A Closer Look at Gravitational Production of Particles during Reheating

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KK, S.M. Lee, K. Oda [JCAP09(2022)018; 2206.10929]

KK, K. Oda [JCAP10(2023)048; 2304.12578]

KK, W. Ke, Y. Mambrini, K. Olive, S. Verner [Phys.Rev.D108, 115027; 2309.15146]

M. Garcia, **KK**, W. Ke, Y. Mambrini, K. Olive, S. Verner [JCAP 06 (2024) 014; 2311.14794]

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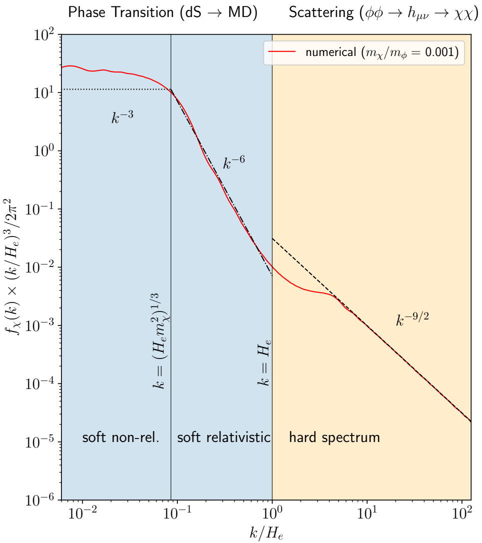
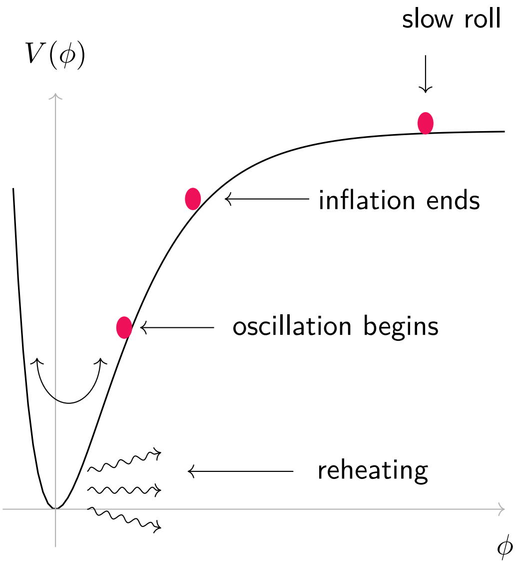
Outline



- Inflation and reheating
- Perturbative aspects of gravity
  - Boltzmann's approach
  - Spin-0 production
  - Other spins (spin-1 and spin-3/2)
- Non-perturbative aspects
  - Bogoliubov's approach
  - Equivalence to Boltzmann's
  - Inequivalence to Boltzmann's
- Summary

- Inflation and reheating

Inflation



$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}(\partial\phi)^2 - V(\phi)\right]$$

1. Inflation (de Sitter) phase

$$\text{K.E.} \ll \text{P.E.} \sim \text{const.}$$

2. Inflation ends ($\text{K.E.} \sim \text{P.E.}$)
3. Oscillation phase

$$\ddot{\phi} + 3H\dot{\phi} + m_\phi^2\phi \simeq 0$$

$\Rightarrow$ Damped oscillation

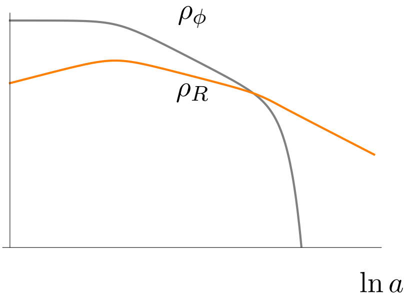
4. Reheating phase

Example of Inflation Model and Reheating

- T-model [Kallosh, Linde, '13]:

$$V(\phi) = 6\lambda M_P^4 \tanh^2\left(\frac{\phi}{\sqrt{6}M_P}\right) \simeq \begin{cases} 6\lambda M_P^4 & (\phi \gg M_P) \\ \frac{1}{2}(2\lambda M_P)\phi^2 & (\phi \ll M_P) \end{cases}$$

- Inflaton coupling to light (SM) particles: $\mathcal{L} \supset y\phi \bar{f} f \quad \Rightarrow \quad \Gamma_\phi = \frac{y^2}{8\pi} m_\phi$
- Decay products of inflaton form the thermal bath (inst. thermalization assumed)



- Boltzmann Eqs.

$$\begin{cases} \dot{\rho}_\phi + 3H\rho_\phi = -\Gamma_\phi \rho_\phi \\ \dot{\rho}_R + 4H\rho_R = +\Gamma_\phi \rho_\phi \end{cases}$$

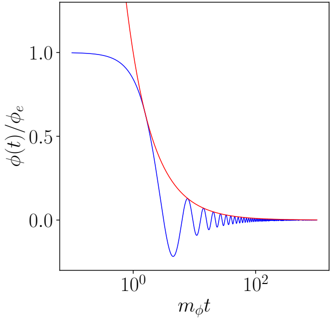
- Reheating temperature:

$$\rho_\phi(a_{\rm RH}) = \rho_R(a_{\rm RH}) \equiv \frac{\pi^2 g_*}{30} T_{\rm RH}^4$$

$$\Rightarrow T_{\rm RH} \simeq 10^{14}\ {\rm GeV} \times y \left(\frac{m_\phi}{10^{13}\ {\rm GeV}}\right)^{1/2}$$

