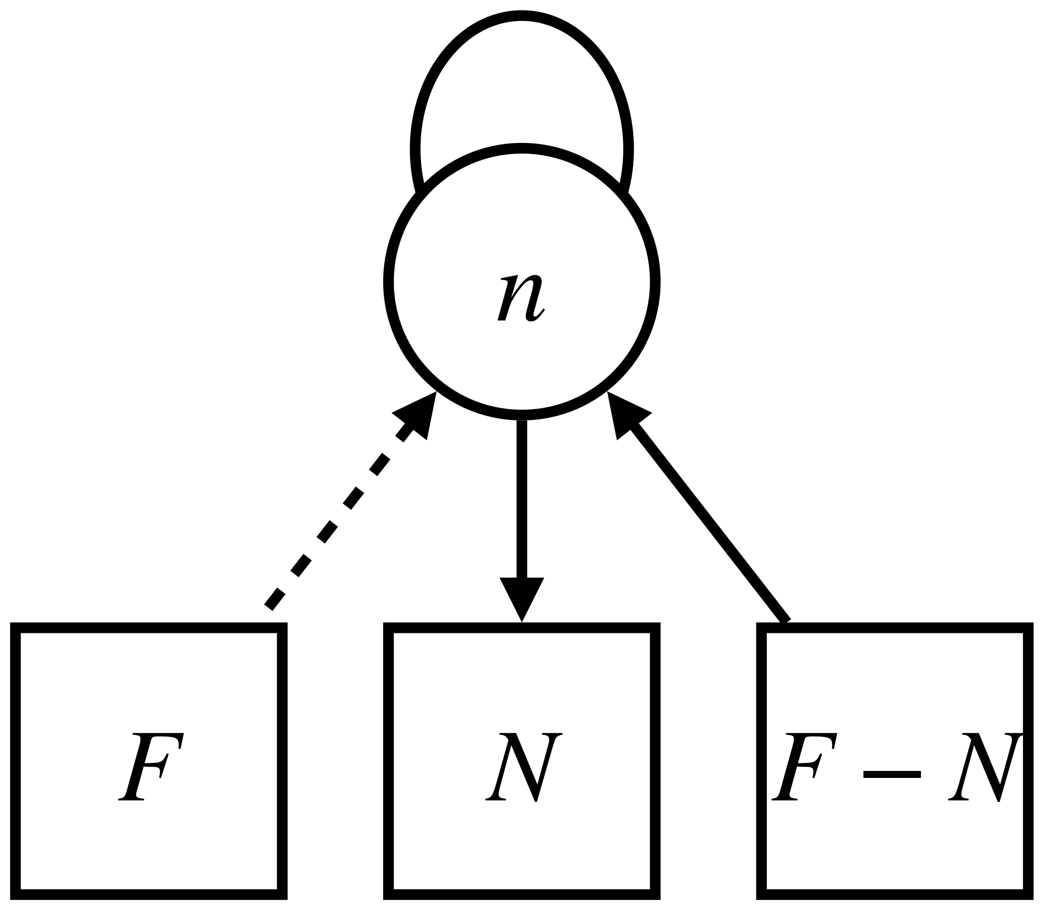

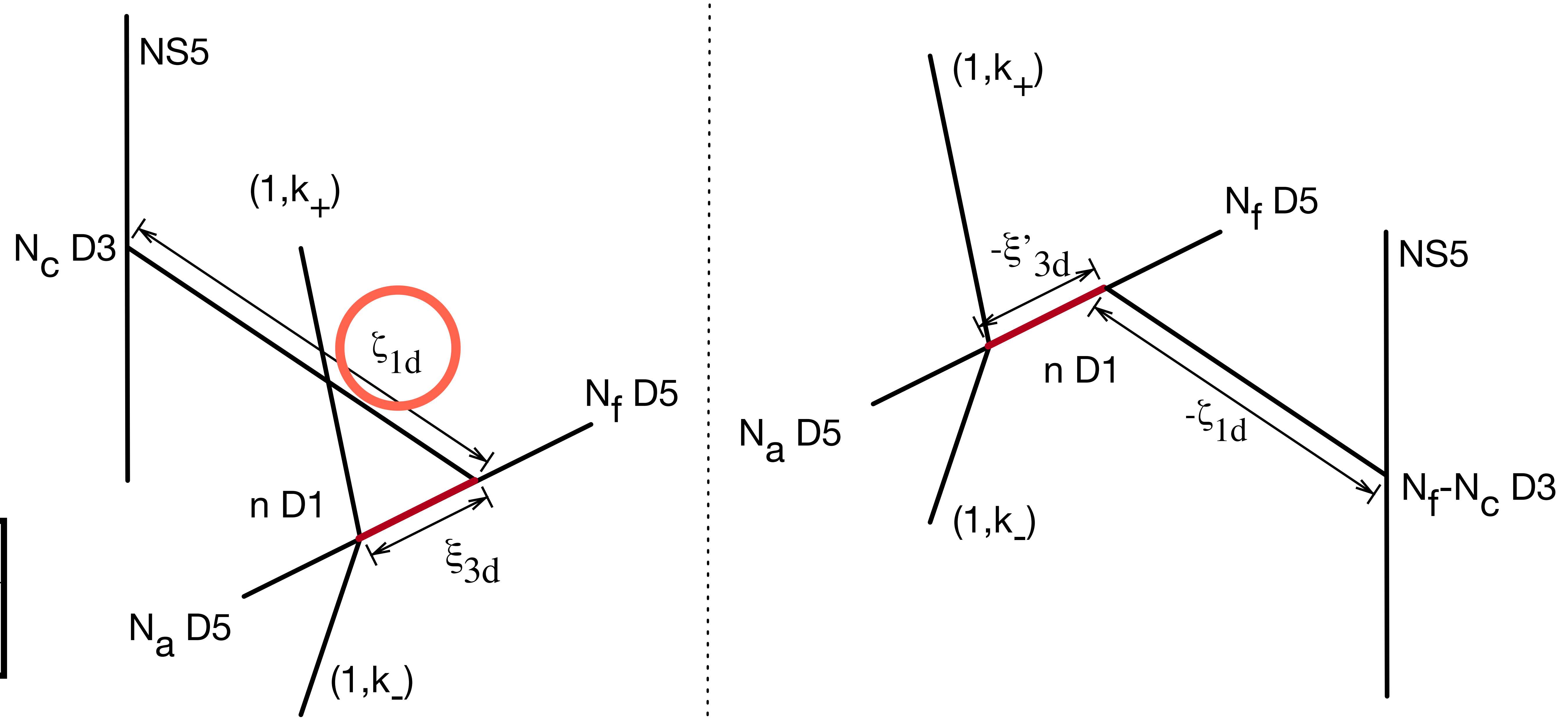

