Exploring the nature of the phase transition with Terminated Supercooling

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Tomasz Dutka

고등과학원
KOREA INSTITUTE FOR ADVANCED STUDY

TPD, Tae Hyun Jung, Chang Sub Shin
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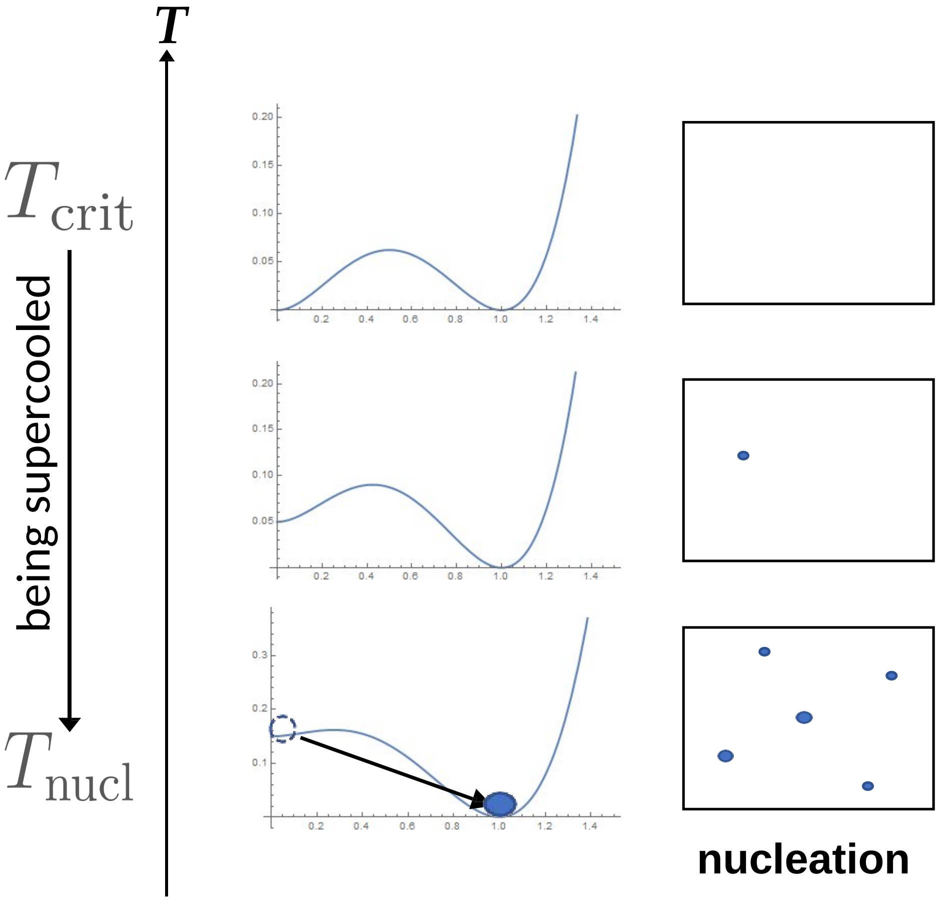
General features of FOPTs

Supercooled: $T_{\rm nucl} \ll T_{\rm crit}$

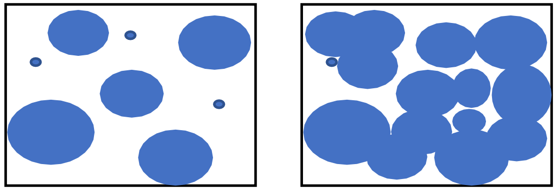


$\Gamma_n \neq 0, \Gamma_n \ll H^4$

Bubble nucleation is possible, but bubbles are diluted away by the spacetime expansion outside bubbles

$\boldsymbol{\Gamma_n \simeq H^4}$

One bubble nucleation per one Hubble patch within one Hubble time



nucleation **expansion** **collision**

Figures courtesy of Tae Hyun Jung

Supercooled First Order Phase Transitions

Recent popularity in studying supercooled phase transitions in cosmological contexts.

- Can produce a secondary, smaller period of ‘thermal inflation’ which can help dilute the abundances of some long-lived BSM particles which could spoil the predictions of BBN, e.g. gravitino, moduli… **hep-ph/9510204, hep-ph/9602263, 0801.4197, 1412.7814....**

- Can produce sizeable gravitational wave signals, through collision and motion of true-vacuum bubbles within the thermal plasma. **1809.08242, 1811.11169, 2007.15586, 2208.11697, 2303.02450....**

- Supercooled phase transitions can be used as explanations for other problems in particle cosmology, e.g. PBH production, dark matter filtering, baryogenesis catalysts.. **1912.04238, 2110.04271, 2206.04691, 2206.09923, 2304.00908, 2305.10759....**

- Constructing supercooled models is not difficult, just require relatively flat potentials, most commonly classically scale invariant or SUSY models.

Simple Toy Model:

A classically scale-invariant potential

$$V_0(\Phi, \dots) \supset \lambda \, |\Phi|^4$$

develops a flat direction at some scale $\lambda(\mu_*) = 0$.

Coupling to other fields (e.g. gauge boson - Coleman-Weinberg, scalar portal - Gildener-Weinberg) induces radiative corrections Coleman, Weinberg 1973, Gildener, Weinberg 1976

$$V_{CW}(m_i^2(\phi)) = (-1)^{2s_i} g_i \frac{m_i^4(\phi)}{64\pi^2} \left[\log\left(\frac{m_i^2(\phi)}{\mu^2} \right) - c_i \right]$$

A minimum forms radiatively $(\langle \Phi \rangle \sim \mu_*)$, $\Delta V / \langle \Phi \rangle^4 \ll 1$

$$V''(\phi)\Big|_{\phi=0} = 0$$

Thermal Corrections

Coupling of field to thermal bath induces corrections

$$V(\phi, T) = V_0(\phi) + \sum_i V_{CW}(m_i^2(\phi)) + \sum_i V_T(m_i^2(\phi))$$

$$V_T(m_i^2(\phi)) = \pm \frac{g_i}{2\pi^2} T^4 J_{\mathrm{B,F}}\left(\frac{m_i^2(\phi)}{T^2}\right)$$

$$J_{\mathrm{B,F}}(y^2) = \int_0^\infty dx\ x^2 \log\left[1 \mp \exp\left(-\sqrt{x^2+y^2}\right)\right]$$