A Closer Look at Inflaton Dynamics during Reheating

- After inflation is over: $S_{\rm inf} \simeq \int d^4x\sqrt{-g}\left[\frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m_\phi^2\phi^2\right]$
- Inflaton = condensate $\Rightarrow$ homogeneous in space ($\phi = \phi(t)$)



- EoM: $\ddot{\phi} + 3H\dot{\phi} + m_\phi^2\phi = 0$
  - $\phi(t) = \phi_0(t)\cdot\mathcal{P}(t)$ $\quad\begin{cases}\phi_0(t) = \phi_e/m_\phi t = \phi_e/a^{3/2}\\ \mathcal{P}(t) = \sin(m_\phi t)\end{cases}$
  - $\rho_\phi = \langle T_\phi^{00}\rangle_{\rm period} = V(\phi_0) = \frac{1}{2}m_\phi^2\phi_0^2$
  - For later convenience, $\mathcal{P}(t) = \sum_{n=-\infty}^{\infty}\mathcal{P}_n e^{-in\omega t}$ $(\omega = m_\phi, \mathcal{P}_{n=\pm1} = i/2, \mathcal{P}_{n\neq\pm1} = 0)$
- Decay: $\mathcal{L}_I = y\phi\bar{f}f$
  - For n-th mode: $\mathcal{M}_n = y\phi_0(t)\mathcal{P}_n\bar{u}(p_A)v(p_B)$
  - Energy transfer rate Γ_ϕ:

$$\Gamma_\phi = \frac{1}{8\pi\rho_\phi}\sum_{n=1}^{\infty}|\mathcal{M}_n|^2\underbrace{E_n}_{(E_n=n\omega)} = \frac{y^2}{8\pi}m_\phi$$

  - Γ_ϕ turns out to be the same as the decay width of a particle at rest (since ϕ is condensate, i.e. $\vec{p}=0$)

- Perturbative Aspects of Gravity

Graviton in General Relativity

- Einstein-Hilbert action (+ matter + gauge fixing): $S = S_{\rm EH} + S_{\rm m} + S_{\rm gf}$
- Introduce graviton $h_{\mu\nu}$: $g_{\mu\nu} = \eta_{\mu\nu} + \frac{2}{M_P} h_{\mu\nu}$

$$S_{\rm EH} + S_{\rm gf} \simeq \int d^4x \left[-\frac{1}{2} h^{\mu\nu} P_{\mu\nu\rho\sigma} \Box h^{\rho\sigma} \right], \quad P_{\mu\nu\rho\sigma} = \frac{\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \eta_{\mu\nu}\eta_{\rho\sigma}}{2}$$

- Graviton propagator

$$\langle 0 | T h_{\mu\nu}(x) h_{\rho\sigma}(y) | 0 \rangle = \int \frac{d^4k}{(2\pi)^4} e^{-ik(x-y)} \, \frac{P_{\mu\nu\rho\sigma}}{k^2 - i\epsilon}$$

- Graviton coupling

$$S_{\rm m} \simeq \int d^4x \left[\mathcal{L}_{\rm m} - \frac{1}{M_P} h_{\mu\nu} T^{\mu\nu} \right], \quad T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L}_{\rm m})}{\delta g^{\mu\nu}}$$

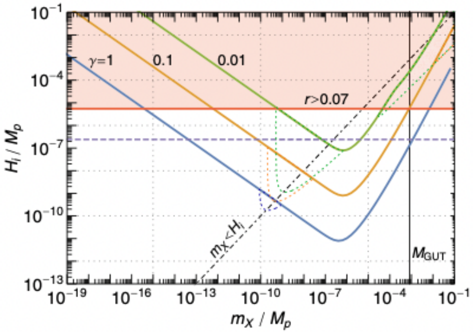
Gravity universally mediates between all matter fields

DM Production from Thermal Bath by Graviton Exchange

- Minimal coupling of DM is gravity:

$$\mathcal{L} = -\frac{M_P^2}{2} R + \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm DM} \quad \Rightarrow \quad \frac{h_{\mu\nu}}{M_P}\left(T^{\mu\nu}_{\rm SM} + T^{\mu\nu}_{\rm DM}\right)$$

- SM + SM $\rightarrow$ graviton $\rightarrow$ DM + DM [Garny, Sandora, Sloth '15]



- Assume scalar DM with mass m_X
- Boltzmann equation:

$$\dot{n}_X + 3Hn_X = R_{{\rm SM}\to X} \sim \frac{T^8}{M_P^4}$$

$$\Rightarrow \frac{n_X}{T^3} \sim \frac{R_{{\rm SM}\to X}}{T^3 H} \propto T_{\rm RH}^3$$

- $T_{\rm RH} \sim \gamma\sqrt{M_P H_i}$
- $\Omega_X h^2 \propto m_X T_{\rm RH}^3 \propto m_X H_i^{3/2}$

DM Production from Inflaton by Graviton Exchange

- Noticed that $\phi \to$ DM dominates over SM $\to$ DM as $\rho_\phi \gg \rho_R$ at early times

