Nonadiabatic quantum Otto engine: frictional effects on performance bounds and operational modes

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Outline of the talk

- Carnot cycle
- Sources of irreversibility in the heat engines
- Definitions of heat and work
- Otto cycle
- Frictional effects in quantum Otto engine

Carnot cycle

Two adiabatic and two isothermal branches [1] :

(1) No heat exchange takes place during the adiabatic branches.

(2) Temperature of the working medium remains constant during the isothermal branches.

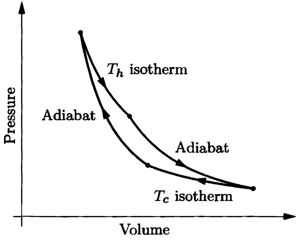


Figure: Carnot cycle for an ideal gas

- Carnot efficiency, $\eta_C = 1 - \frac{T_c}{T_h}$, as theoretical upper bound.
- Maximum efficiency, vanishing power output $(P = W/\tau)$.

[1] D. Kondepudi and I. Prigogine, *Modern thermodynamics*, John Wiley & Sons, UK (2014).

Effect of entropy production on the efficiency of a engine

Finite-time Thermodynamics

- Ideal heat engines; vanishing power output.
- Irreversible heat engines (finite entropy production); finite power output.

Efficiency of a engine coupled to two thermal baths:

$$\eta = \eta_C - \frac{T_c}{Q_h}\sigma \leq \eta_C .$$

- Due to positive entropy production σ, efficiency of the engine is smaller than Carnot efficiency.

Sources of irreversibility in the engine

Sources of irreversibility in the engine

- Heat leaks. *Usually not considered while studying heat engines.*
- Finite rate of heat transfer between the working medium and the external heat baths. *Approach employed to study first model of finite-time heat engine known as endoreveresible engine* [2].

$$\eta_{\mathrm{CA}} = 1 - \sqrt{\frac{T_c}{T_h}}.$$

- Finite-time driving of the working fluid of the engine. *Source of inner friction* [3] [4].

[2] F. L. Curzon and B. Ahlborn, Am. J. Phys. **43**, 22 (1975).
[3] Y. Rezek and R. Kosloff, New. J. Phys. **8**, 83 (2006).
[4] F. Plastina *et al.*, Phys. Rev. Lett. **113**, 260601 (2014).

Definitions of heat and work [5] [6]:

$$U = \langle E \rangle = Tr[\rho H] \implies dU = Tr[\rho\, dH] + Tr[d\rho\, H] \equiv đW + đQ$$

11.9 Entropy and Heat

We must now see how entropy is related to the other thermodynamic quantities we have previously introduced. Suppose that a system in statistical equilibrium undergoes an infinitesimal transformation as a result of its interaction with its surroundings. The interaction results in a change of the partition numbers n_i and of the possible energy states E_i. Since $U = \sum_i n_i E_i$, we then have that

$$dU = \sum_i E_i\, dn_i + \sum_i n_i\, dE_i. \qquad (11.32)$$

This equation holds whether the process is reversible or not. However, we shall

$$đW = -\sum_i n_i\, dE_i. \qquad (11.33)$$

Equation (11.33) gives the work done by the system in terms of the change in the energy levels, resulting, for example, from a change in volume. This puts the statistical definition of the work done by a system on a firmer basis than it was in our preliminary definition, given in Section 11.3. By introducing Eqs. (11.32) and (11.33) into $dU = đQ - đW$, we conclude that we must write

$$đQ = \sum_i E_i\, dn_i \qquad (11.34)$$

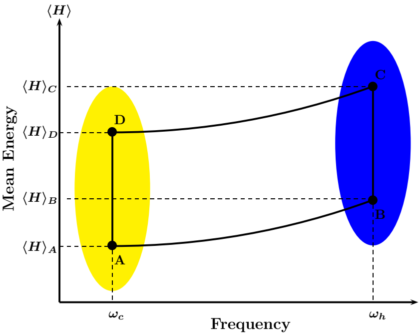
[5] R. Alicki, J. Phys. A **12**, L103 (1979).

[6] Alonso and Finn, *Fundamental University Physics: Volume III*, Addition-Wesley, USA (1968).