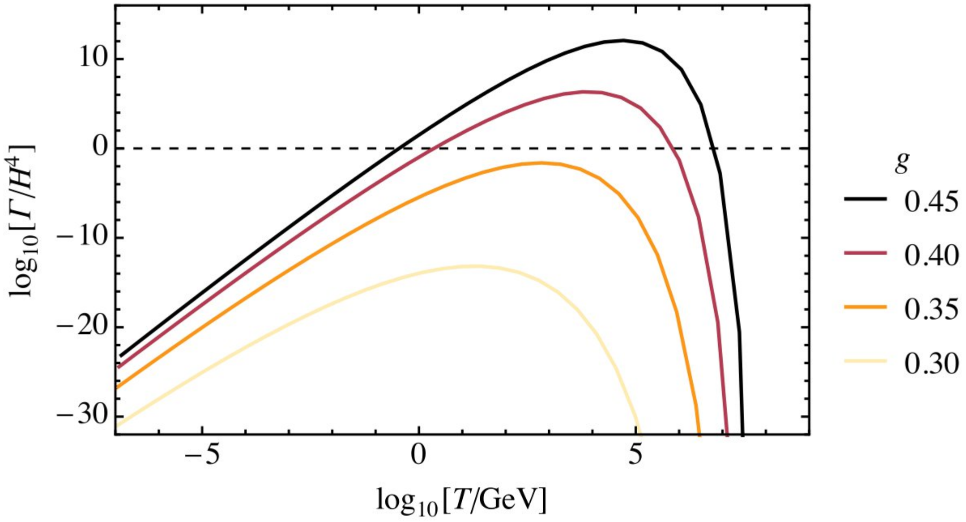
Induces temperature-dependent corrections, particularly for small field values including the extremum at the origin

$$V''(\phi)\Big|_{\phi=0} = 0 + \frac{g^2}{12} T^2 > 0$$

A minimum around the origin forms. At some temperature will become meta-stable. Tree-level conformal invariance -> barrier remains for entire thermal history

Dangers with ‘scaleless’ Supercooling

Barrier persists eternally. Can lead to scenarios with ‘eternal inflation’ for not-that-small values of couplings (bubble nucleation cannot proceed due to expansion).



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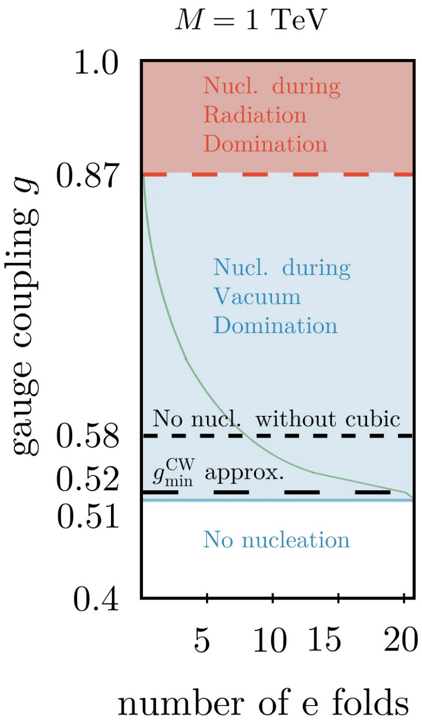

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Introduce an Additional Scale

Realistically, completely reasonable to expect the existence of additional scales (e.g. soft breaking terms) which can change the simple picture somewhat

E.g.

$$V_0 = \lambda^2 \phi^2 S^2 + M_s^2 S^2$$

$$V_{\rm CW}(\phi) \to \frac{(\lambda^2\phi^2 + M_s^2)^2}{64\pi^2}\left[\log\left(\frac{\lambda^2\phi^2+M_s^2}{\mu^2}\right) - \frac{3}{2}\right] \qquad V_T(\phi) \to \frac{T^4}{2\pi^2} J_B\left(\frac{\lambda^2\phi^2+M_s^2}{T^2}\right)$$

Destabilisation effect on the origin at zero temperature

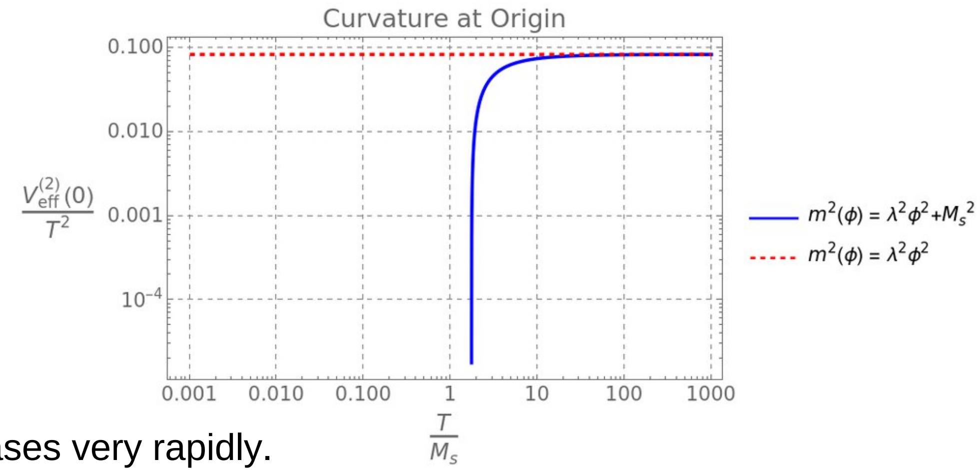
$$V''_{\rm eff}\Big|_{\phi=0,T=0} = \frac{\lambda^2 M_s^2}{16\pi^2}\left(2\log\left(\frac{M_s}{\mu}\right) - 1\right) < 0$$

Introduce an Additional Scale

The thermal corrections to the curvature are (obviously) significant at sufficiently large temperatures

$$V''_{\text{eff}}\Big|_{\phi=0} = \frac{\lambda^2}{16 M_s \pi^2}\left[M_s^3\left(2\log\left(\frac{M_s}{\mu}\right)-1\right)+8T^3 J'_B\left(\frac{M_s}{T}\right)\right]$$

At high temperatures behaviour unchanged but needs to be a sign flip at some scale -> phase transition always completes. Terminated Supercooling.



In fact, barrier decreases very rapidly.

Implications for the Transition

- (i) $T_{\mathrm{phase}} > M_s$

  If the phase transition can complete, and completes at large temperatures, extra mass scale does not change the phase transition dynamics.

- (ii) $T_{\mathrm{phase}} < M_s$

  Field remains trapped until after the barrier disappears. Will roll down to the true vacuum. Hard to believe as field is coupled relatively strongly to the thermal bath.