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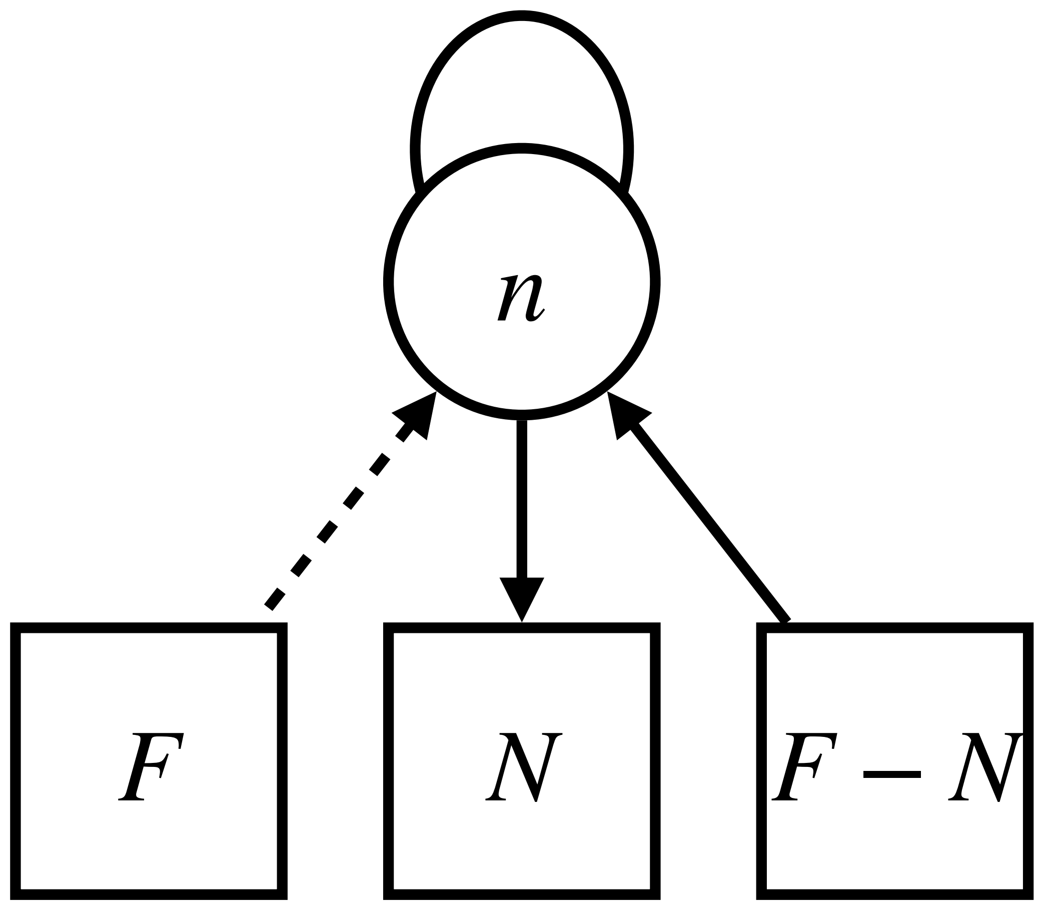

