PHASE-ORDERING KINETICS IN FERROMAGNETIC SYSTEMS WITH LONG-RANGE INTERACTIONS

Seoul,
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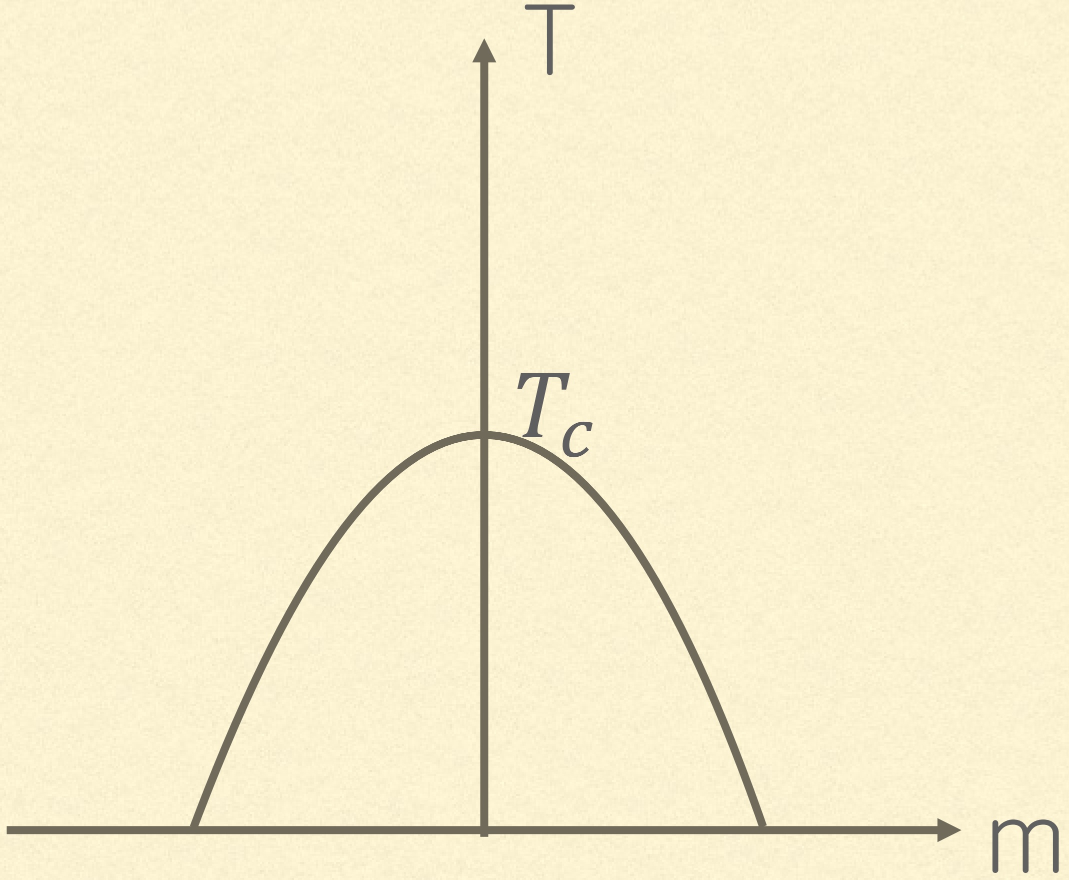
Work with:

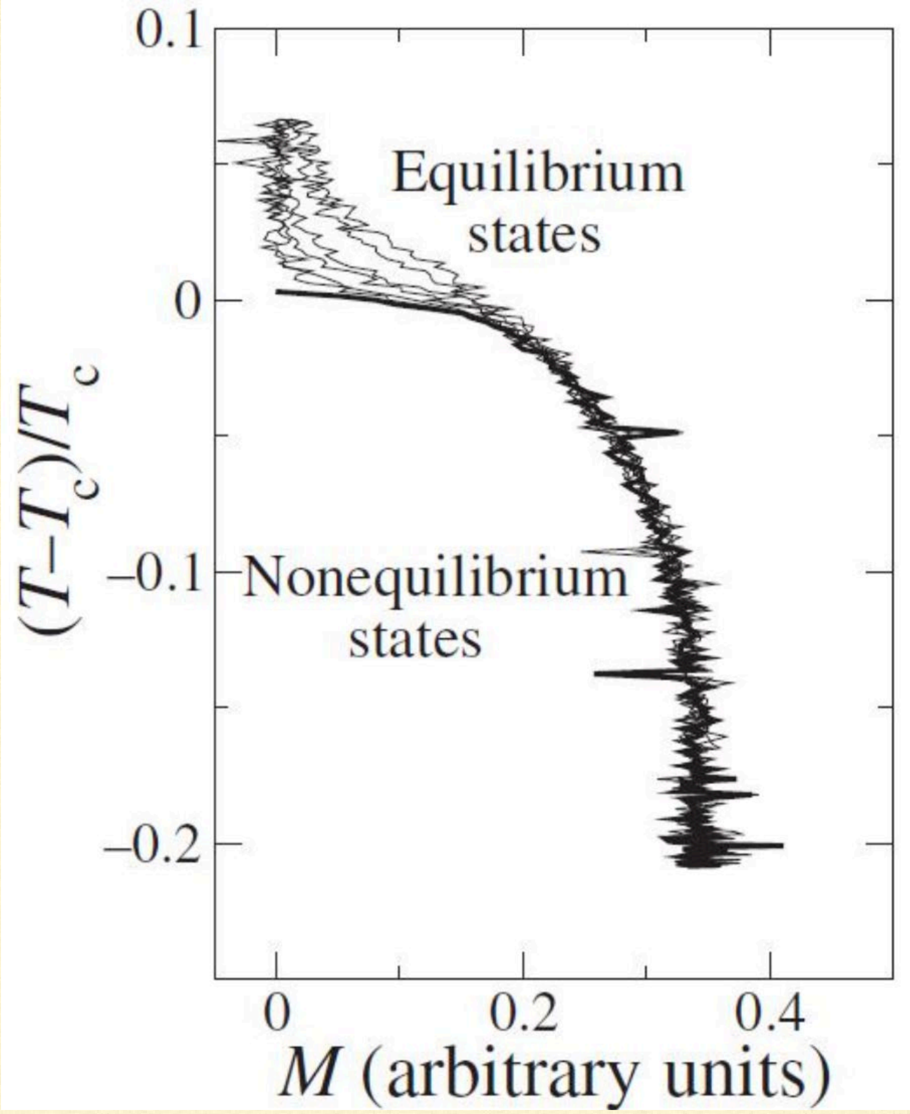
audio Castellano (Institute for Complex Systems - CNR, Rom