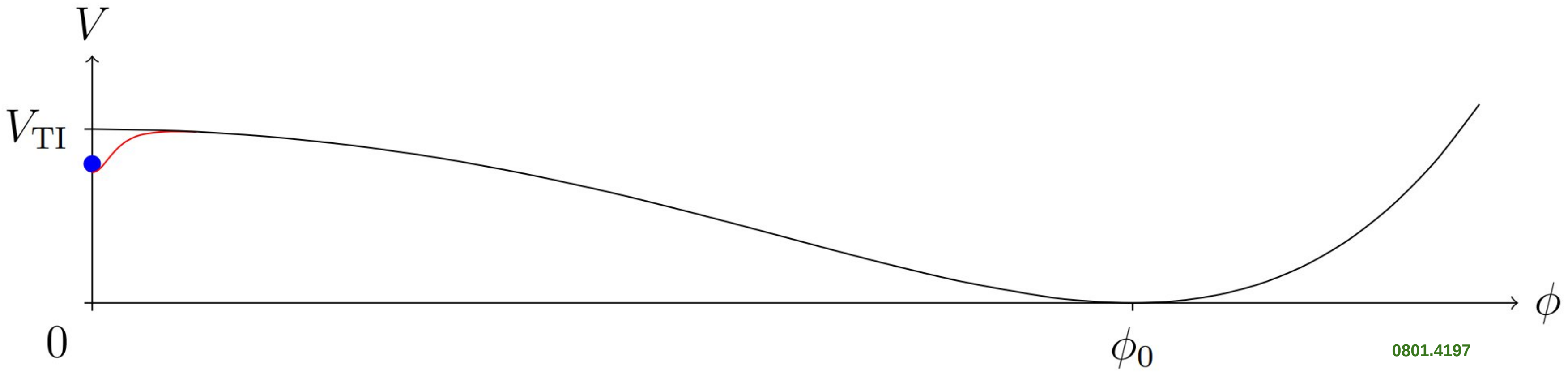
- (iii) $T_{\mathrm{phase}} \simeq M_s$

  The phase transition proceeds when a potential barrier still exists around the meta-stable origin. **The barrier is rapidly disappearing and thermal fluctuations ∝ T persist.**

  Still bubble formation? Prospects for gravitational wave signals?

Barrier Size vs Temperature

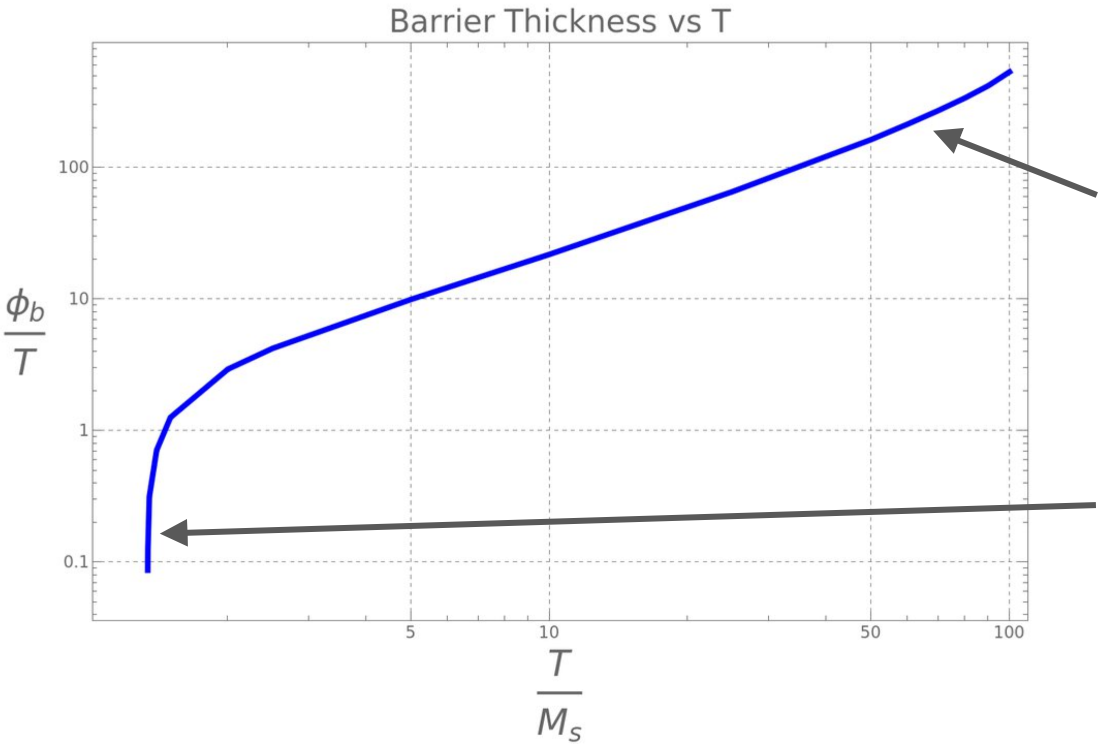
Consider now a SUSY motivated picture of thermal inflation with some soft SUSY breaking scale for a more concrete example of the problem.



At relatively large temperatures the scalar field is held at the origin (from finite temperature effects) and there is a very wide potential barrier.

Barrier size vs temperature

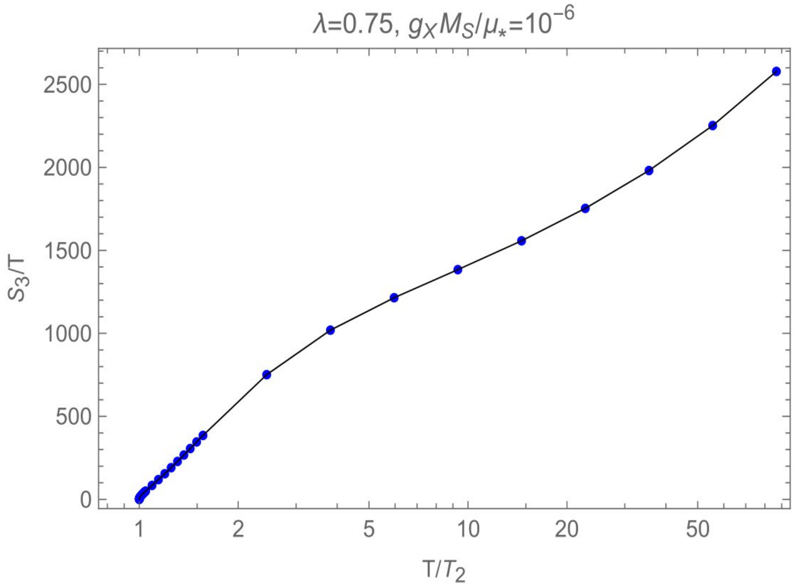
Consider now a SUSY motivated picture of thermal inflation with some soft SUSY breaking scale for a more concrete example of the problem.

