Vacuum Stability of Orbifold Gauge Breaking in 5D

Application to asymptotic GUTs

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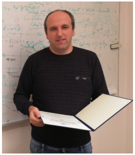

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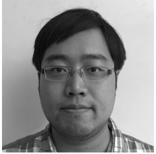

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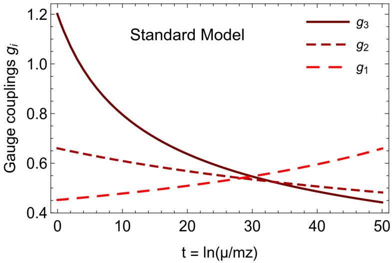
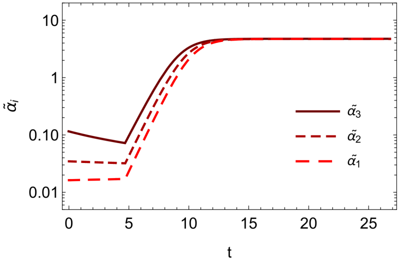
Motivation: Asymptotic Unification

- The Standard Model relies on local gauge symmetries: $SU(3)_c \times SU(2)_L \times U(1)_Y$
- At $\Lambda_{GUT} \sim 10^{16}$ GeV, the gauge couplings meet: enlarge gauge structure ($SU(5)$, $SO(10)$...)
- Traditional GUTs have limitations (proton decay, doublet-triplet splitting, Landau poles...)
- New paradigm: No exact unification, gauge couplings tend to the same UV fixed point





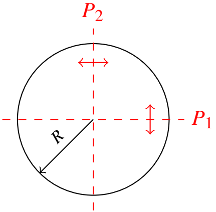
- Need power law running of the couplings → introduce 1 **extra-dimension**

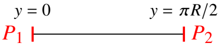
Content

I. Introduction to 5D Orbifolds

Adding 1 extra dimension

- Higher dimensional QFT:
  - Gauge group $\mathcal{G}$
  - $\mathcal{M} = \mathbb{R}^4 \times C$, coordinates $\{x^\mu, y\}$
- Extra dimension compactified on a circle/ interval, radius R
- Mod the theory by $\mathbb{Z}_2$ on each boundary





- Action of the parity on the fields Φ:

$$P_1 : \ \Phi(x^\mu, y) \sim P_1 \, \Phi(x^\mu, -y), \qquad P_2 : \ \Phi(x^\mu, y') \sim P_2 \, \Phi(x^\mu, -y')$$

- Choose diagonal basis for P_1 and P_2: $\quad P_{1,2} \, \Phi(x^\mu, -y) = \pm\Phi(x^\mu, y)$
- Classify fields according to their eigenvalues $(P_1, P_2) = (\pm, \pm)$

- KK Decomposition:

$$\phi(x,y) = \sum_{n=0}^{+\infty} \phi^{(n)}(x) f_n(y) \quad \text{with} \quad f_n(y) = \begin{cases} (++) & \frac{1}{\sqrt{2\pi R}} + \frac{1}{\sqrt{\pi R}} \cos\left(\frac{ny}{R}\right) & \longrightarrow m_n = \frac{n}{R} \\ (+-) & \frac{1}{\sqrt{\pi R}} \cos\left(\frac{(n+1/2)y}{R}\right) & \longrightarrow m_n = \frac{n+1/2}{R} \\ (-+) & \frac{1}{\sqrt{\pi R}} \sin\left(\frac{(n+1/2)y}{R}\right) & \longrightarrow m_n = \frac{n+1/2}{R} \\ (--) & \frac{1}{\sqrt{\pi R}} \sin\left(\frac{(n+1)y}{R}\right) & \longrightarrow m_n = \frac{n+1}{R} \end{cases}$$

- Low energy effective theory: KK towers integrated out, only zero modes coming from (++) states remain

- Each parity breaks the GUT gauge group $\mathcal{G}$ at low energies:

$$\left.\begin{array}{l} P_1 : \mathcal{G} \to \mathcal{H}_1 \\ P_2 : \mathcal{G} \to \mathcal{H}_2 \end{array}\right\} (P_1, P_2) : \mathcal{G} \to \mathcal{H}_1 \cap \mathcal{H}_2$$