Luca Smaldone (Salerno University)

Salvatore dello Russo (Salerno University)

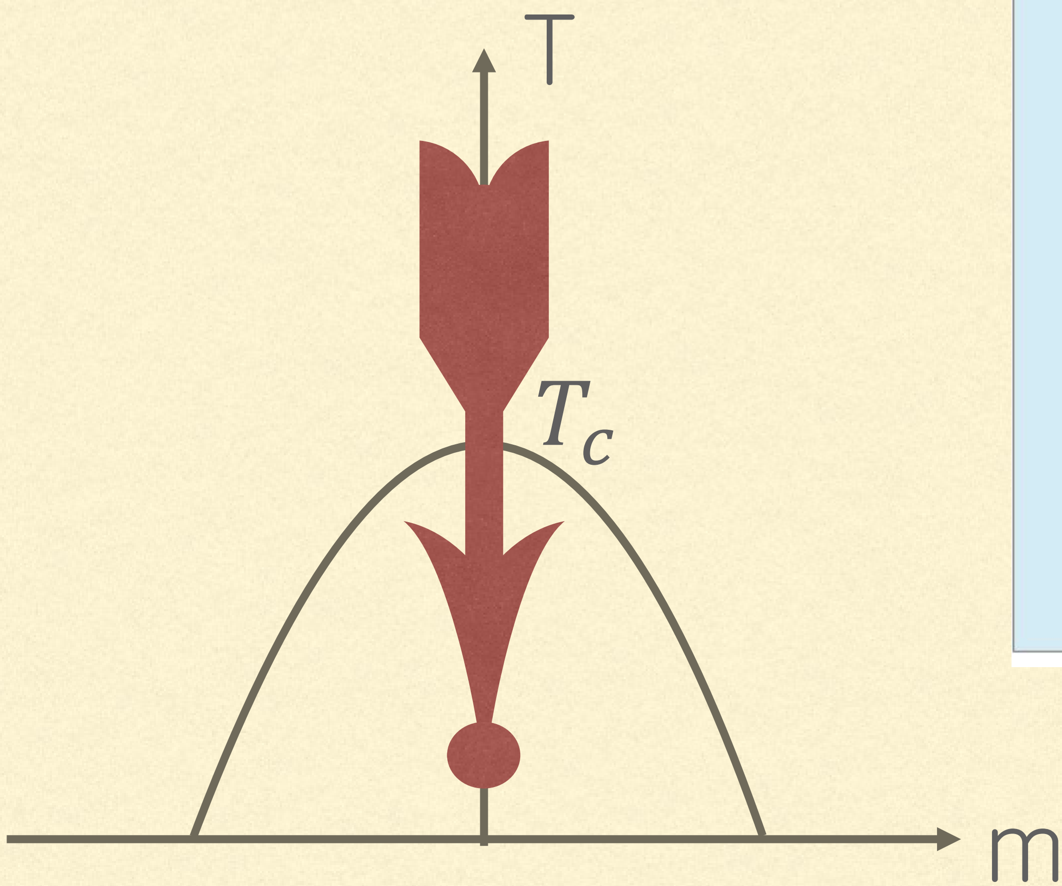
EQUILIBRIUM

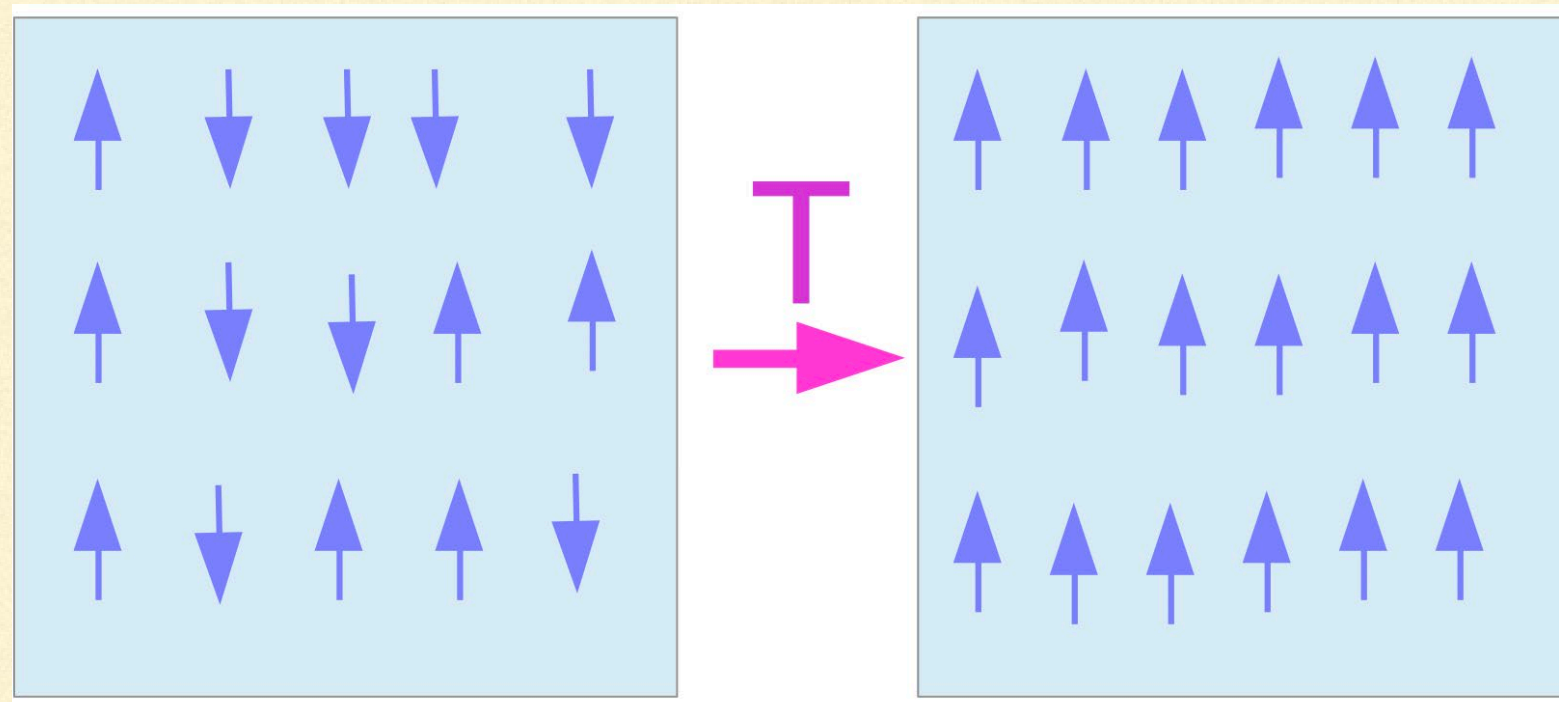




Didascalia

NON-EQUILIBRIUM DYNAMICS (QUENCH)