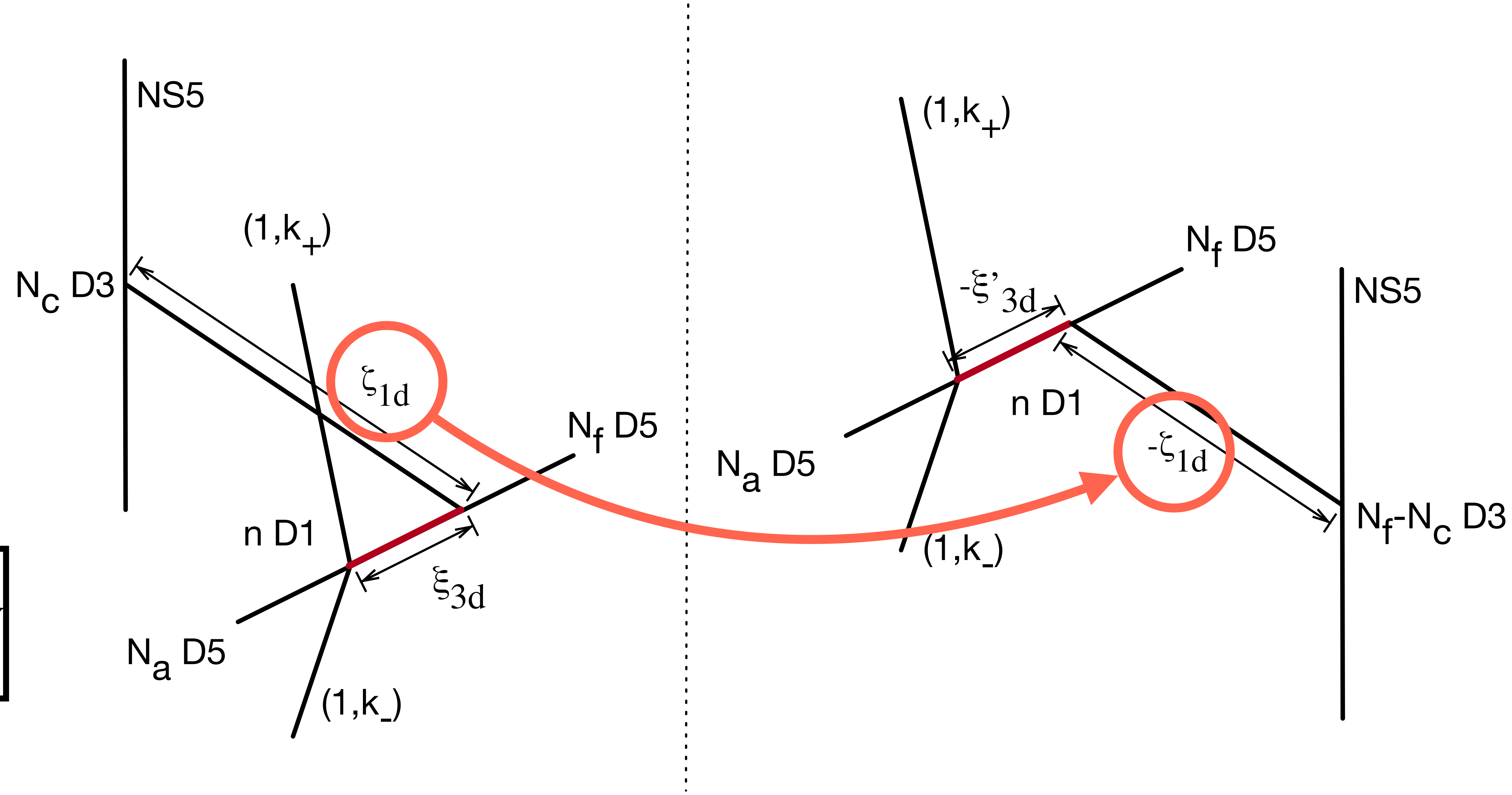

The Aharony Duality and Vortex