  [Ema, Jinno, Mukaida, Nakayama '15,'16, Ema, Nakayama, Tang '18, Mambrini, Olive '21]

- Energy-momentum tensor of $\phi(t)$: $T^\phi_{\mu\nu} = (\rho_\phi + P_\phi) u_\mu u_\nu - \eta_{\mu\nu} P_\phi$

$$(T^\phi_{\mu\nu}) = \begin{pmatrix} \rho_\phi & & & \\ & P_\phi & & \\ & & P_\phi & \\ & & & P_\phi \end{pmatrix} \quad \Rightarrow \quad \begin{cases} T^\phi_{\mu\nu} = \displaystyle\sum_{n=-\infty}^{\infty} T^\phi_{n,\mu\nu} e^{-in\omega t}, \quad (\omega = m_\phi) \\ T^\phi_{n,\mu\nu} = 2K_n u_\mu u_\nu - \eta_{\mu\nu}(K_n - V_n) \\ u_\mu = (1,0,0,0)_\mu \\ (T^\phi_{n,00} = K_n + V_n, T^\phi_{n,i=j} = K_n - V_n) \end{cases}$$

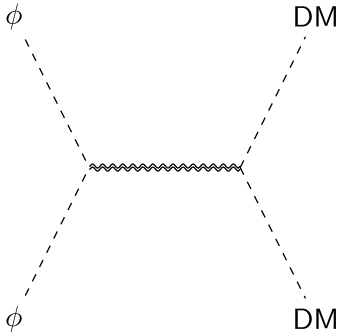
- E.g. spin-0 DM: $\phi\phi \to \chi\chi \Longleftrightarrow$ annihilation of n-th modes in $T^\phi_{\mu\nu}$

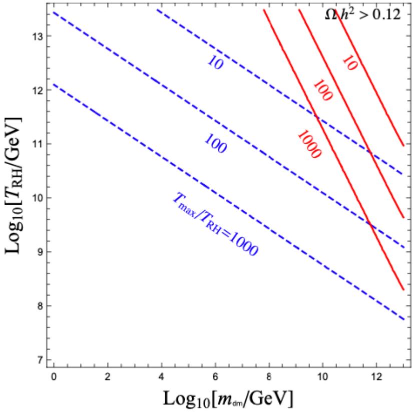
$$\chi = \text{ real scalar : } \qquad \mathcal{M}_n = \frac{V_n}{M_P^2}\left(1 + \frac{2m_\chi^2}{(n\omega)^2}\right) \sim \frac{\rho_\phi}{M_P^2}$$

$$R_{\phi\phi\to\chi\chi} = \sum_{n=1}^{\infty} \int \frac{d^3 p_A}{(2\pi)^3 2p_A^0} \frac{d^3 p_B}{(2\pi)^3 2p_B^0} |\mathcal{M}_n|^2 (2\pi)^4 \delta(n\omega - p_A^0 - p_B^0) \delta^3(\vec{p}_A + \vec{p}_B)$$

Spin-0 DM

- Scalar DM [Mambrini, Olive '21]: $T^{\chi}_{\mu\nu} = \partial_\mu\chi\partial_\nu\chi - \eta_{\mu\nu}\left(\frac{1}{2}\partial^\alpha\chi\partial_\alpha\chi - \frac{1}{2}m_\chi^2\chi^2\right)$

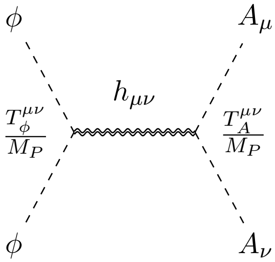




- Boltzmann Equation: $\dot{n}_\chi + 3Hn_\chi = R_{\phi\to\chi} \sim \frac{\rho_\phi^2}{M_P^4}$ with $\rho_\phi \propto a^{-3}$
- Easily find $\Omega_\chi h^2 \sim \left(\frac{T_{\rm RH}}{10^{10}\ \rm GeV}\right)^3 \left(\frac{T_{\rm max}/T_{\rm RH}}{100}\right)^4 \left(\frac{m_\chi}{10^{10}\ \rm GeV}\right)$

Spin-1 DM

- Spin-1 DM production during reheating (not include production during inflation)
  [Garcia, **KK**, Ke, Mambrini, Olive, Verner, 2311.14794]
- $\mathcal{L}_A = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m_A^2 A^\mu A_\mu \quad \Rightarrow \quad T_A^{\mu\nu} = -F^{\mu\alpha}F_\alpha{}^\nu + m_A^2 A^\mu A^\nu - \eta^{\mu\nu}\mathcal{L}_A$
- Interaction: $\sqrt{-g}\mathcal{L}_{\rm int} = -\frac{1}{M_P}h_{\mu\nu}(T_{\rm SM}^{\mu\nu} + T_\phi^{\mu\nu} + T_A^{\mu\nu})$



- Helicity amplitudes:
  - $\mathcal{M}_{n,\rm LL} = \frac{V_n}{M_P^2}\left(1 + \frac{2m_A^2}{(n\omega)^2}\right)$ (cf. minimally-coupled scalar)
  - $\mathcal{M}_{n,\rm TT} = \frac{V_n}{M_P^2}\frac{2m_A^2}{(n\omega)^2} \ll \mathcal{M}_{n,\rm LL}$ for $m_A \ll m_\phi$
- From Boltzmann equation

$$\Omega_L h^2 \simeq 0.12 \times \left(\frac{T_{\rm RH}}{10^{10}\ \rm GeV}\right)\left(\frac{m_A}{10^7\ \rm GeV}\right)$$