Didascalia

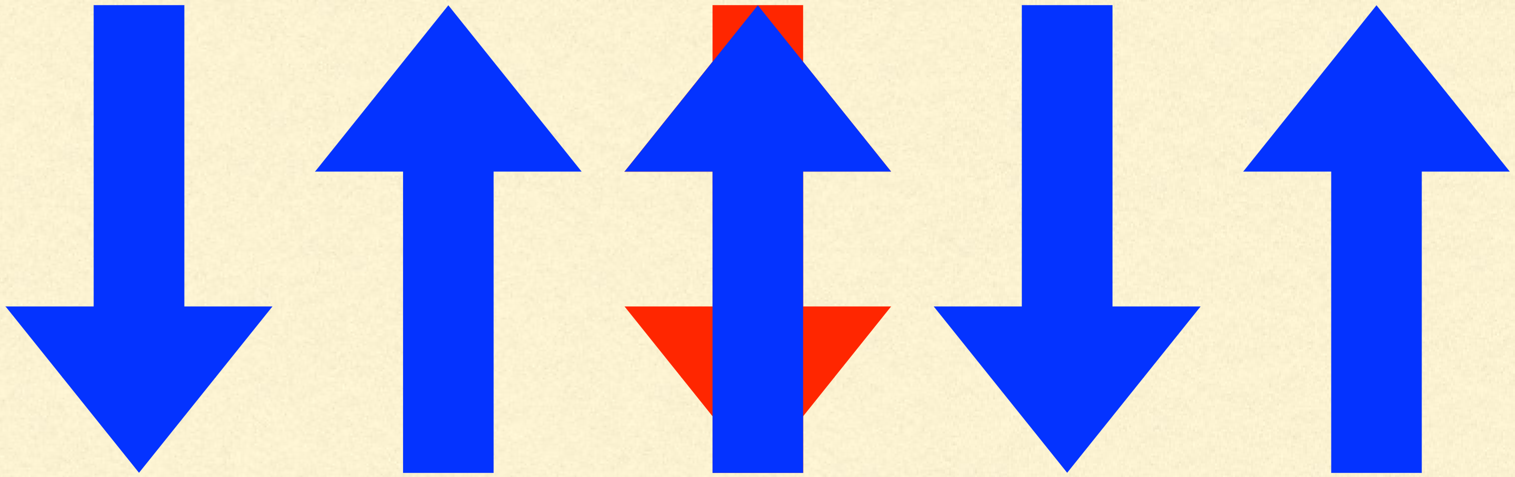
DISCRETE MODELS

ISING

$$-\mathcal{H}(\{s_i\}) = \sum_i \sum_{\vec{r}} J(r) s_i s_{i+\vec{r}}$$

$$\omega_{\uparrow\downarrow}$$

$$\frac{\omega_{\uparrow\downarrow}}{\omega_{\downarrow\uparrow}} = e^{-\beta\Delta E}$$