Wall-Crossing

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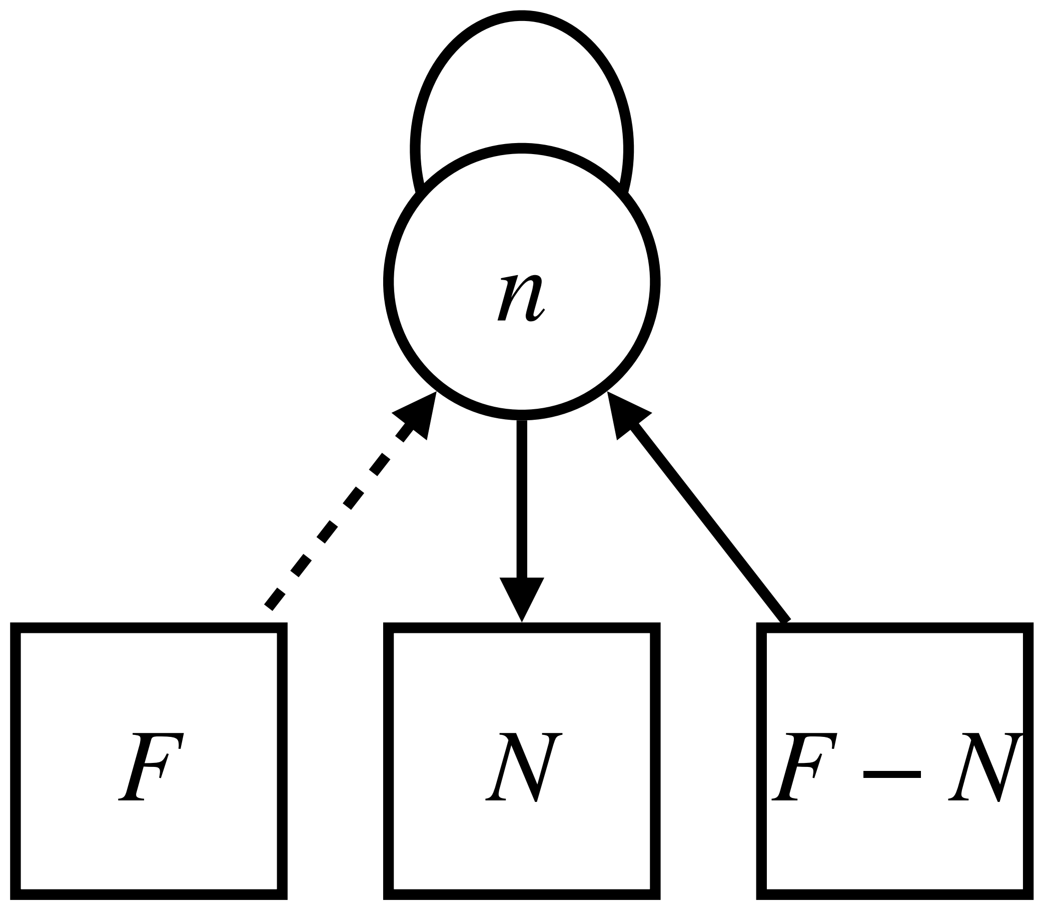
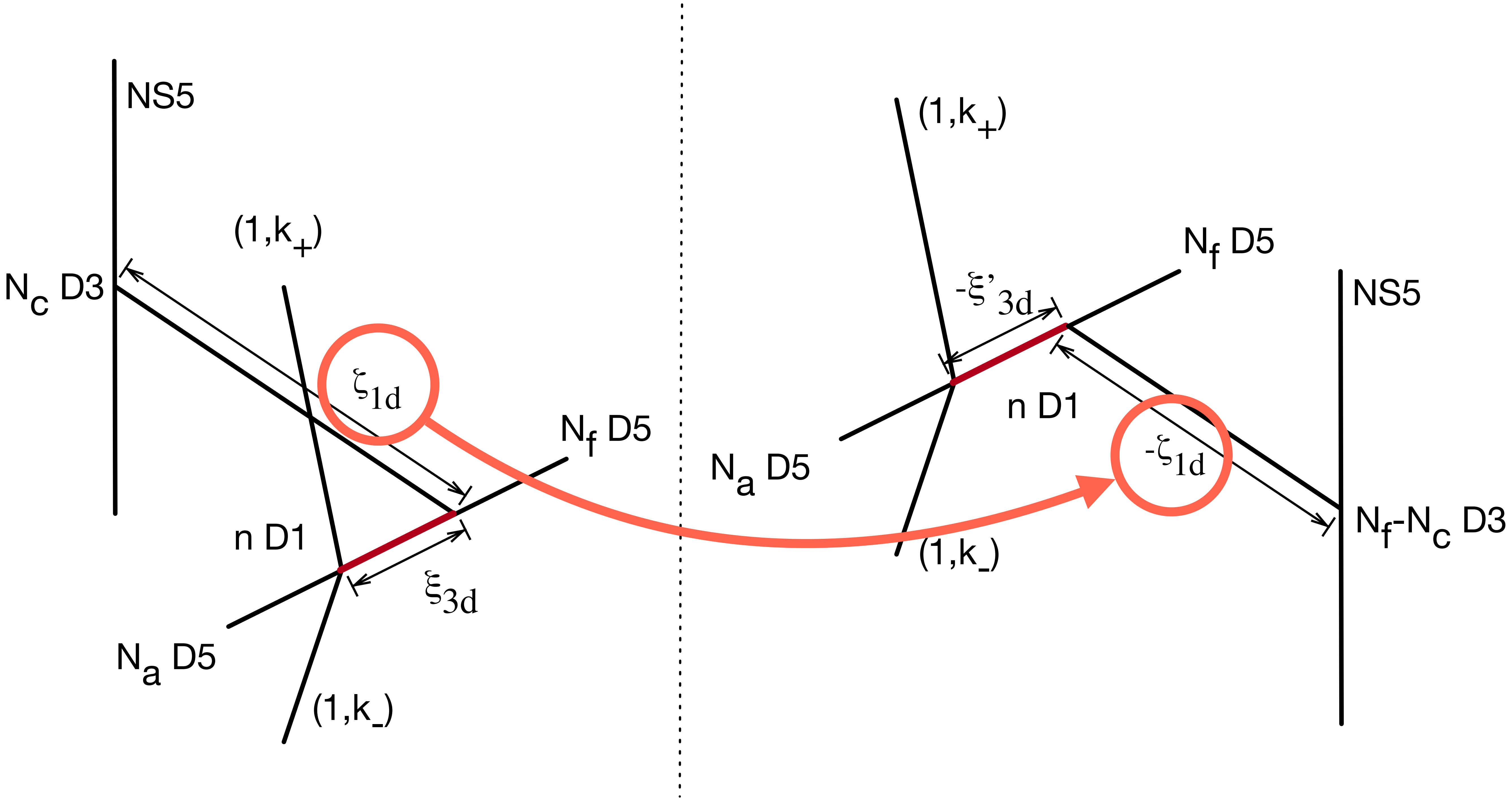