I. Introduction to 5D Orbifolds

Parities for $\mathcal{G} = \mathrm{SU}(N)$

- Most general parities ($p + q + r + s = N$):

$$\begin{aligned} P_1 &= \mathrm{diag}(+1, \cdots, +1, +1, \cdots, +1, -1, \cdots, -1, -1, \cdots, -1), \\ P_2 &= \mathrm{diag}(\underbrace{+1, \cdots, +1}_{p}, \underbrace{-1, \cdots, -1}_{q}, \underbrace{+1, \cdots, +1}_{r}, \underbrace{-1, \cdots, -1}_{s}), \end{aligned}$$

- Action of the parities on A_μ:

$$(P_1, P_2)(A_\mu) = \begin{array}{c} \begin{matrix} p & q & r & s \end{matrix} \\ \begin{pmatrix} (+,+) & (+,-) & (-,+) & (-,-) \\ (+,-) & (+,+) & (-,-) & (-,+) \\ (-,+) & (-,-) & (+,+) & (+,-) \\ (-,-) & (-,+) & (+,-) & (+,+) \end{pmatrix} \end{array} \begin{matrix} p \\ q \\ r \\ s \end{matrix}$$

- At low energy, only the $(++)$ degrees of freedom remain:

$$\mathrm{SU}(N) \to \mathrm{SU}(p) \times \mathrm{SU}(q) \times \mathrm{SU}(r) \times \mathrm{SU}(s) \times \mathrm{U}(1)^3$$

II. Gauge-Higgs Unification

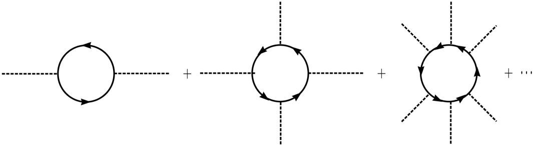
Potential for the gauge-scalar

- The 5D gauge field A_M decomposes as a 4D vector field A_μ and a scalar A_5

- Gauge invariance forbids tree level potential for A_5 but consider quantum corrections $\rightarrow$ Higgs mechanism, Gauge-Higgs unification

- 1-loop effective potential (Coleman Weinberg) for I fields:

$$V_{\text{eff}}(A_5) = \frac{1}{2}\sum_I (-1)^{F_I} \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \log\left(p^2 + m_I^2\right) \quad \text{with} \quad F_I = \{0, 1\}$$



[Quiros 1999]

- Bulk field: $m_{n,I}^2 = \frac{(n+ca)^2}{R^2} \quad \rightarrow \quad V_{\text{eff}}(a) = \frac{\mp 1}{32\pi^2}\frac{1}{(\pi R)^4}\mathcal{F}(ca) \quad \text{with} \quad \mathcal{F}(a) = \sum_{n=1}^{\infty}\frac{\cos(2\pi n a)}{n^5}$

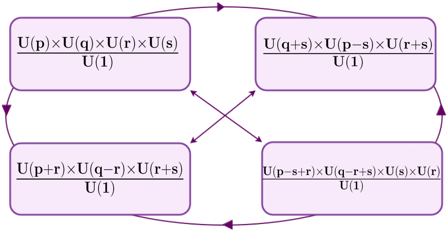
$\rightarrow$ **Finite potential**

II. Gauge-Higgs Unification

Gauge Transformation and VEV

- If A_5 develops a VEV, can use gauge transformations to cancel it
- However, gauge transformations modify the boundary conditions, can lead to different symmetry breaking patterns
- Use gauge transformation to build equivalence class of parities:

$$\mathrm{SU}(N): \quad (p,q,r,s) \sim (p-1,q+1,r+1,s-1) \sim (p+1,q-1,r-1,s+1)$$



III. Orbifold stability

- Contributions to the effective potential coming from all kind of fields:

$$V_{\text{eff}}(A_5) = V_{\text{eff}}^{\text{gauge}}(A_5) + V_{\text{eff}}^{\text{fermion}}(A_5) + V_{\text{eff}}^{\text{scalar}}(A_5) \tag{1}$$

- $V_{\text{eff}}^{\text{gauge}}(A_5)$ can't lead to the breaking of the gauge group ($\langle A_5 \rangle \neq 0$) as it would make the theory inconsistent