NCOP

DISCRETE MODELS

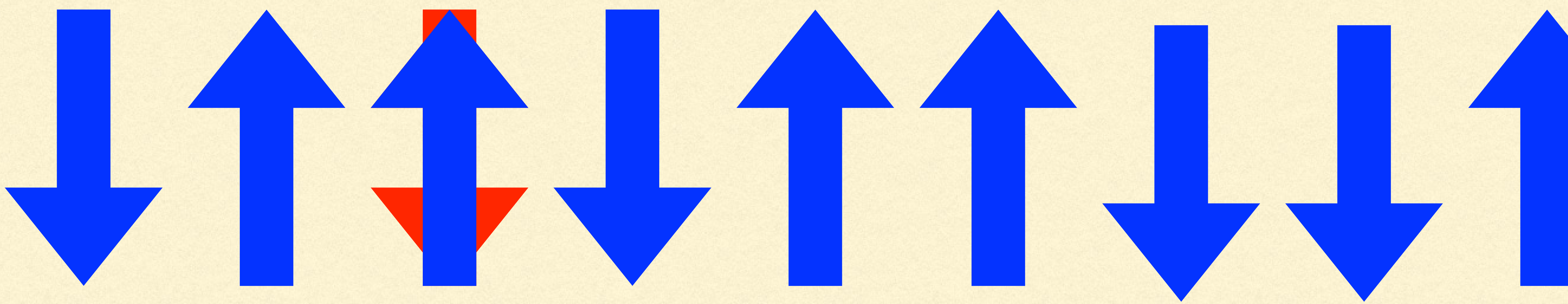
VOTER

$-\mathcal{H}(\{s\}) = \sum_{i}\sum_{\vec{r}} J(r) s_i s_{i+\vec{r}}$

$w(S_i) = \frac{1}{2N}\sum_{r} P(r) \sum_{k=i\pm r} (1 - S_i S_k)$

$\omega_{\uparrow\downarrow}$

$\frac{\omega}{\omega} = e^{-\beta\Delta E}$

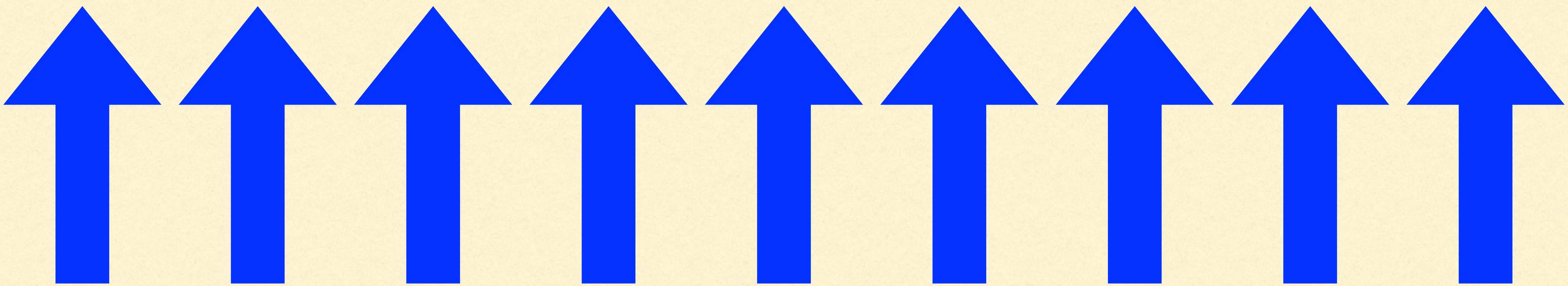


T=0

In D=1 with n.n. maps onto Ising $P(r) \sim J(r)$

Add Temperature

ABSORBING STATES (VOTER)

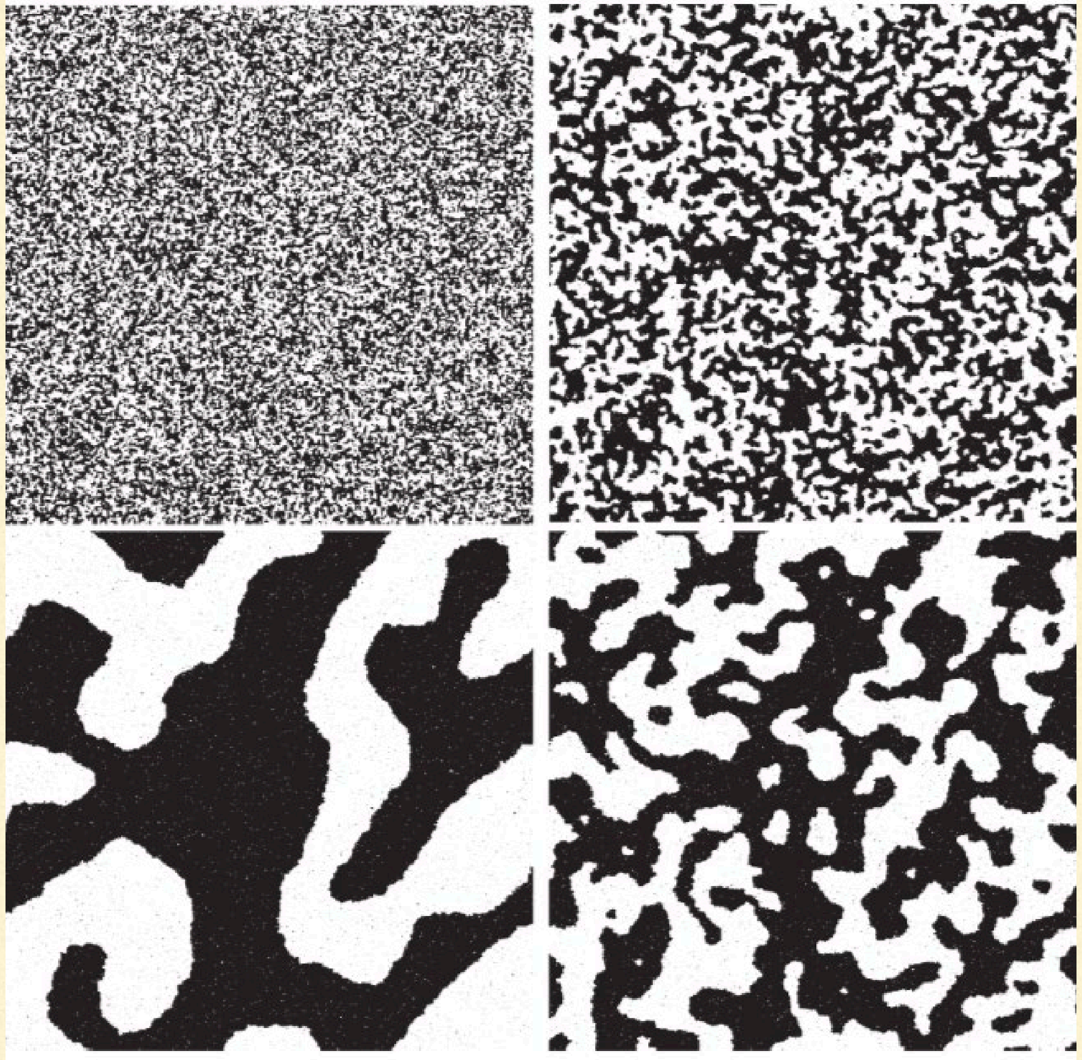


Fully ordered, consensus (up or down)

But, in some cases, there are stationary states without consensus, with lifetime diverging with size

NEXT NEAREST NEIGHBOUR

ISING D=2



Time of order N^z to equilibrate

$$L(t) \sim t^{1/2}$$

BEYOND NEAREST NEIGHBOURS (NEW RESULTS)

$$\mathcal{H}(\{s_i\}) = \sum_i \sum_{\vec{r}} J(r) s_i s_{i+\vec{r}}$$

$$J(r) = \begin{cases} J_0 e^{-r/R} \\ \frac{1}{Z} r^{-\alpha} \end{cases} \qquad Z = \sum_r r^{-\alpha}$$

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Growth laws for phase ordering

A. J. Bray and A. D. Rutenberg
Department of Theoretical Physics, The University, Manchester M13 9PL, England
(Received 5 March 1993)

$$\mathcal{H}[\vec{\Phi}] = \int d^dx\,[(\nabla\vec{\Phi})^2 + V(\vec{\Phi})] + \int d^dx \int d^dr\, \frac{[\vec{\Phi}(\vec{x}+\vec{r}) - \vec{\Phi}(\vec{x})]^2}{r^{d+\sigma}}$$

$\alpha = 0$, Mean field (MF)

$\alpha = \infty$, Nearest neighbourghs (NN)

Only (well organised) results we to our knowledge
(continuum theory asymptotic ($\alpha > d$)

F. Corberi, E. Lippiello e P. Politi, Europhys. Lett. **119**, 26005 (2017).
F. Corberi, E. Lippiello e P. Politi, J. Stat. Phys. **176**(3), 510 (2019).
F. Corberi, E. Lippiello e P. Politi, J. Stat. Mech. 074002 (2019).
F. Corberi, E. Lippiello e P. Politi, Phys. Rev. E **102**, 020102(R) (2020).
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F. Corberi, M. Kumar, E. Lippiello, e P. Politi, Chaos, Solitons and Fractals **173**, 113681 (2023).