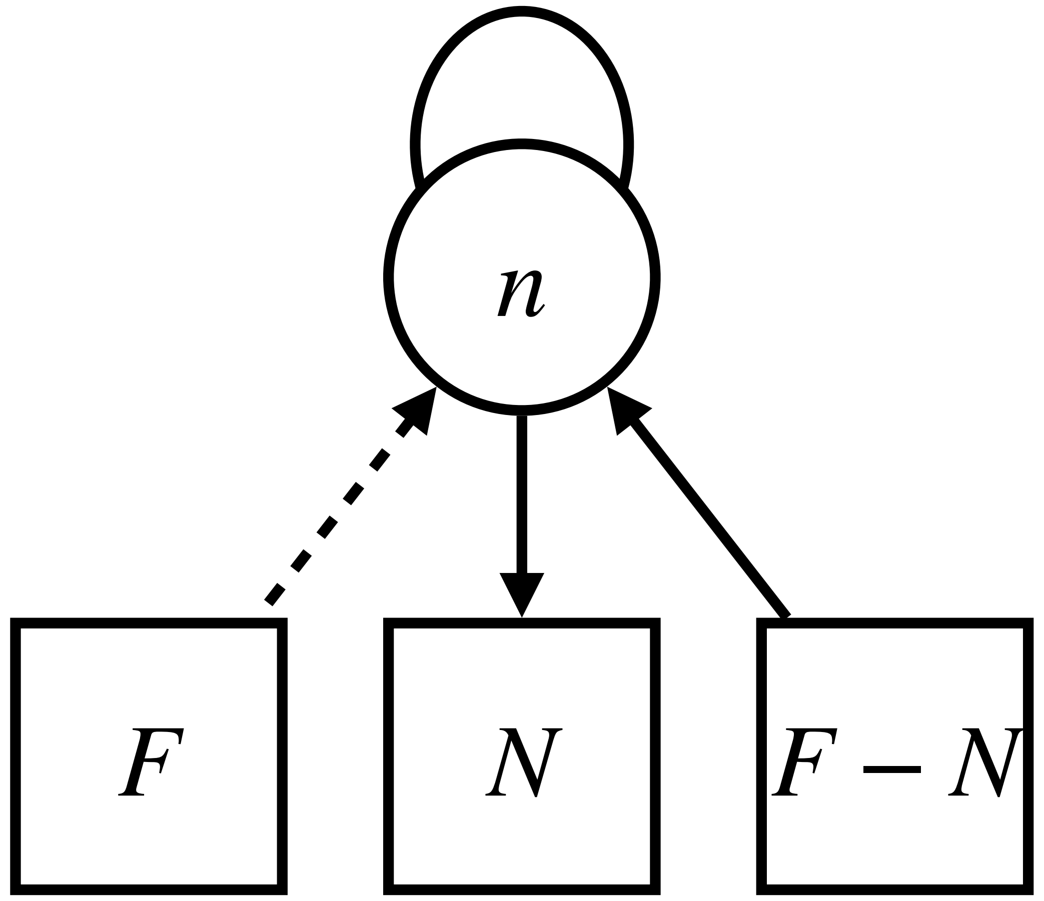
The Aharony duality of a 3d gauge theory = the wall-crossing of a 1d vortex GLSM