- If it does, can use gauge transformation to remove the VEV $\longrightarrow$ modification on the parities, different breaking patterns

Orbifold Stability

$V_{\text{eff}}^{\text{gauge}}(A_5)$ must have its minimum $\langle A_5 \rangle = 0$

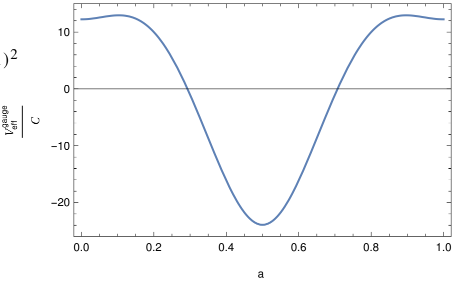
III. Orbifold Stability

Example with SU(6)

$(p = 2,\ q = 3,\ s = 1\ )$

$SU(6) \to SU(3) \times SU(2) \times U(1)^2$

$\phi_{A_5} = (\mathbf{1}, \mathbf{2})_{1/2,3}$



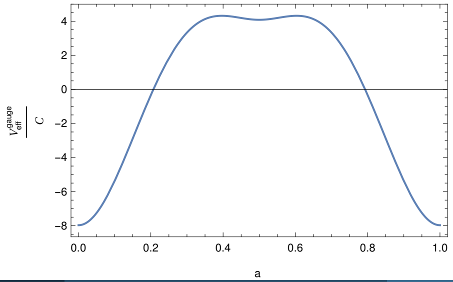
→ **UNSTABLE**

↓ **GAUGE TF**

$(p = 1,\ q = 4,\ r = 1\ )$

$SU(6) \to SU(4) \times U(1) \times U(1)$

$\phi_{A_5} = \mathbf{4}_{5,1}$



→ **STABLE**

III. Orbifold Stability

Classification of stable orbifolds

Model	Breaking pattern	Stability criteria
$SU(N)$	$SU(N) \to SU(A) \times SU(N-A) \times U(1)$	stable $\forall$ A
	$SU(N) \to SU(p) \times SU(q) \times SU(s) \times U(1)^2$	$p \geq N/2$
$Sp(2N)$	$Sp(2N) \to Sp(2A) \times Sp(2(N-A))$	stable $\forall$ A
	$Sp(2N) \to Sp(2p) \times Sp(2q) \times Sp(2s)$	$p \geq N/2$
	$Sp(2N) \to SU(p) \times SU(q) \times U(1)^2$	stable $\forall p, q$
$SO(2N)$	$SO(2N) \to SO(2A) \times SO(2(N-A))$	stable $\forall$ A
	$SO(2N) \to SO(2p) \times SO(2q) \times SO(2s)$	$p \geq N/2$
	$SO(2N) \to SU(p) \times SU(q) \times U(1)^2$	stable $\forall p, q$

IV. Example: SU(6)

- Choice of parities leading to a stable orbifold:

$$\begin{aligned} P_1 &= \mathrm{diag}(+1 \cdots, +1, +1, \cdots, +1, -1, \cdots, -1)\,, \\ P_2 &= \mathrm{diag}(\underbrace{+1, \cdots, +1}_{p=3}, \underbrace{-1, \cdots, -1}_{q=2}, \underbrace{-1, \cdots, -1}_{s=1})\,. \end{aligned} \tag{2}$$

- Orbifold breaking: $\mathrm{SU}(6) \longrightarrow \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)^2$

- Gauge-Higgs: $\phi_{A_5} = (\mathbf{3}, \mathbf{1})_{-1/3,3}$

- Fermion representations leading to SM zero modes:

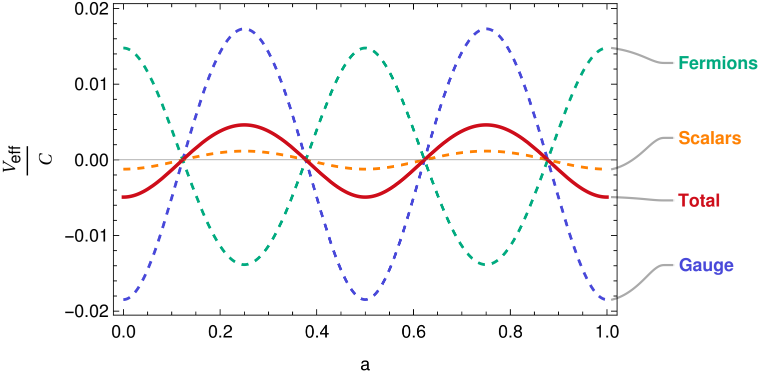
$$\Psi_{\mathbf{15}}^{(+,-)} \supset q_L + l_L^c \quad \text{and} \quad \Psi_{\overline{\mathbf{15}}}^{(-,-)} \supset u_R + e_R + d_R^c \tag{3}$$