BEYOND NEAREST NEIGHBOURS (NEW RESULTS, $1D$ VOTER)

F. C. and C. Castellano, J.Phys. Compl. 5, 025021 (2024

F. C. and L. Smaldone, Phys. Rev. E 109, 034133 (2024)

F. C., S. dello Russo e Luca Smaldone, arxiv.org/abs/24(

$$w(S_i) = \frac{1}{2N}\sum_r P(r) \sum_{k=i\pm r} (1 - S_i S_k)$$

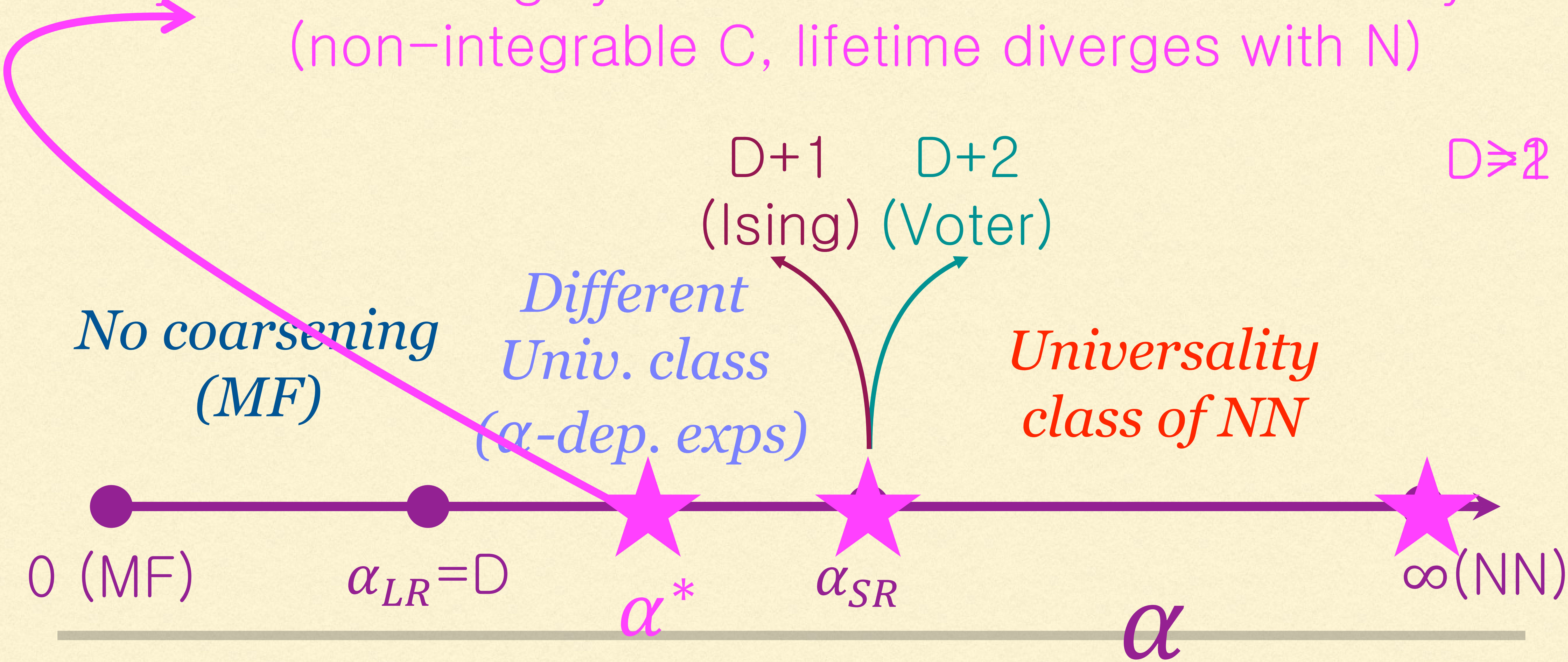
Solvable also for any D

$$\frac{d}{dt}\langle S_i S_j \rangle = -2\langle S_i S_j [w(S_i) + w(S_j)]\rangle$$

$$\frac{d}{dt}C(r,t) = -2C(r,t) + 2\sum_\ell P(\ell)[C(|r-\ell|,t) + C(r+\ell,t)]$$

GENERAL COARSENING PROPERTIES

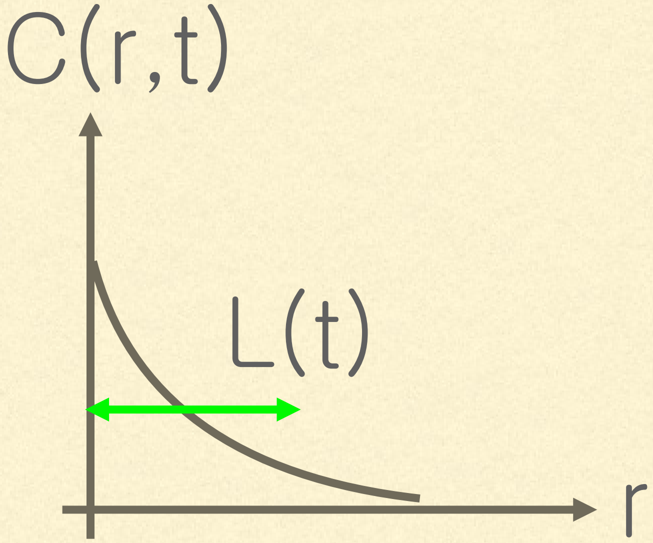
Only for voter: Highly correlated disordered stationary stat
(non-integrable C, lifetime diverges with N)