Otto cycle with HO as the working medium

Two work strokes and two heat strokes: [7]

(1) Compression (work) stroke $A(\omega_c)$ to $B(\omega_h)$: To begin with, the system is at inverse temperature β_c. The system is isolated and frequency of the oscillator is changed from ω_c to ω_h. Work is done on the system in this stage.



$$\langle H\rangle_A = \left(\langle n_c\rangle + \frac{1}{2}\right)\omega_c,\ \langle n_c\rangle = \frac{1}{e^{\beta_c\omega_c}-1}$$

$$\Rightarrow \langle H\rangle_A = \frac{\omega_c}{2}\coth\left(\frac{\beta_c\omega_c}{2}\right),$$

$$\langle H\rangle_B = \frac{\omega_h}{2}\lambda_{\mathrm{AB}}\coth\left(\frac{\beta_c\omega_c}{2}\right).$$

λ is the adiabaticity parameter. [8]

$$W_{AB} = \langle H\rangle_B - \langle H\rangle_A.$$

[7] O. Abah, Phys. Rev. Lett. **109**, 203006 (2012), O. Abah, Science **352**, 325 (2016.)

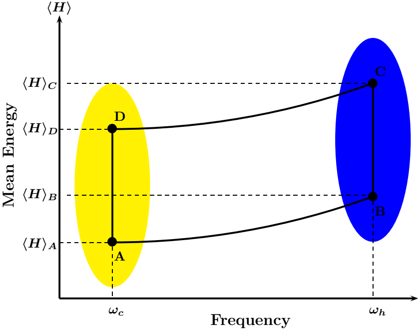
[8] K. Husimi, Prog. Theor. Exp. Phys. **9**, 238 (1953).

Otto cycle with HO as the working medium

(2) Hot isochore B to C: The oscillator is coupled to a hot thermal reservoir at inverse temperature β_h and allowed to relax to a thermal state.

(3) **Expansion (work) stroke $C(\omega_h)$ to $D(\omega_c)$:** In this stage, frequency of the oscillator is changed from ω_h to ω_c. Work is done by the system.

(4) **Cold isochore D to A:** The oscillator is placed in contact with the cold **thermal** reservoir at inverse temperature β_c and thermalization takes place.



$$\langle H\rangle_C = \frac{\omega_h}{2}\coth\left(\frac{\beta_h\omega_h}{2}\right).$$

$$\langle H\rangle_D = \frac{\omega_c}{2}\lambda_{CD}\coth\left(\frac{\beta_h\omega_h}{2}\right).$$

$$\langle H\rangle_A = \frac{\omega_c}{2}\coth\left(\frac{\beta_c\omega_c}{2}\right).$$

$$Q_h = \langle H_C\rangle - \langle H_B\rangle.$$

$$Q_c = \langle H_A\rangle - \langle H_D\rangle.$$

$$W_{CD} = \langle H_D\rangle - \langle H_A\rangle.$$

Operation of Otto cycle as a heat engine:

$$W_{\text{ext}} = -W = -(W_{AB} + W_{CD}) \geq 0, \quad Q_h \geq 0, \quad Q_c \leq 0$$

.

Work output and efficiency for sudden-switch protocol $\left(\lambda = \frac{\omega_c^2+\omega_h^2}{2\omega_c\omega_h}\right)$ [9] :

$$W_{\text{ext}} = \frac{\omega_h^2 - \omega_c^2}{4\omega_c\omega_h}\left[\omega_c \coth\left(\frac{\beta_h\omega_h}{2}\right) - \omega_h \coth\left(\frac{\beta_c\omega_c}{2}\right)\right],$$

$$\eta = \left[\frac{2}{1-\frac{\omega_c^2}{\omega_h^2}} + \frac{1}{\frac{\omega_c}{\omega_h}\coth\left(\frac{\beta_h\omega_h}{2}\right)\tanh\left(\frac{\beta_c\omega_c}{2}\right) - 1}\right]^{-1} \equiv \left(\frac{2}{\Delta_1} + \frac{1}{\Delta_2}\right)^{-1}. \qquad (1)$$