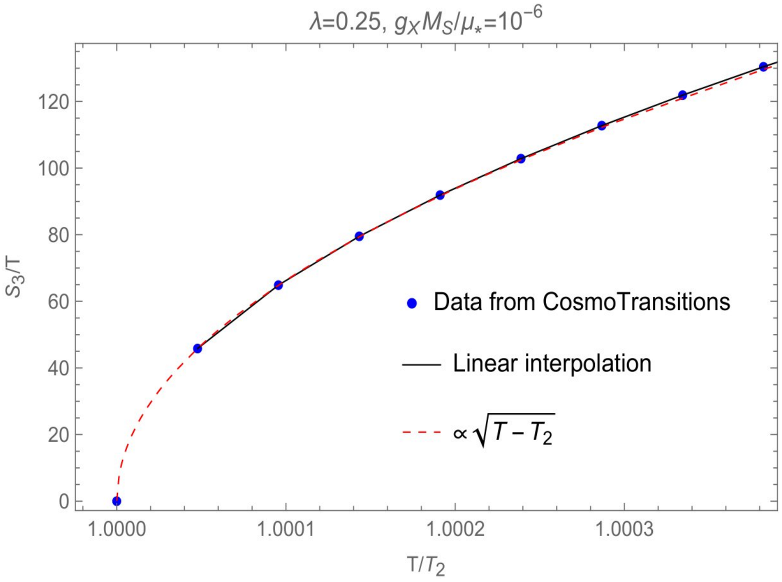


At high temperatures the field is strongly trapped. Good candidate for supercooling. Can generate a number of e-folds.

Barrier disappears so eternal inflation is prevented

When would Bubbles Nucleate?



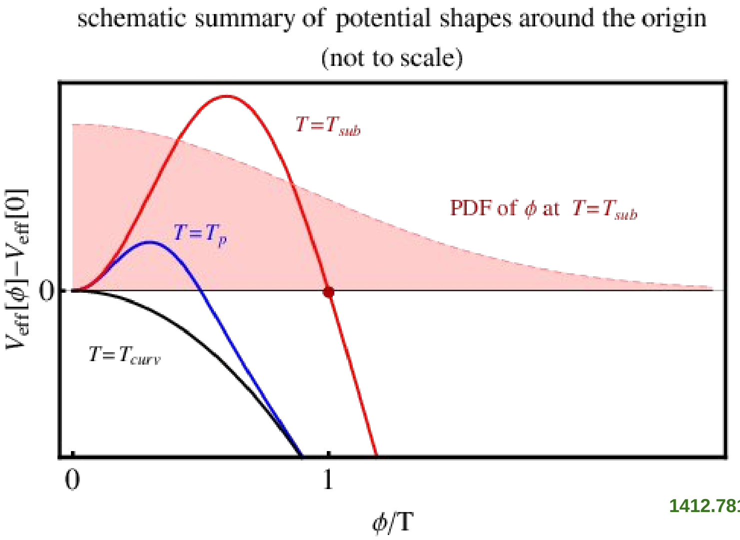


Roughly speaking $\frac{S_3}{T} \simeq \mathcal{O}(100)$ is required for sizeable bubble nucleation.

In such models, this coincides with the period where the thermal barrier has a thickness comparable to temperature (actually, almost certainly less).

Thermal fluctuations vs Bubble Formation

As thermal fluctuations will lead to field-value distributions beyond the thermal barrier, it has been conjectured that bubble formation will not occur.



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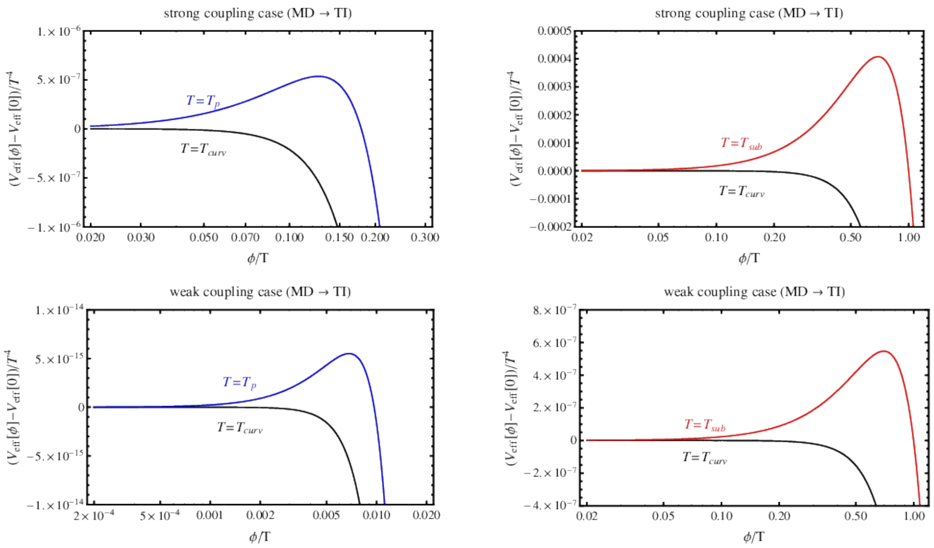


Instead this might lead to something like ‘phase-mixing’ where there is an inhomogeneous steady-state coexistence of the two phases -> gravitational signals from bubble collisions would then not occur.

The case for Bubble Formation

A phase transition proceeding by bubble formation may still be a likely explanation:

- Potential is very flat $|V'(\phi)| < T^3\Big|_{\phi<\mathcal{O}(10)T}$

  Expectation: field remains trapped around the origin (even though field value is beyond the barrier thickness), due to thermal random walk, until a critical bubble can form.

- The three-dimensional Euclidean action of the O(3) symmetric bounce solution (critical bubble) remains large e.g.

$$\frac{S_3}{T}\Big|_{\phi_b \simeq 0.1T} > \mathcal{O}(10)$$

  Might naïvely expect that the action should reflect that critical bubble formation becomes ill-defined but still might expect deviations from this prediction.

Lattice Simulation with Thin Barriers

We wish to address this question by simulating a potential with similar characteristics numerically on a lattice and determine the nature of the phase transition.