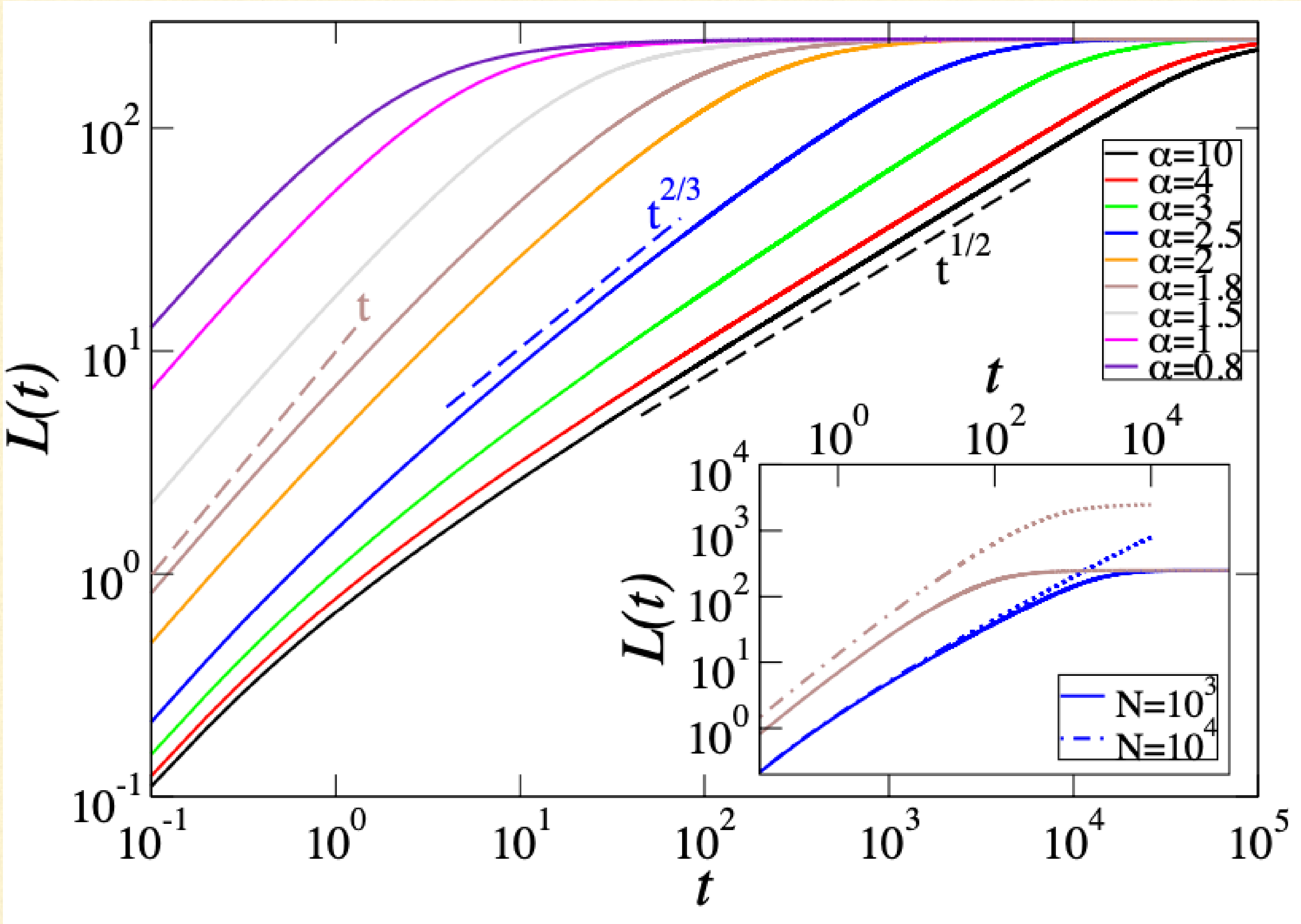
VOTER

$$\frac{d}{dt}C(r,t) = -2C(r,t) + 2\sum_{\ell} P(\ell)\left[C(|r-\ell|,t) + C(r+\ell,t)\right]$$