- Scalar that contains the SM Higgs field:

$$\Phi_{\mathbf{15}}^{(-,+)} \supset \phi_h \tag{4}$$

IV. Example: SU(6)

- Effective potential:



- Gauge Higgs doesn't break the gauge group, $SU(3) \times SU(2) \times U(1)^2$ preserved

IV. Example: SU(6)

- Compute mass of the gauge scalar ϕ_{A_5}:

$$m^2_{\phi_{A_5}} = \frac{R^2}{2} \frac{\partial^2}{\partial a^2} V_{\text{eff}}(a)\bigg|_{a=0} = \frac{3}{16}\zeta(3)\frac{1}{\pi^4 R^2}\,, \tag{5}$$

- Bulk gauge interactions allow leptoquark coupling with the gauge scalar:

$$\mathcal{L} \supset \overline{\Psi}^{(+,-)}_{\mathbf{15}} iD_M \Gamma^M \Psi^{(+,-)}_{\mathbf{15}} \supset \overline{q}_L \phi_{A_5} l^c_L \tag{6}$$

- Leptoquark searches at LHC: $m_{\phi_{A_5}} \geq 2$ TeV

- Constraints on the compactification scale:

$$m_{KK} = \frac{1}{R} \geq 50\text{TeV} \tag{7}$$

Conclusion

- aGUTs: alternative to traditional GUTs, can be realized in 5D
- Orbifolds come in handy to build realistic models
- Stable Orbifold: the Gauge-Higgs scalar has to preserve the gauge part of the theory
- Only a few scenario are compatible with the orbifold stability criteria, constrains on aGUTs theories that can be built

Thank you for your attention

Asymptotic Unification: α running

- In 5D, α carries a mass dimension, define effective t'Hooft coupling:

$$\tilde{\alpha} = \mu R \alpha \tag{8}$$

- 1-loop 5D Beta function:

$$2\pi \frac{\mathrm{d}\tilde{\alpha}}{\mathrm{d}\ln\mu} = 2\pi\tilde{\alpha} - b_5\tilde{\alpha}^2 \tag{9}$$

$$b_5 = \frac{7}{3}C(\mathcal{G}) - \frac{4}{3}\sum_f T(R_f) - \frac{1}{3}\sum_s T(R_s) \tag{10}$$

- UV fixed point for $b_5 > 0$:

$$\tilde{\alpha}^* = \frac{2\pi}{b_5} \tag{11}$$

- Similar for Yukawa couplings, RGE given by:

$$2\pi \frac{\mathrm{d}\tilde{\alpha}_y}{\mathrm{d}\ln\mu} = 2\pi\tilde{\alpha}_y + c_y\tilde{\alpha}_y^2 - d_y\tilde{\alpha}\tilde{\alpha}_y \tag{12}$$

- Fixed point when $d_y > 0$, $c_y > 0$ and $d_y\tilde{\alpha}^* > 2\pi$:

$$\tilde{\alpha}_y^* = \frac{d_y\tilde{\alpha}^* - 2\pi}{c_y} \tag{13}$$

Asymptotic Unification: SU(6)

- SU(6) Beta function:

$$b_5 = \frac{61 - 16n_g}{3} \tag{14}$$

- $b_5 > 0$ for $n_g \leq 3$
- Yukawa term: $\mathcal{L} \supset -Y_u \overline{\Psi}_{\overline{\mathbf{15}}} \Phi_{\mathbf{15}} \Psi_{\mathbf{15}}$
- We get the following fixed point:

$$d_y = 28, \qquad c_y = 144 \qquad \tilde{\alpha}_y = \frac{23 + 16n_g}{72(61 - 16n_g)} \pi \tag{15}$$

- We can have at most $n_g \leq 3$