Maximum achievable efficiency: [Phys. Rev. E **102**, 062123 (2020)]

$$W_{\text{ext}} \geq 0 \quad \Rightarrow \quad \frac{\omega_c}{\omega_h}\coth\left(\frac{\beta_h\omega_h}{2}\right)\tanh\left(\frac{\beta_c\omega_c}{2}\right) - 1 \geq 0, \quad \boxed{\text{i. e., } \Delta_2 \geq 0.}$$

Using $\Delta_2 \geq 0$ in Eq. (1), and then using $0 \leq \Delta_1 \leq 1$,

$$\boxed{\eta \leq \frac{\Delta_1}{2} \leq \frac{1}{2}}$$

[9] K. Husimi, Prog. Theor. Exp. Phys. **9**, 238 (1953).

Key Points

- The maximum efficiency that our engine can achieve is 1/2 only.
- This result is also valid for the Otto engine with squeezed thermal reservoir.
- This is in contrast to all previous studies (valid for the quasi-static regime) claiming unit efficiency under the effect of a squeezed reservoir.
- In the sudden switch regime, the sudden quench of the frequency of the harmonic oscillator induces **nonadiabatic transitions** between its energy levels, thereby causing the system to develop coherence in the energy basis. In such a case, the energy entropy increases and an additional parasitic internal energy is stored in the working medium [10]. The additional energy corresponds to the waste (or excess) heat which is dissipated to the heat reservoirs during the proceeding isochoric stages of the cycle.

[10] F. Plastina *et al.*, Phys. Rev. Lett. **113**, 260601 (2014).

Upper bound on the efficiency of the heat engine

Upper bound on the efficiency and efficiency at maximum work

$$\eta_{SS}^{\rm up} = \frac{[3-2\sqrt{2(1-\eta_C)}-\eta_C]\eta_C}{(1+\eta_C)^2}, \quad \eta_{SS}^{\rm MW} = \frac{1-\sqrt{1-\eta_C}}{2+\sqrt{1-\eta_C}}.$$

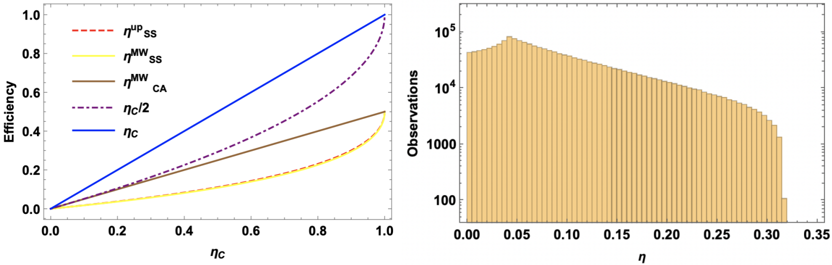


Figure: Left panel: Efficiency versus η_C. Right panel: Histogram of sampled values of η_{SS} for random sampling over a region of the parametric space (ω_c, ω_h). Here, $\eta^{\rm up} = 0.316$.

The upper bound obtained here is much tighter than the corresponding Carnot bound η_C (valid for the quasi-static regime).

Nonadiabatic Otto cycle in the low-temperature limit

JNET2024, https://doi.org/10.1515/jnet-2024-0034

Operation of Otto cycle as a heat engine:

$$W_{\text{ext}} = -W = -(W_{AB} + W_{CD}) \geq 0, \quad Q_h \geq 0, \quad Q_c \leq 0$$

.

Work output for sudden-switch protocol $\left(\lambda = \frac{\omega_c^2 + \omega_h^2}{2\omega_c \omega_h}\right)$ [11] :

$$W_{\text{ext}} = \frac{\omega_h^2 - \omega_c^2}{4\omega_c \omega_h} \left[\omega_c \coth\left(\frac{\beta_h \omega_h}{2}\right) - \omega_h \coth\left(\frac{\beta_c \omega_c}{2}\right)\right].$$

PWC in the low-temperature limit $(\coth(\beta_k \omega_k)/2 \approx 1)$:

$$W_{\text{ext}} \geq 0 \quad \boxed{\Rightarrow \omega_c \geq \omega_h}$$

Positive cooling condition for the refrigerator mode:

$$Q_c \geq 0 \quad \boxed{\Rightarrow (\omega_h - \omega_c)^2 \leq 0}$$

Harmonic Otto cycle with sudden work strokes cannot work as a heat engine or **refrigerator** in the low-temperature limit due to **frictional effects**.

