Unified Hierarchical Relationship Between Thermodynamic Tradeoff Relations

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Recent years have witnessed a surge of discoveries in the studies of thermodynamic inequalities: the thermodynamic uncertainty relation (TUR) and the entropic bound (EB) provide a lower bound on the entropy production (EP) in terms of nonequilibrium currents; the classical speed limit (CSL) expresses the lower bound on the EP using the geometry of probability distributions; the power-efficiency (PE) tradeoff dictates the maximum power achievable for a heat engine given the level of its thermal efficiency. In this study, we show that there exists a unified hierarchical structure encompassing all of these bounds, with the fundamental inequality given by a novel extension of the TUR (XTUR) that incorporates the most general range of current-like and state-dependent observables. By selecting more specific observables, the TUR and the EB follow from the XTUR, and the CSL and the PE tradeoff follow from the EB. Our derivations cover both Langevin and Markov jump systems, with the first proof of the EB for the Markov jump systems and a more generalized form of the CSL. We also present concrete examples of the EB for the Markov jump systems and the generalized CSL.

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