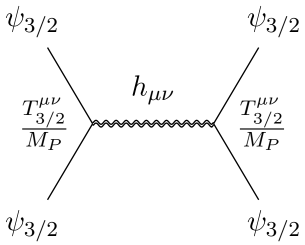
$$\Omega_T h^2 \simeq 10^{-16} \times \left(\frac{T_{\rm RH}}{10^{10}\ \rm GeV}\right)\left(\frac{10^{13}\ \rm GeV}{m_\phi}\right)^4\left(\frac{m_A}{\rm EeV}\right)^5$$

Spin-3/2 DM

- 0, 1/2, 1, 2, why not 3/2? $\Rightarrow$ Rarita-Schwinger field (e.g. gravitino)
  [**KK**, Ke, Mambrini, Olive, Verner, Phys.Rev.D108, 115027; 2309.15146]
  - $\mathcal{L}_{3/2} = \bar{\psi}_\mu (i\gamma^{\mu\rho\nu}\partial_\rho + m_{3/2}\gamma^{\mu\nu})\psi_\nu$
  - $T^{\mu\nu}_{3/2} = \frac{i}{2}\left[-\frac{1}{2}\bar{\psi}_\rho\gamma^{(\mu}\overset{\leftrightarrow}{\partial}{}^{\nu)}\psi^\rho + \bar{\psi}^{(\nu}\gamma^{\mu)}\overset{\leftrightarrow}{\partial}_\rho\psi^\rho + \bar{\psi}^\rho\gamma^{(\mu}\overset{\leftrightarrow}{\partial}_\rho\psi^{\nu)}\right]$
- Helicity amplitudes:
  - $\mathcal{M}_{n,\mathrm{LL}} = \frac{V_n}{M_P^2}\frac{n\omega}{m_{3/2}}\left(\frac{2}{3} - \frac{m_{3/2}^2}{2(n\omega)^2}\right)\sqrt{1-\frac{m_{3/2}^2}{(n\omega)^2}}$ (diverge when $m_{3/2}\to 0$?)
  - $\mathcal{M}_{n,\mathrm{TT}} = \frac{V_n}{M_P^2}\frac{m_{3/2}}{2n\omega}\sqrt{1-\frac{m_{3/2}^2}{(n\omega)^2}} \ll \mathcal{M}_{n,\mathrm{LL}}$ for $m_{3/2} \ll m_\phi$
- DM abundance:
  - $\Omega_L h^2 \simeq 10^9 \left(\frac{T_{\mathrm{RH}}}{10^{10}\ \mathrm{GeV}}\right)\left(\frac{m_\phi}{10^{13}\ \mathrm{GeV}}\right)^2\left(\frac{\mathrm{EeV}}{m_{3/2}}\right) \Rightarrow m_{3/2} \gtrsim 10\ \mathrm{PeV}$ ($T_{\mathrm{RH}} \gtrsim 4$ MeV)
  - $\Omega_T h^2 \simeq 10^{-8} \left(\frac{T_{\mathrm{RH}}}{10^{10}\ \mathrm{GeV}}\right)\left(\frac{m_\phi}{10^{13}\ \mathrm{GeV}}\right)^2\left(\frac{\mathrm{EeV}}{m_{3/2}}\right)^3$

Issues on Spin-3/2?

- $m_{3/2}$ can not be zero due to unitarity bound



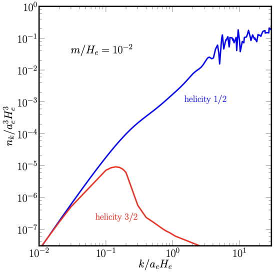
- Scattering of longitudinal modes: $\psi_\mu \sim \frac{\partial_\mu}{m_{3/2}}$
- Amplitude at $E \sim m_\phi$:

$$\mathcal{M} \sim \frac{m_\phi^4}{m_{3/2}^2 M_P^2} \gtrsim 1 \quad \text{when} \quad m_{3/2} \lesssim 40 \text{ EeV}$$

- If SUSY, $\mathcal{M} \sim m_\phi^3/m_{3/2}M_P^2 \gtrsim 1 \Rightarrow m_{3/2} \lesssim 0.1$ EeV

[Antoniadis, Guillen, Rondeau, '22]

- Catastrophic production [Hasegawa et al, '17; Kolb, Long, McDonough, '21]



- Sound speed of DM: $c_s^2 = \frac{(P_\phi - 3m_{3/2}^2 M_P^2)^2}{(\rho_\phi + 3m_{3/2}^2 M_P^2)^2}$
- Non-perturbative production of DM when $c_s = 0$
- $f_{3/2}(k) \sim$ const. $\Rightarrow n_{\rm DM} = \int \frac{d^3k}{(2\pi)^3} f_{3/2}(k) \propto \Lambda^3$
- Divergent? Backreaction?