Within the simulation we assume we have a scalar field coupled to a thermal bath with temperature T and a potential

$$V(\Phi) = \frac{1}{2}m^2\Phi^2 - \frac{\lambda}{4!}\Phi^4 + \frac{\epsilon}{6!}\Phi^6$$

subject to the constraint

$$m^2 = \frac{\lambda}{12}\phi_b^2 \ , \quad \phi_b = aT$$

such that the potential barrier width and height is fixed: $V(0) = V(\phi_b)$.

Lattice Simulation with Thin Barriers

Although this potential has similar characteristics to potentials appearing in thermal inflation the main advantage for lattice simulations is a accurate semi-analytic expression for the bounce action

$$V(\Phi) = \frac{1}{2}\frac{\lambda}{12}\phi_b^2\Phi^2 - \frac{\lambda}{4!}\Phi^4 + \frac{\epsilon}{6!}\Phi^6 \qquad \left(\epsilon T^2 < 1\right)$$

This can allow for (ideally) a controllable and predictable rate of bubble nucleation to compare with the results of the simulation:

Linde, 1983

$$\frac{S_3}{T} \simeq 19\sqrt{3}\frac{\phi_b}{\sqrt{\lambda}T}$$

Aside: There is also a difficult to calculate the numerical prefactor entering in the rate which we find using numerical packages **2308.15652**

$$\log(\Gamma_{\rm nuc}) = \log(A) - \frac{S_3}{T}$$

Including thermal fluctuations

Introduce thermal fluctuations by incorporating random-walk effects to the equation of motion for the scalar field (Langevin dynamics) e.g. hep-ph/9410235, hep-lat/9607026, 0711.1866, 2407.06263,

$$\partial_t^2 \Phi(\mathbf{x}, t) + \eta \partial_t \Phi(\mathbf{x}, t) - \nabla^2 \Phi(\mathbf{x}, t) + \partial_\Phi V(\Phi) = \xi(\mathbf{x}, t)$$

with a friction and (colourless) stochastic noise term:

$$\langle \xi(\mathbf{x}, t) \xi(\mathbf{x}', t') \rangle = D \delta(t - t') \delta^3(\mathbf{x} - \mathbf{x}')$$

related through the fluctuation dissipation theorem

$$D = 2T\eta$$

Fast equilibration

The friction coefficient is relevant for the timescale for the simulation to reach equilibrium which we expect to be related to temperature (a strong enough coupling to thermal bath is required anyway for supercooling). 1412.7814

$$\eta \sim T$$

Implies that the simulation equilibration time scale should be significantly faster than the expansion rate

$$\frac{t_{\rm equil}}{\rm Hubble} \sim \frac{\eta^{-1}}{H^{-1}} \ll 1$$

In both a radiation dominated or vacuum dominated scenario. A fairly robust assumption for most choices of friction coefficient.

Therefore we neglect the effect of Hubble expansion within the simulation.

Some 3D-simulation specifics

Potential shape characteristics are related to the temperature of the thermal bath so the lattice values are also chosen as a function of temperature. E.g.

$$\delta t \sim \frac{\mathcal{O}(0.01)}{T} \qquad a \sim \frac{\mathcal{O}(0.5)}{T} \qquad L \sim \frac{\mathcal{O}(100)}{T}$$

In general we perform a number of different simulations with

$$N_{\text{lattice}}^3 = \left(\frac{L}{A}\right)^3 \sim (200 - 400)^3$$

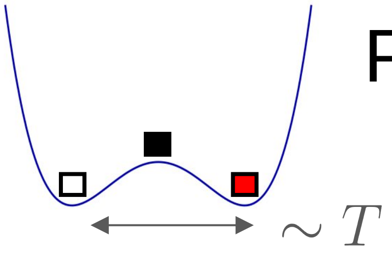
with good agreement between different simulations of the same potential.

We start each simulation with the unphysical initial condition

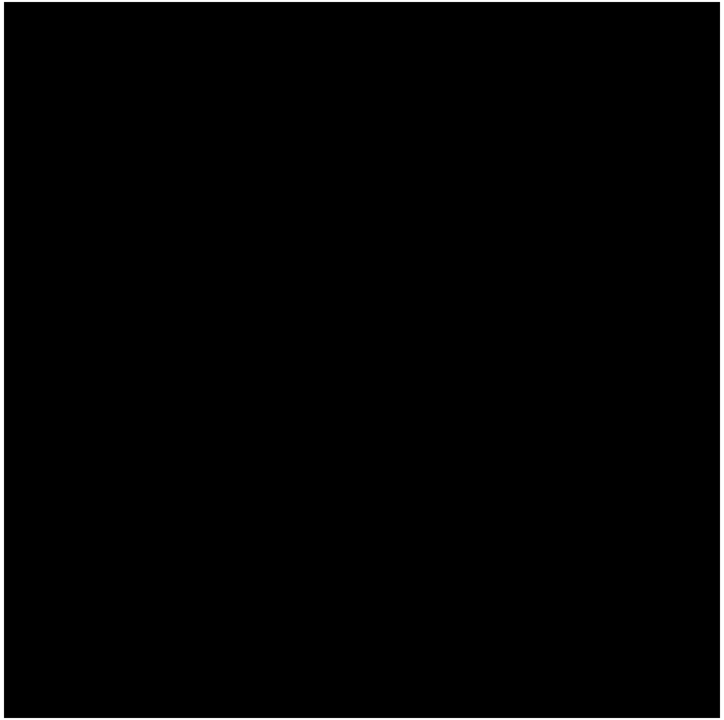
$$\phi(\mathbf{x}, 0) = 0, \dot{\phi}(\mathbf{x}, 0) = 0$$

but we find the time for thermalisation around the origin is always faster than the (eventual) transition.

Some simulation results



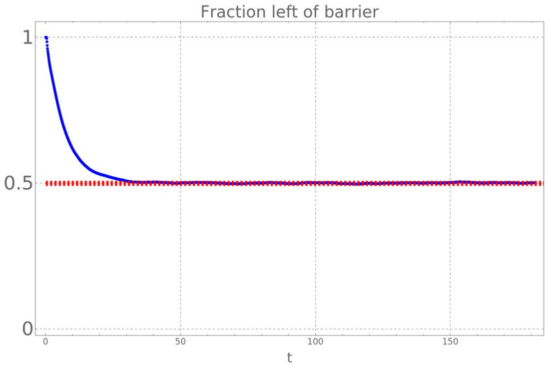
Phase-mixing Demonstration (diff. pot.)