$$I = \sum_{\text{Higgs vacua}} Z_{pert}\, Z_{vortex}\, \overline{Z}_{pert}\, \overline{Z}_{vortex}$$

$$\Downarrow$$

$$Z_{vortex} = \sum_n w^n Z_n$$

$$Z_n = \frac{1}{W}\,\text{JK-Res}_{\vec{\eta}=\zeta\vec{1}}\left[g(u)\, d^n u\right]$$



$$g^n(u) = \frac{\left(\prod_{i\neq j}^n \sinh\frac{u_i-u_j}{2}\right)\left(\prod_{j=1}^n\prod_{a=1}^F \sinh\frac{u_j-\tilde{m}_a+\mu-\gamma}{2}\right)}{\left(\prod_{i,j}^n \sinh\frac{u_i-u_j-2\gamma}{2}\right)\left(\prod_{i=1}^n\prod_{b=1}^N \sinh\frac{u_i-m_b-\mu-\gamma}{2}\right)\left(\prod_{j=1}^n\prod_{a=N+1}^F \sinh\frac{-u_j+m_a+\mu-\gamma}{2}\right)}$$

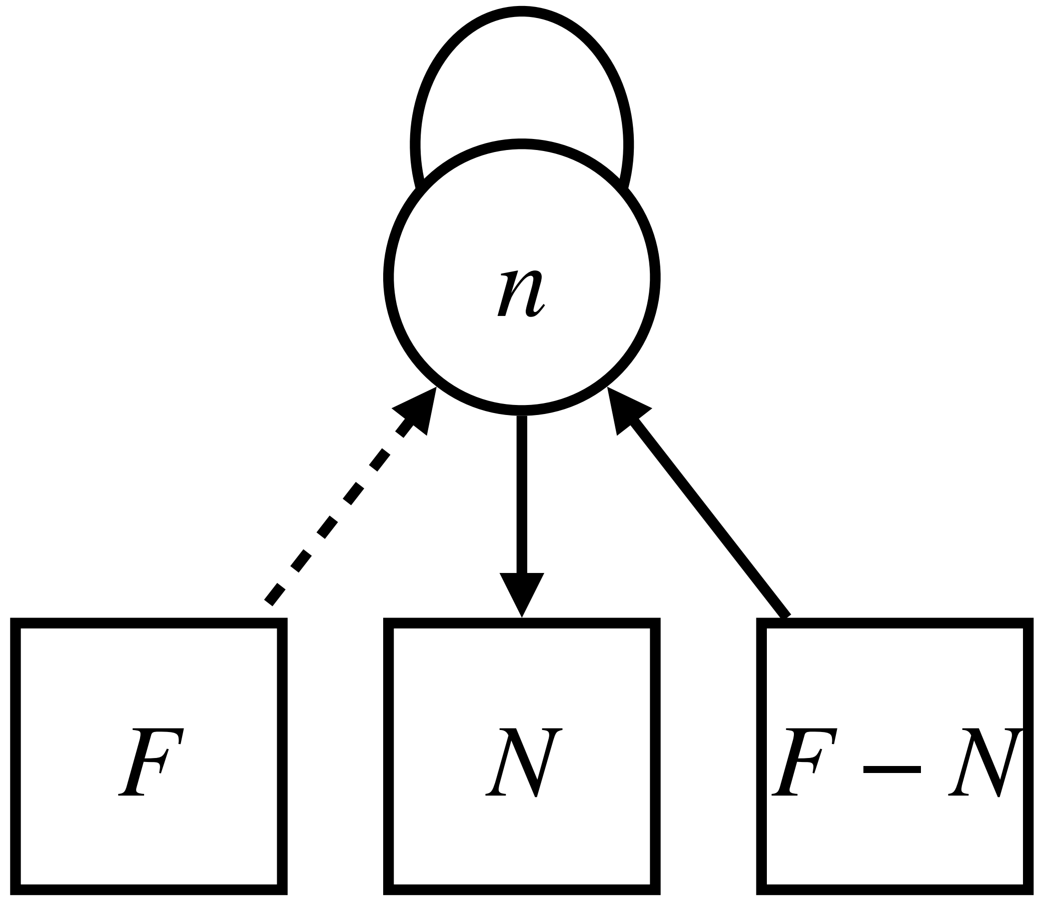
$$I = \sum_{\text{Higgs vacua}} Z_{pert}\, Z_{vortex}\, \overline{Z}_{pert}\, \overline{Z}_{vortex}$$

$$\Downarrow$$

$$Z_{vortex} = \sum_{n} w^n\, Z_n$$

The contribution of each vortex number can be computed using the Jeffrey-Kirwan residue method [Hori-Kim-Yi 14, CH-Kim-Kim-Park 14].

$$Z_n = \frac{1}{W}\, \text{JK-Res}_{\vec{\eta}=\zeta\vec{1}} \left[g(u)\, d^n u \right]$$



$$g^n(u) = \frac{\left(\prod_{i\neq j}^{n} \sinh \frac{u_i - u_j}{2}\right)\left(\prod_{j=1}^{n}\prod_{a=1}^{F} \sinh \frac{u_j - \tilde{m}_a + \mu - \gamma}{2}\right)}{\left(\prod_{i,j}^{n} \sinh \frac{u_i - u_j - 2\gamma}{2}\right)\left(\prod_{i=1}^{n}\prod_{b=1}^{N} \sinh \frac{u_i - m_b - \mu - \gamma}{2}\right)\left(\prod_{j=1}^{n}\prod_{a=N+1}^{F} \sinh \frac{-u_j + m_a + \mu - \gamma}{2}\right)}$$

- For each vortex sector, it is shown that

$$Z_n(\zeta) \; = \; Z_n(-\zeta) + Z_n^{wall-crossing}$$

$$\sum_n w^n Z_n = Z_{vortex}$$

$$\sum_n w^n \left(Z_n(-\zeta) + Z_n^{wall-crossing} \right) = \tilde{Z}_{vortex} \tilde{Z}_V$$