- Non-perturbative Aspects

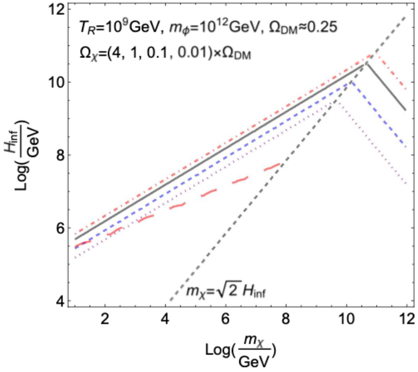
Another Approach to Gravitational DM Production

- Different way to describe the gravitational DM production

  [Ema, Jinno, Mukaida, Nakayama '15,'16, Ema, Nakayama, Tang '18]

- DM production scenario:
  1. Inflaton (damped) oscillation $\Rightarrow$ oscillation of the metric (or H)
  2. Oscillation in DM kinetic term ($\sqrt{-g}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi$) $\Rightarrow$ DM production

[Ema, Nakayama, Tang '18]



- Equation of motion of DM (χ):

$$\chi_k'' + \left[k^2 + a^2 m_\chi^2 + \frac{1}{6}a^2 R\right]\chi_k = 0$$

- Numerically compute ρ_χ, based on Bogoliubov transformation:

$$\begin{aligned}\rho_\chi =& \frac{1}{2}\int \frac{d^3k}{(2\pi)^3 a^4}[|\chi_k'|^2 \\ &+ (k^2 + a^2 m_\chi^2 + a^2 H^2)|\chi_k|^2 \\ &- aH(\chi_k\chi_k^{*\prime} + \chi_k^*\chi_k')]\end{aligned}$$

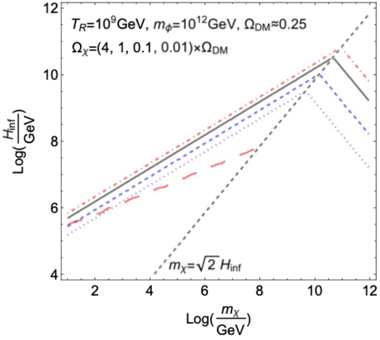
Different Interpretation?

- ϕ oscillates $\Rightarrow$ R oscillates

$$\begin{cases} \chi_k'' + \omega_k^2 \chi_k = 0 \\ \omega_k^2 = k^2 + a^2 m_\chi^2 + \frac{1}{6} a^2 R \end{cases}$$

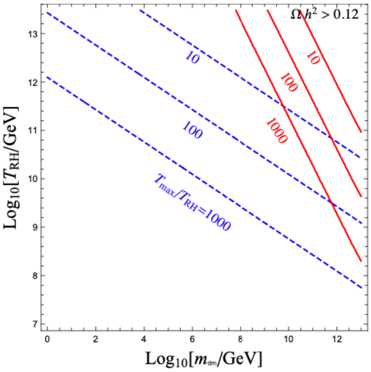
- Bogoliubov approach

[Ema, Nakayama, Tang '18]



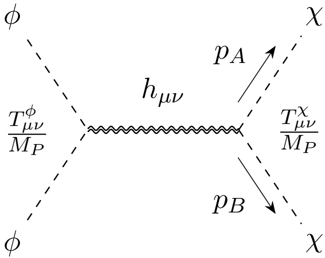
- $\phi\phi$ →graviton→ $\chi\chi$
- Production rate $R_{\phi\to\chi} \sim \frac{\rho_\phi^2}{M_P^4}$
- Boltzmann approach

[Mambrini, Olive '21]



Are these two approaches (Boltzmann and Bogoliubov) equivalent or not?

Phase Space Distribution of Spin-0 in Boltzmann's Approach



- Graviton exchange diagram

$$S \simeq \int d^4x \left[\mathcal{L}_{h,\mathrm{kin}} + \mathcal{L}_\phi + \mathcal{L}_\chi - \frac{1}{M_P} h^{\mu\nu} (T^{\phi}_{\mu\nu} + T^{\chi}_{\mu\nu}) \right]$$

- Boltzmann equation for $f_\chi(t, \vec{p}_A)$:

$$\frac{\partial f_\chi}{\partial t} - H |\vec{p}_A| \frac{\partial f_\chi}{\partial |\vec{p}_A|} = C[f_\chi]$$

- Collision term (when $\chi\chi \to \phi\phi$ is negligible):

$$C[f_A] = \frac{\pi \rho_\phi^2}{16 \beta m_\phi^2 M_P^4} \left(1 + \frac{m_\chi^2}{2 m_\phi^2} \right)^2 \delta(|\vec{p}_A| - \beta m_\phi), \qquad \beta \equiv \sqrt{1 - \frac{m_\chi^2}{m_\phi^2}}$$

- Phase space distribution (comoving momentum $k = |\vec{p}_A| a(t)$):

$$f_\chi(t, |\vec{p}_A|) = \frac{9\pi}{64} \left(\frac{H_e}{m_\phi} \right)^3 \left(\frac{m_\phi}{k} \right)^{9/2} \left(1 - \frac{m_\chi^2}{m_\phi^2} \right)^{5/4} \left(1 + \frac{m_\chi^2}{2 m_\phi^2} \right)^2 \propto k^{-9/2}$$

How to Address the Question

- What do we want? $f_\chi^{(\text{Boltzmann})}$ vs $f_\chi^{(\text{Bogoliubov})}$ to see if they are equivalent
- The question may be stated as follows.