To demonstrate qualitative behaviour of a scenario with phase mixing.

Potential has two degenerate minima with a short (in temperature) distance between them.

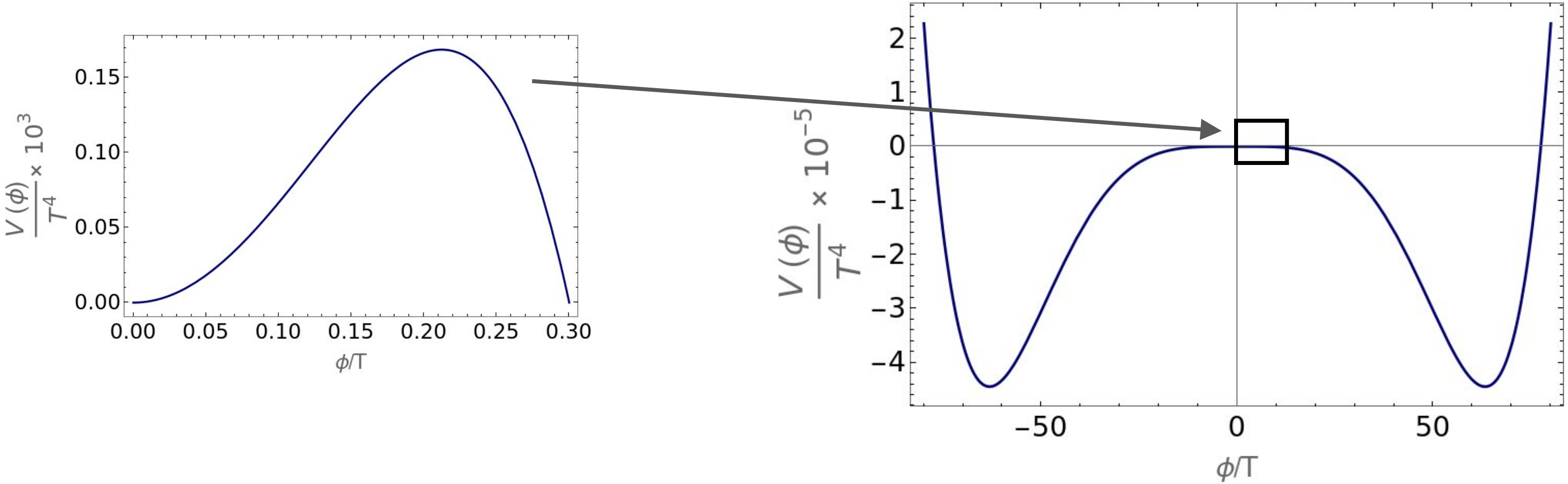
Previous candidate for the EW phase transition. **hep-ph/9410235**



Supercooling-inspired Potential on the Lattice

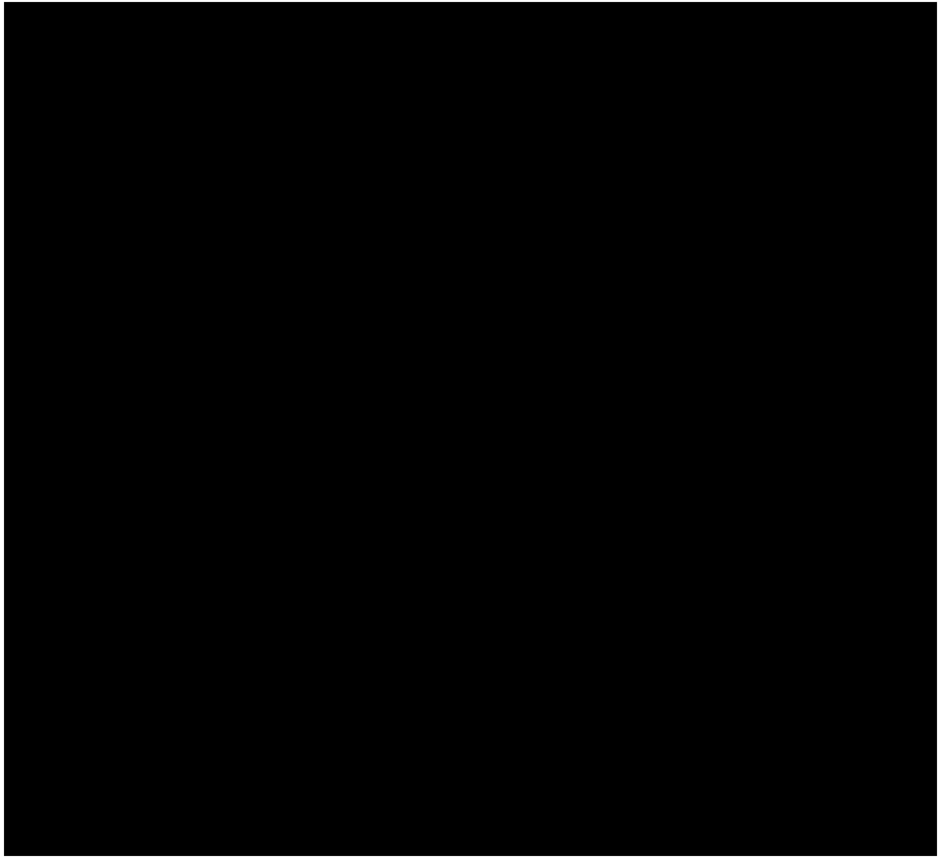
We performed a number of simulations of our toy-model potential taking the potential barrier to be an order of magnitude smaller than the temperature scale of the thermal bath

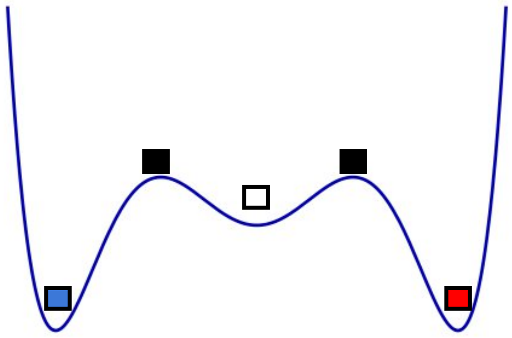
$$\phi_b \sim (0.1 - 0.5)T$$

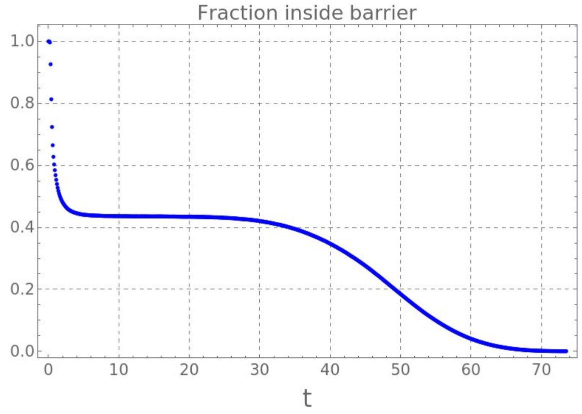


Aside: This toy-potential features two degenerate minima, realistic models should avoid this, e.g. U(1) complex scalar, etc.