$$\alpha \le \alpha_{SR}$$

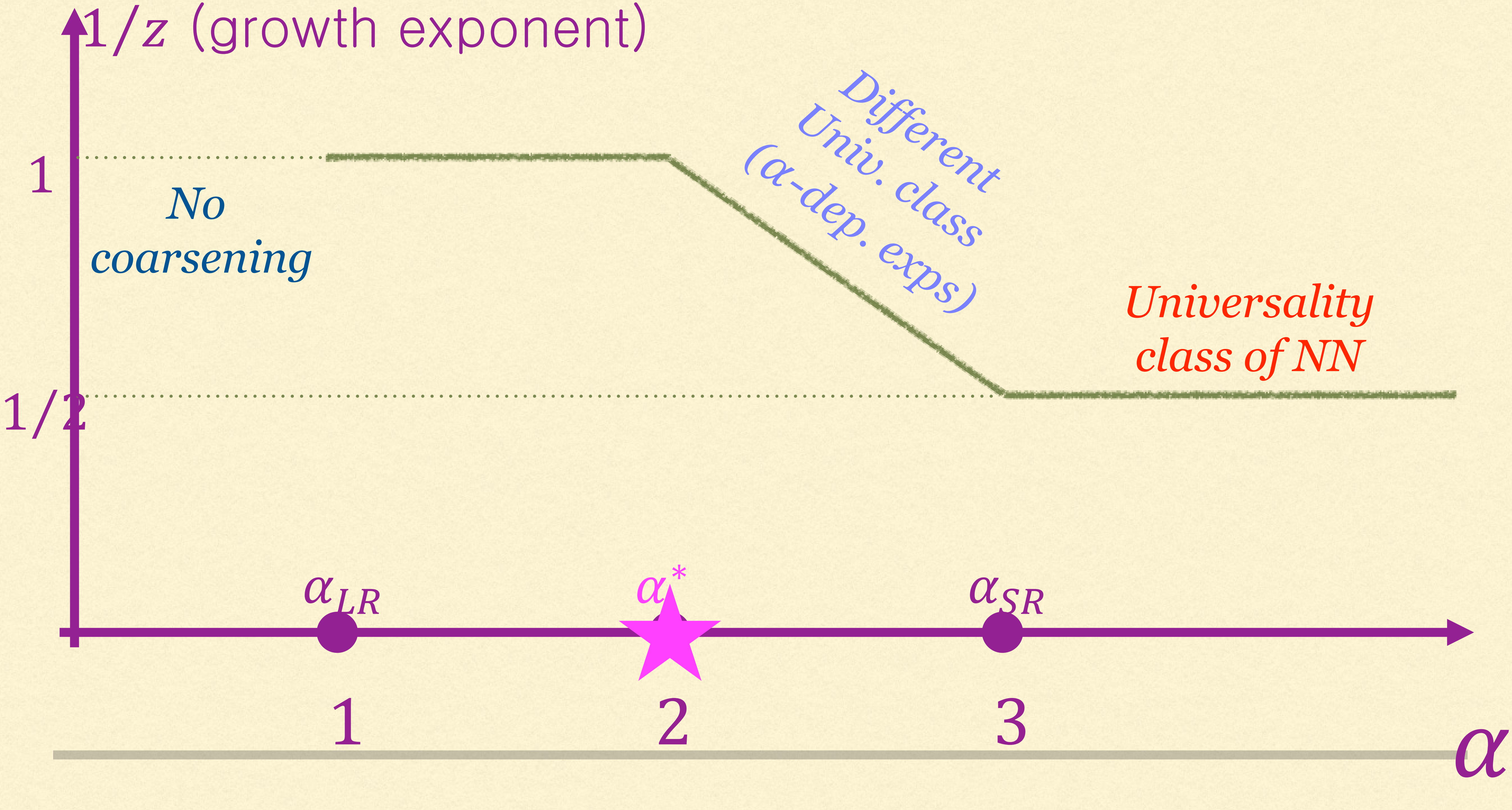


VOTER



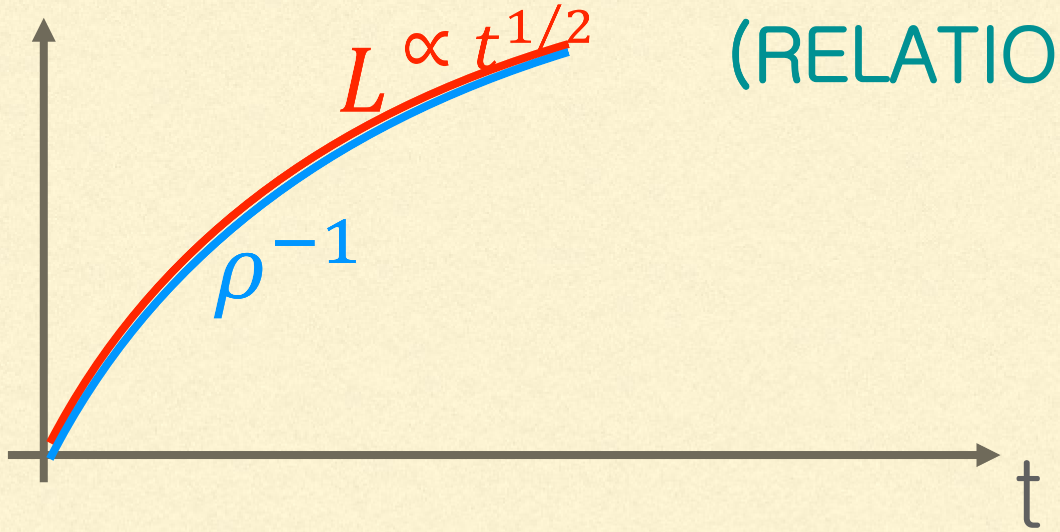
VOTER $D = 1$

SUMMARY OF GROWTH-LAW

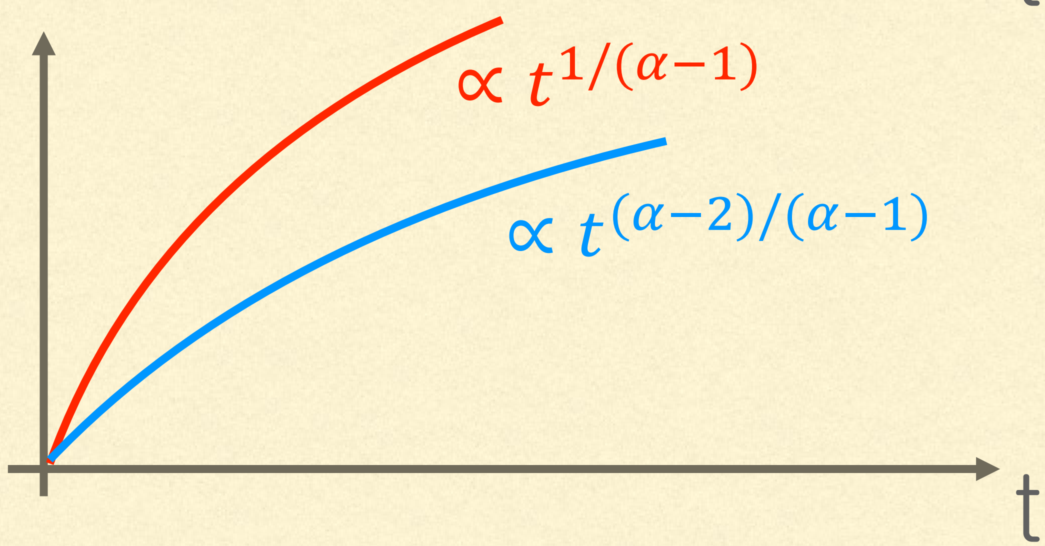


VOTER

(RELATION $L(t) \leftrightarrow \rho^{-1}(t)$)