Boltzmann approach

- Based on QFT in Minkowski spacetime
- Particle production is described by collision terms in the Boltzmann Eqs

Bogoliubov approach

- Based on QFT in curved spacetime
- Particle production is described by time dependence in effective frequencies

- Therefore, there seems **no guarantee** that they are equivalent

QFT in Curved Spacetime in a Nutshell

- Use $a d\eta = dt$ and $\widetilde{\chi} \equiv a^{-1}\chi$ to write

$$S_\chi = \int d^4x\sqrt{-g}\mathcal{L}_\chi = \int d^4x \left[\frac{1}{2}(\widetilde{\chi}')^2 - \frac{1}{2}\widetilde{\chi}\omega^2\widetilde{\chi}\right], \quad \omega^2 \equiv \nabla^2 + a^2 m_\chi^2 + \frac{1}{6}a^2 R$$

- Frequency ω is time dependent, so

$$\boxed{\widetilde{\chi}^{(\mathrm{past})} \neq \widetilde{\chi}^{(\mathrm{future})} \quad \Longleftrightarrow \quad |0^{(\mathrm{past})}\rangle \neq |0^{(\mathrm{future})}\rangle}$$

- Introduce Bogoliubov coefficients:

$$\begin{pmatrix} b_{\vec{k}} \\ b^\dagger_{-\vec{k}} \end{pmatrix}_{\mathrm{future}} = \begin{pmatrix} \alpha_k^* & -\beta_k^* \\ -\beta_k & \alpha_k \end{pmatrix} \begin{pmatrix} a_{\vec{k}} \\ a^\dagger_{-\vec{k}} \end{pmatrix}_{\mathrm{past}}$$

- Define the initial vacuum by $a_{\vec{k}}|0^{(1)}\rangle = 0$, then

$$n_k^{(2)} = \langle 0^{(1)}|b^\dagger_{\vec{k}} b_{\vec{k}}|0^{(1)}\rangle = |\beta_k|^2 \neq 0$$

Bogoliubov Coefficients in Perturbative Regime

- From $\widetilde{\pi} = \widetilde{\chi}'$ and EoM $\Rightarrow \begin{cases} \alpha_k' = -i\omega_k\alpha_k + \dfrac{\omega_k'}{2\omega_k}\beta_k \\ \beta_k' = i\omega_k\beta_k + \dfrac{\omega_k'}{2\omega_k}\alpha_k \end{cases}$
- For $|\omega_k'/\omega_k^2| \ll 1 \quad \Rightarrow \quad \beta_k(\eta) \simeq \displaystyle\int_{\eta_i}^{\eta} d\eta' \left[\frac{\omega_k'}{2\omega_k}\right] e^{-2i\int_{\eta_i}^{\eta'} d\eta''\omega_k(\eta'')}$

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- For $|\omega_k'/\omega_k^2| \ll 1 \quad \Rightarrow \quad \beta_k(\eta) \simeq \displaystyle\int_{\eta_i}^{\eta} d\eta' \left[\frac{a^3 H m_\chi + (a^2 R)'/6}{k^2 + a^2 m_\chi^2 + a^2 R/6}\right] e^{-2i\int_{\eta_i}^{\eta'} d\eta'' \omega_k(\eta'')}$

Bogoliubov Coefficients in Perturbative Regime

- From $\widetilde{\pi} = \widetilde{\chi}'$ and EoM $\Rightarrow \begin{cases} \alpha_k' = -i\omega_k \alpha_k + \dfrac{\omega_k'}{2\omega_k}\beta_k \\ \beta_k' = i\omega_k \beta_k + \dfrac{\omega_k'}{2\omega_k}\alpha_k \end{cases}$
- For $|\omega_k'/\omega_k^2| \ll 1 \quad \Rightarrow \quad \beta_k(\eta) \simeq \int_{\eta_i}^{\eta} d\eta' \left[\dfrac{a^3 H m_\chi + (a^2 R)'/6}{k^2 + a^2 m_\chi^2 + a^2 R/6} \right] e^{-2i \int_{\eta_i}^{\eta'} d\eta'' \omega_k(\eta'')}$
- Use $\phi(t) = (\phi_e/m_\phi t)\,\sin(m_\phi t)$ and $\overline{H} = H_e(a/a_e)^{-3/2}$ (slowly-moving):

$$\beta_k \simeq \int_\eta \left[\frac{3}{4} \frac{(a\overline{H})^3 (m_\phi/\overline{H})}{k^2 + a^2 m_\chi^2} \left(1 + \frac{m_\chi^2}{2m_\phi^2}\right) \underbrace{\sin(2m_\phi t)}_{\text{fast oscillation of } \phi} + \ldots \right] \underbrace{e^{-2i\int_\eta \omega_k}}_{\text{change in } \chi\text{'s frequency}}$$

for $k \gg a\overline{H} \sim a\sqrt{\overline{R}}$ (otherwise, adiabaticity is broken) and $\overline{H}/m_\phi \ll 1$