Supercooling-inspired Potential on the Lattice



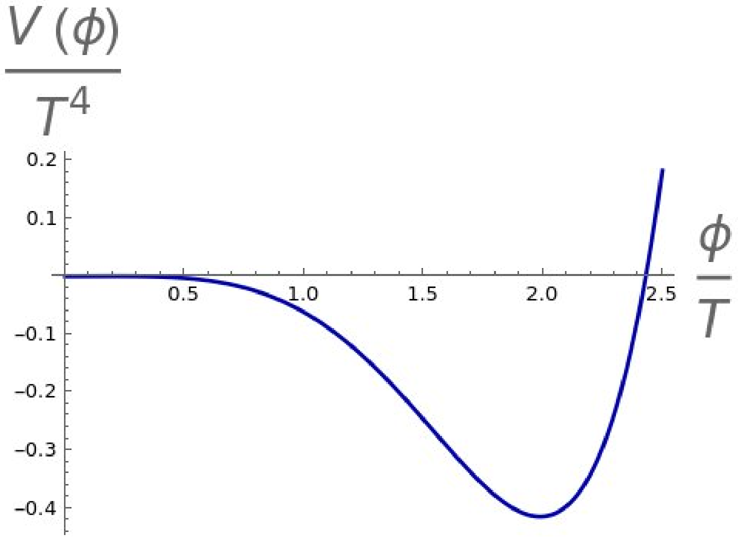




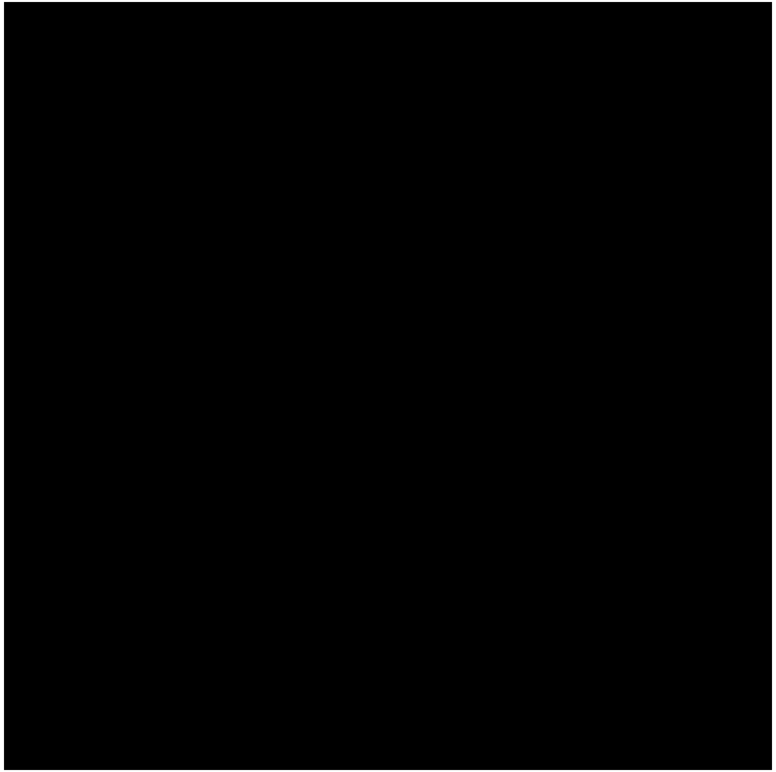
Transitioning to a phase-mix-like scenario

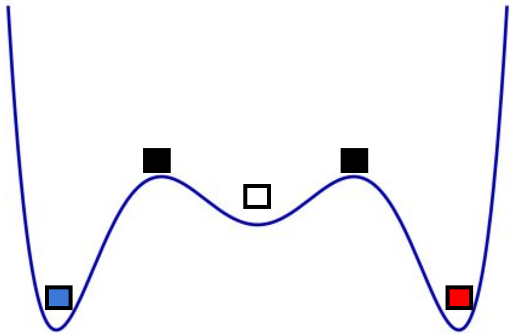
- Taking $\epsilon T^2 \gg 1$ causes the stable minimum to move towards the origin.
- **This does not provide a good example of a supercooled transition** but can observe deformations of the bubble formation hypothesis in such cases.

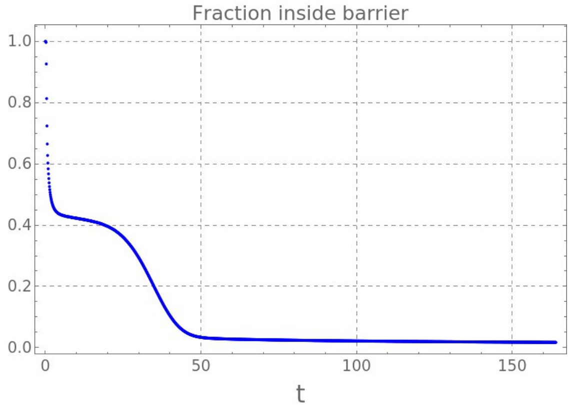
e.g.