$$Z_{vortex} \; = \; \tilde{Z}_{vortex} \tilde{Z}_V$$

$$\Downarrow$$

$$I \; = \; \tilde{I}$$

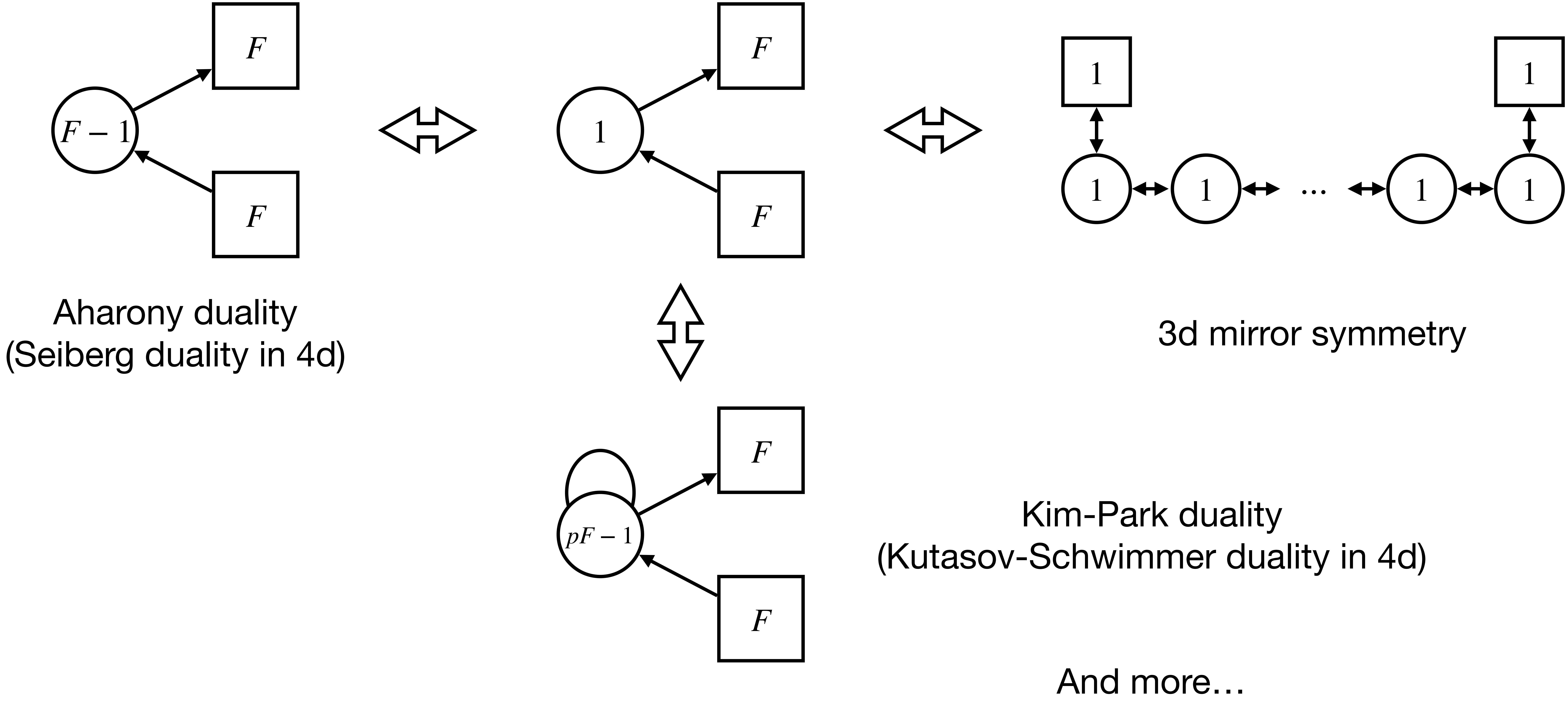
Provides a proof of the index identity motivated by a physical D-brane picture

- **Furthermore, this index identity for the Aharony duality also derives the identity for the *deconfined* Kim-Park duality!**

- Index identities for BBP? -> Open questions

- Together with BBP, whose index identities are still conjectural, the deconfinement of the $\mathbb{D}_p[SU(N)]$ can also be proved.

Conclusion


F
F − 1
F
1
F
F
1
1
1
1
1
1
1
⋯
Aharony duality
(Seiberg duality in 4d)
3d mirror symmetry
F
pF − 1
F
Kim-Park duality
(Kutasov-Schwimmer duality in 4d)
And more…

The Aharony duality is a mother duality of various 3d supersymmetric dualities.

Many possible generalizations

- Relaxing the conditions among the parameters
- Monopole deformation
- Multiple adjoints with ADE-type superpotentials
- Non-supersymmetric counterparts?
    - Many versions of 3d bosonization/particle-vortex dualities, resembling supersymmetric mirror symmetry, and generalized level-rank dualities of Chern-Simons-matter theories

$$W_A = \mathrm{tr}\left(X^{p+1} + Y^2\right)$$ Kim-Park 13

$$W_D = \mathrm{tr}\left(X^{p+1} + X\,Y^2\right)$$ CH-Kim-Park 13

$$W_{E_6} = \mathrm{tr}\left(Y^3 + X^4\right)$$

$$W_{E_7} = \mathrm{tr}\left(Y^3 + YX^3\right)$$

$$W_{E_8} = \mathrm{tr}\left(Y^3 + X^5\right)$$

Thank you