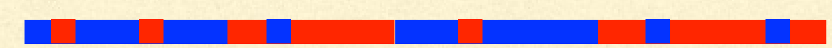


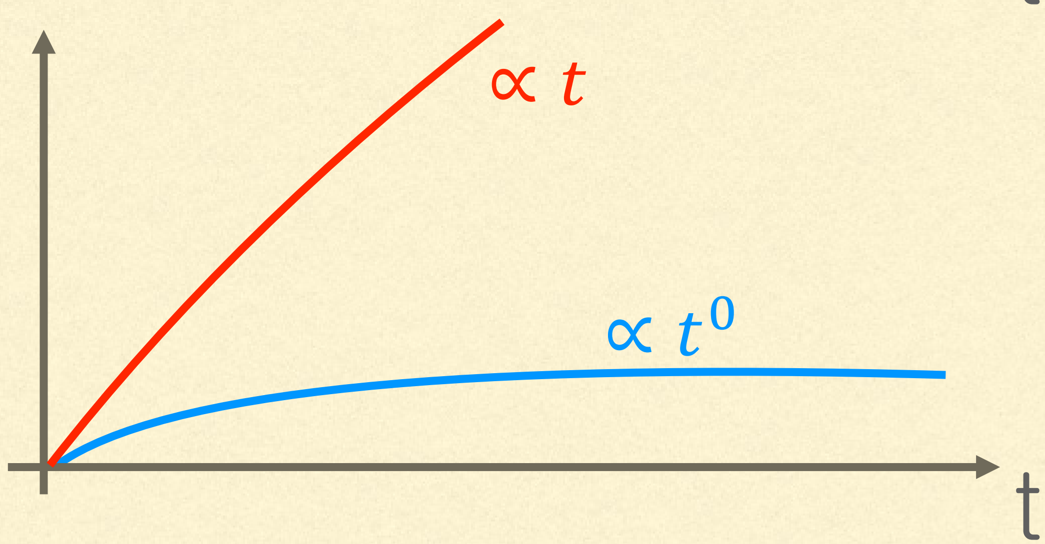
$\alpha > \alpha_{SR}$



$2 < \alpha \leq \alpha_{SR}$

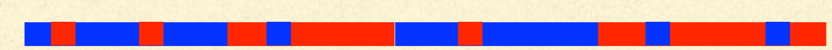
holes number vanishes





$\alpha < \alpha^* = 2$

holes remain



VOTER

(STATIONARY STATES)

$$C_{stat}(r) \propto r^{-(D-2)}, for\ \alpha > \alpha_{SR} (for\, D \geq 3)$$

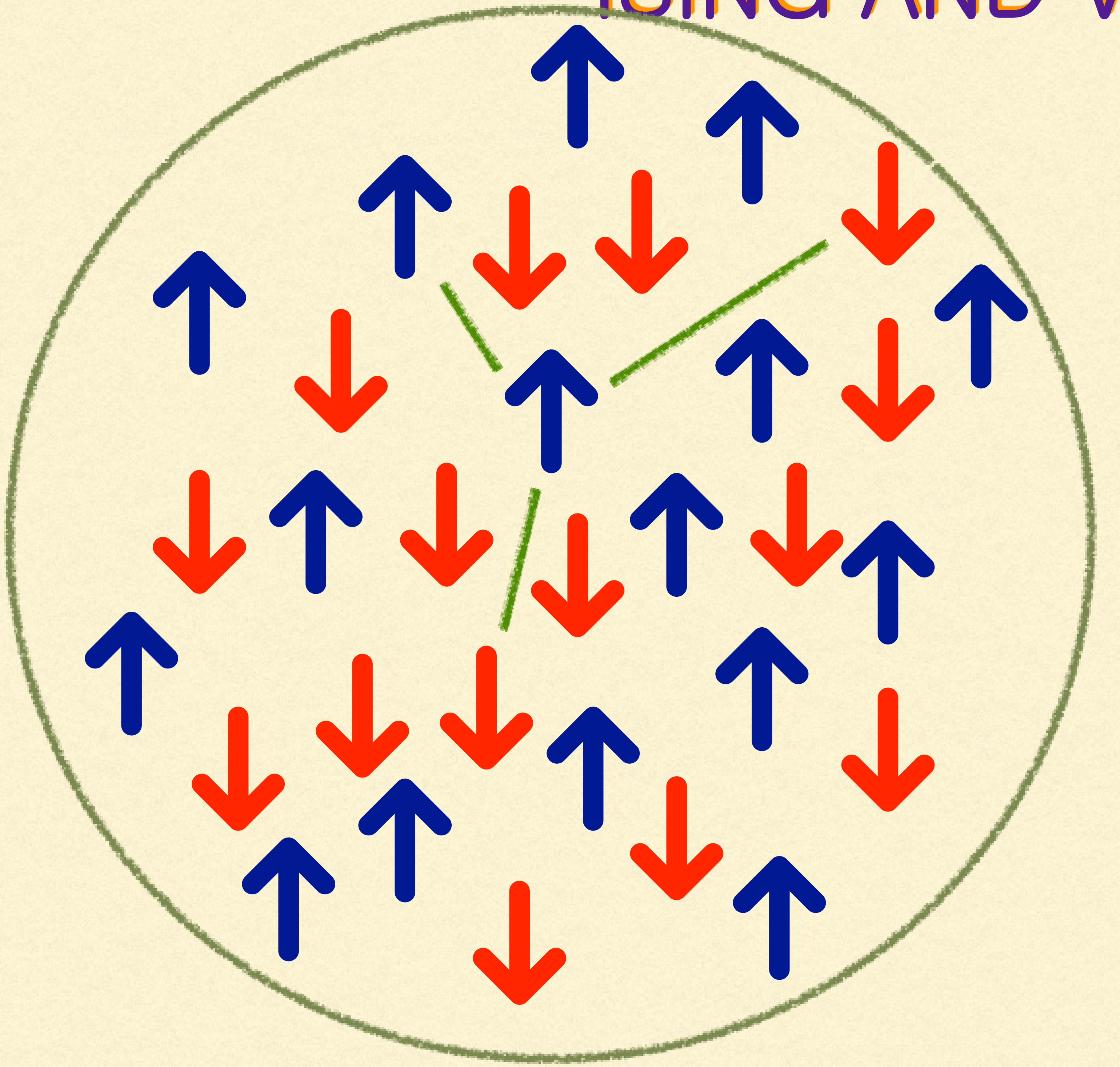
$$C_{stat}(r) \propto r^{-(2D-\alpha)}, for\ \alpha_{LR} < \alpha \leq min(\alpha_{SR}, \alpha^*)$$

$$C_{stat}(r) \propto r^{-\alpha}, for\ \alpha \leq \alpha_{LR}$$

$\lim_{N\to\infty} \int d^D r\, C_{stat}(r) = \infty$ → Strongly correlated

$\lim_{N\to\infty} \int d^D r\, r C_{stat}(r) = \infty$ → reached by coarsening

INTERPOLATING BETWEEN ISING AND VOTER



Ising

Voter

p−voter

INTERPOLATING BETWEEN ISING AND VOTER

- $p = 1$ is voter
- $p = 2$ maps onto voter
- $p \geqq 3$ Ising

Computational time ~ pN

CONCLUSIONS

- Space dependent interactions profoundly modify the coarsening process

- Much more remains to be done beyond determination of L(t

- Thank you