Bogoliubov Coefficients in Perturbative Regime

- From $\widetilde{\pi} = \widetilde{\chi}'$ and EoM $\Rightarrow \begin{cases} \alpha_k' = -i\omega_k \alpha_k + \dfrac{\omega_k'}{2\omega_k}\beta_k \\ \beta_k' = i\omega_k \beta_k + \dfrac{\omega_k'}{2\omega_k}\alpha_k \end{cases}$
- For $|\omega_k'/\omega_k^2| \ll 1 \quad \Rightarrow \quad \beta_k(\eta) \simeq \int_{\eta_i}^{\eta} d\eta' \left[\dfrac{a^3 H m_\chi + (a^2 R)'/6}{k^2 + a^2 m_\chi^2 + a^2 R/6}\right] e^{-2i\int_{\eta_i}^{\eta'} d\eta'' \omega_k(\eta'')}$
- Use $\phi(t) = (\phi_e/m_\phi t)\sin(m_\phi t)$ and $\overline{H} = H_e(a/a_e)^{-3/2}$ (slowly-moving):

$$\beta_k \simeq \int_\eta \left[\frac{3}{4}\frac{(a\overline{H})^3(m_\phi/\overline{H})}{k^2 + a^2 m_\chi^2}\left(1 + \frac{m_\chi^2}{2m_\phi^2}\right)\underbrace{\sin(2m_\phi t)}_{\text{fast oscillation of }\phi} + \ldots\right]\underbrace{e^{-2i\int_\eta \omega_k}}_{\text{change in }\chi\text{'s frequency}}$$

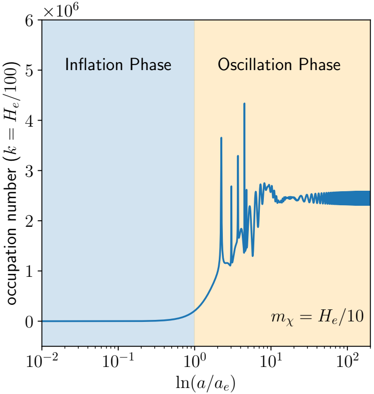
for $k \gg a\overline{H} \sim a\sqrt{R}$ (otherwise, adiabaticity is broken) and $\overline{H}/m_\phi \ll 1$

- Use stationary phase approximation to find

$$\boxed{|\beta_{k\gg a\overline{H}}|^2 \simeq \frac{9\pi}{64}\left(\frac{H_e}{m_\phi}\right)^3\left(\frac{m_\phi}{k}\right)^{9/2}\left(1 - \frac{m_\chi^2}{m_\phi^2}\right)^{5/4}\left(1 + \frac{m_\chi^2}{2m_\phi^2}\right)^2 \propto k^{-9/2}}$$

which is **identical** to f_χ obtained in the Boltzmann approach

Gravitational Particle Production from Phase Transition



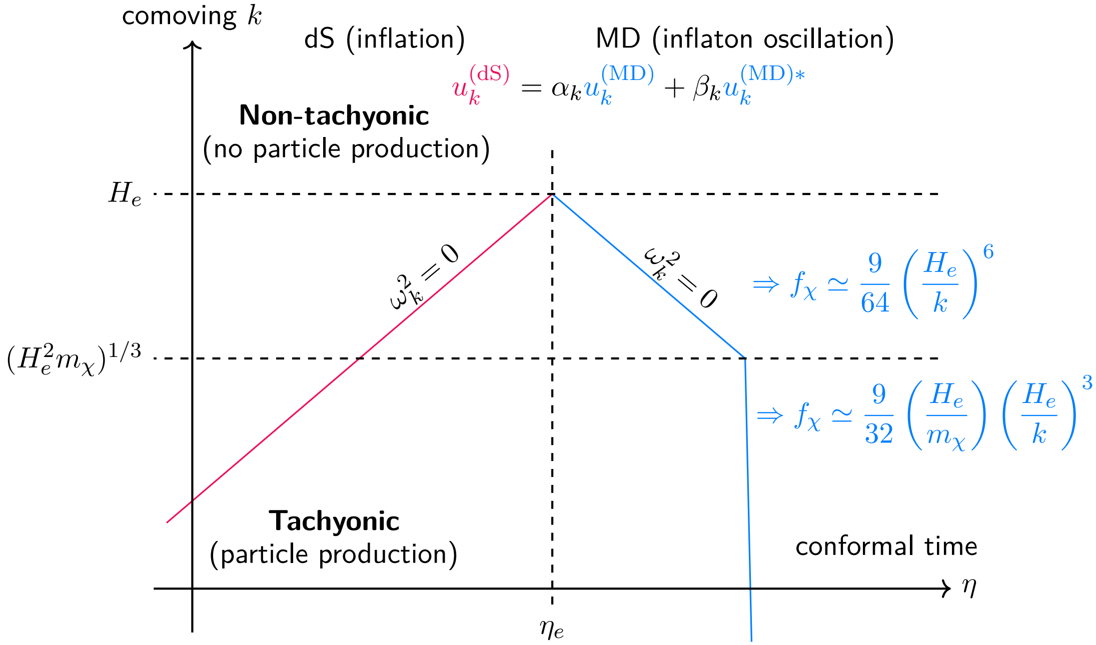
- Particle production through abrupt phase transition (dS $\to$ MD)
- Initial condition: Bunch-Davies vacuum (no particle)
- $\omega_\chi^{(\mathrm{dS})} \neq \omega_\chi^{(\mathrm{MD})}$, producing particles
- Phase space distribution:

$$f_\chi(k) \simeq \begin{cases} \frac{9}{64}\left(\frac{H_e}{k}\right)^6 & (k > k_*) \\ \frac{9}{32}\left(\frac{H_e}{m_{\chi,\mathrm{eff}}}\right)\left(\frac{H_e}{k}\right)^3 & (k < k_*) \end{cases}$$

