Time-cost-error trade-off relation in stochastic thermodynamics: third-law like constraint

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Elucidating fundamental limitations inherent in physical systems is a central quest in physics. For generic processes such as information erasure and cooling, resources such as time and energetic cost must be consumed to accomplish the task up to a predetermined error. In the present study, we uncover a three-way trade-off relation between time, cost, and error for this general class of thermodynamic processes, rigorously showing that these incompatible quantities cannot be simultaneously small. Remarkably, this relation leads to crucial implications in nonequilibrium thermodynamics, including a quantification for the third law of thermodynamics in the form of the unattainability principle, a limitation on the preparation of separate states, and a no-go theorem for exact classical copying. Furthermore, we generalize the findings to the quantum regime, including both Markovian and non-Markovian scenarios. The generalization implies that heat dissipation becomes infinite as the quantum system is exactly cooled down to the ground state or perfectly reset to a pure state, thereby resolving an open question regarding the thermodynamic cost of information erasure.