

# How does PDE order affect the convergence of PINNs?

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This talk addresses the inverse relationship between the order of partial differential equations (PDEs) and the convergence of gradient descent in physics-informed neural networks (PINNs) utilizing Rectified Power Unit (RePU) activation. By integrating PDE constraints into the loss function, PINNs inherently require the computation of derivatives up to the order of the PDE. While empirical observations suggest that PINNs often struggle with convergence for high-order or high-dimensional PDEs, a thorough theoretical explanation has been lacking. This talk provides a theoretical foundation for these challenges, showing that the gradient flow is less likely to converge as the PDE order increases. Additionally, we explore the impact of dimensionality on convergence, which is further compounded by higher PDE orders. To mitigate these issues, we propose a variable splitting technique that decomposes high-order PDEs into a system of lower-order PDEs. We demonstrate that this approach improves the likelihood of convergence to the global optimum. Numerical experiments are presented to support our theoretical findings and highlight the practical implications of our method.