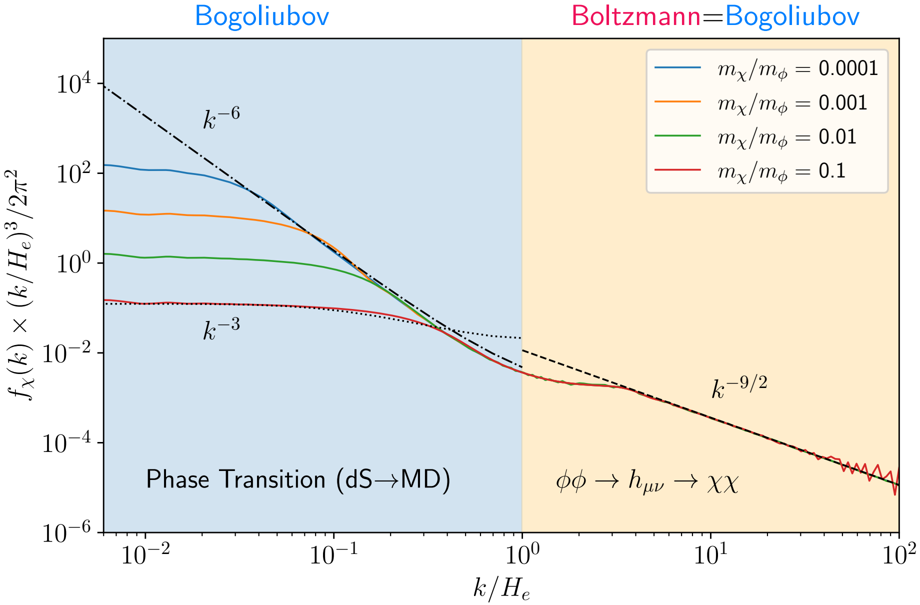
$$k_* \simeq \left(H_e^2 m_{\chi,\mathrm{eff}}\right)^{1/3}$$

- Only $k < H_e$ may be excited

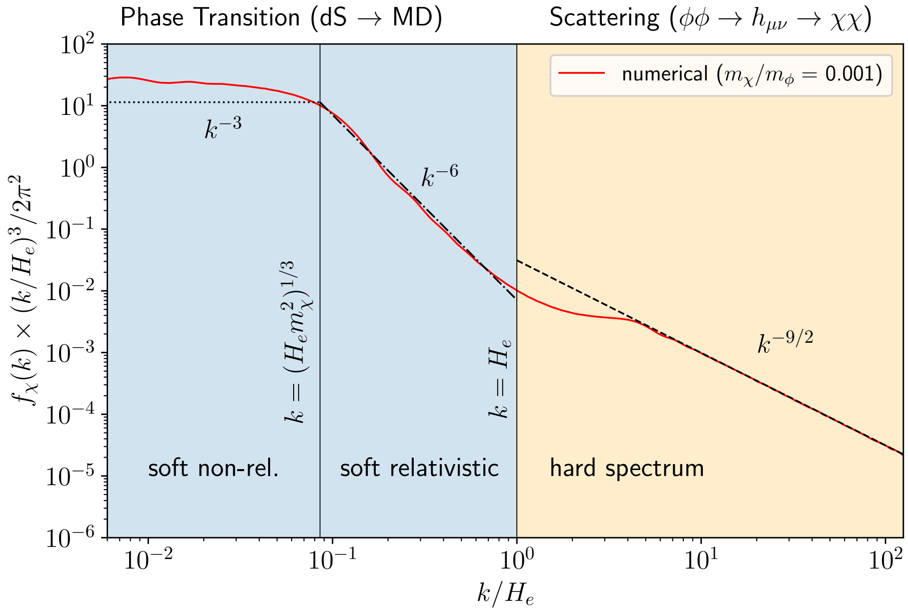
Schematic Picture of Particle Production



Two Different Sources of Gravitational Production

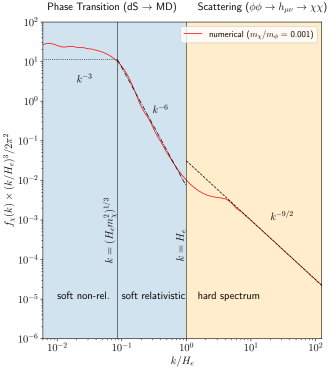


Three Regimes in Spectrum



- Summary

Summary



Two regimes in gravitational particle production

- $k > m_\phi$: Boltzmann = Bogoliubov
- $k < m_\phi$: Bogoliubov

Main characteristics

- $k > m_\phi$:
  - Unavoidable and easy to estimate
  - Depends on spins and inflaton oscillation
- $k < m_\phi$:
  - Can not be negligible in many cases
  - Depends on spins, whole shape of inflaton potential, and (DM) self interactions