Transitioning to a phase-mix-like scenario



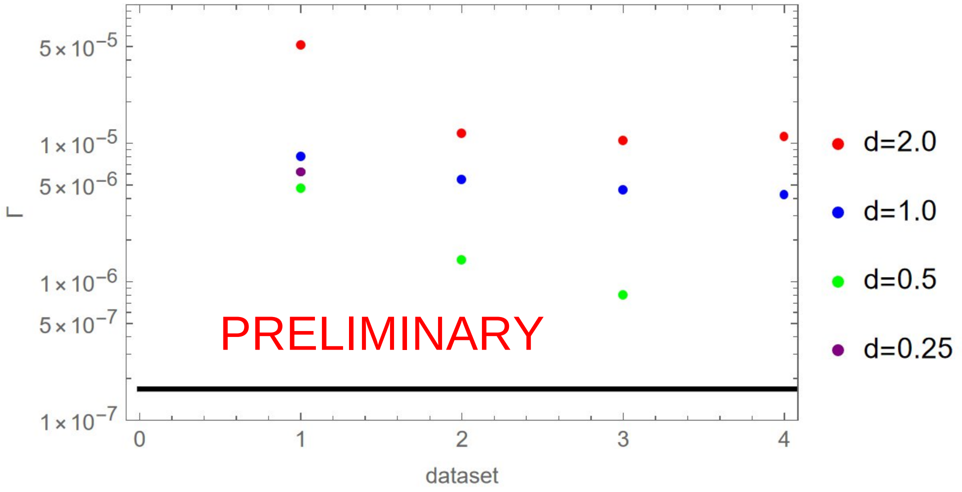




Ideally would like to define some criteria when bubble-formation description will be valid - ongoing work.

Future Aspirations

- These simulations appear to confirm a bubble-catalysed prediction for the PT, qualitatively.
- There is current work in progress to use these simulations to estimate and compare the numerical phase transition dynamics compared to the analytic predictions.
- Currently the simulation has very positive behaviour as a function of the potential shape e.g. taking potentials with slower predicted nucleation -> bubble formation takes parametrically longer. Stability of the simulation for different lattice choices, etc.
- Ongoing improvements..



Conclusion

- Our lattice simulations seem to confirm that supercooled phase transitions which exhibit a rapid termination in their supercooling, still proceed via bubble formation.

- Robust simulation results when lattice dimensions and parameters in the scalar potential are varied.

- Simulation exhibits very good qualitative behaviour compared to the predictions of the bubble nucleation rate for the given potential. Ongoing work to verify simulation results compared to analytic/numerical predictions.

- In future would like to extend these simulations to realistic potentials to obtain information about phase-transition parameters, e.g. bubble radius at collision, nucleation rates, grav. wave signals etc..

Fin.

Backup

Some simulation details

In order to efficiently numerically evolve Φ in time we perform a three-dimensional simulation of a 3-dimensional effective field theory containing the light bosonic field(s) (the zero Matsubara mode), which we write as ϕ with mass-dimension 1, with the remaining heavy fields integrated out

The parameters within the dimensionally reduced theory, from herein denoted as X3, are related to the 3 + 1 dimensional quantum field theory through powers of temperature, e.g.

$$m_3^2 = m^2$$
$$\lambda_3 = \lambda T$$
$$\epsilon_3 = \epsilon T^2$$

The other relevant lattice implementations within the Langevin simulation are

$$\nabla^2 \phi(\mathbf{x}) = \frac{1}{12a^2} \sum_i \left(-\phi(\mathbf{x}-2i) + 16\phi(\mathbf{x}-i) - 30\phi(\mathbf{x}) + 16\phi(\mathbf{x}+i) - \phi(\mathbf{x}+2i)\right)$$

$$\langle \xi(\mathbf{x},t)\xi(\mathbf{x}',t')\rangle = D\delta(t-t')\delta^3(\mathbf{x}-\mathbf{x}') \overset{\text{lattice}}{\rightarrow} \frac{D}{(\Delta t)a^3}\delta_{x_i,x_j}\delta_{t_a,t_b} \qquad \xi(\mathbf{x}_i, t_a) = \sqrt{\frac{D}{\Delta t a^3}}\mathcal{G}_{i,a}$$

Some Simulation Details

Mapping the 4D continuum theory to this 3D simulation requires some care due to UV divergences associated with the lattice spacing a: e.g., hep-lat/0209144

$$V(\Phi) \to V_3(\phi) = \frac{1}{2} Z_\phi Z_m (m_3^2 + \delta m_3^2)\phi^2 - \frac{1}{4!} Z_\phi^2 (\lambda_3 - \delta\lambda_3)\phi^4 + \frac{1}{6!} Z_\phi^3 (\epsilon_3 + \delta\epsilon_3)\phi^6$$

$$\nabla^2 \Phi \to Z_\phi \nabla^2 \phi$$

$$\delta m_3^2 = -\frac{\Sigma\lambda_3}{8\pi a} + \mathcal{O}(\lambda_3^2 a^0)$$

$$\delta\lambda_3 = -\frac{\Sigma\epsilon_3}{8\pi a} + \mathcal{O}(\epsilon_3^2 a^0) + \mathcal{O}(\lambda_3^2 a)$$

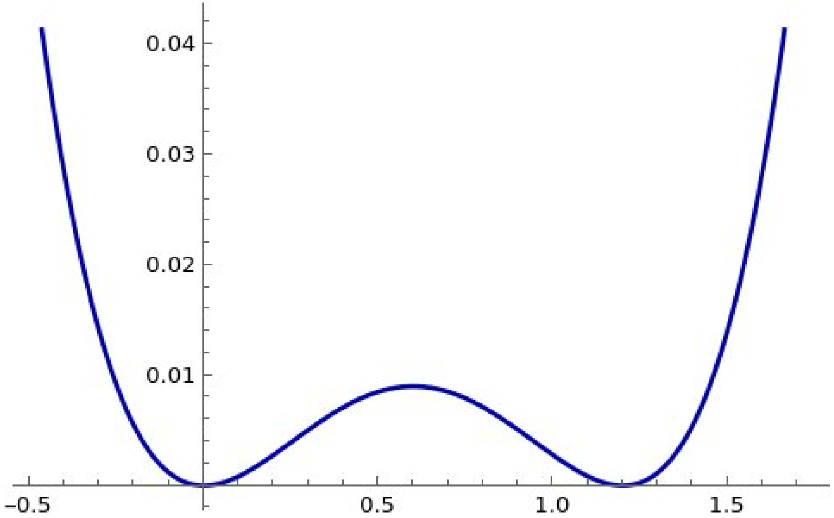
$$Z_\phi = 1 + \mathcal{O}(a^2) \qquad Z_m = 1 + \mathcal{O}(a) \qquad \delta\epsilon_3 = 0 + \mathcal{O}(\epsilon_3^2 a)$$

This ensures consistent equilibrium behaviour for the scalar field in the continuum limit (a->0)

Phase-mixing case

Simulate a ‘Higgs-like’ case at the critical temperature:

$$V(\phi) = \frac{1}{2}m^2\phi^2 - \frac{1}{3!}\phi^3 + \frac{1}{4!}\frac{T^2}{3m^2}\phi^4$$



$$m^2 = \frac{T^2}{5}$$

Lattice Results (supercooling-like case)