[11] K. Husimi, Prog. Theor. Exp. Phys. **9**, 238 (1953).

Different operational modes of nonadiabatic Otto cycle

Possible operational modes of a thermal machine

$$\begin{aligned} &\text{Engine :} && W_{\text{ext}} \geq 0, Q_h \geq 0, Q_c \leq 0, \\ &\text{Refrigerator :} && W_{\text{ext}} \leq 0, Q_h \leq 0, Q_c \geq 0, \\ &\text{Heater :} && W_{\text{ext}} \leq 0, Q_h \leq 0, Q_c \leq 0, \\ &\text{Thermal accelerator :} && W_{\text{ext}} \leq 0, Q_h \geq 0, Q_c \leq 0, \end{aligned}$$

Different operational regimes of nonadiabatic Otto cycle in high-temperature limit

Engine	Accelerator	Heater	Refrigerator
$z \geq \sqrt{\tau}$	$\sqrt{\frac{\tau}{2-\tau}} \leq z \leq \sqrt{\tau}$	$\sqrt{2\tau - 1} \leq z \leq \sqrt{\frac{\tau}{2-\tau}}$	$z \leq \sqrt{2\tau - 1}, \quad \tau \geq \frac{1}{2}$

Phase-diagram

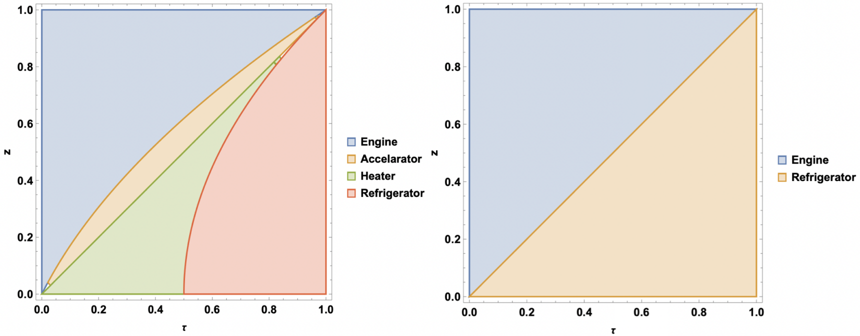


Figure: Left panel: Otto cycle with sudden work strokes. Right panel: Adiabatic Otto cycle.

Power-efficiency curves

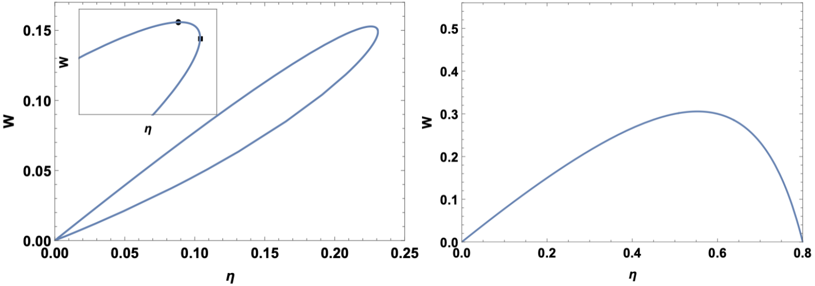


Figure: Left panel: Loop shaped work versus efficiency curve, characteristic of nonadiabatic Otto engines. Right panel: Adiabatic Otto engine.

Conclusions

- Harmonic Otto cycle with sudden work stroke can not work as a heat engine or a refrigerator in the low-temperature regime due to frictional nature of the work strokes.
- The frictional effects in work strokes give rise to richer structure of the phase diagram because of the emergence of heater and accelerator modes, which are inaccessible for the adiabatically driven engine.
- The maximum efficiency that our engine can achieve is 1/2 only.
- The upper bounds on the efficiency (COP) obtained here are much tighter than the corresponding Carnot bound η_C (ζ_C).
- Heat cannot be extracted from the cold reservoir unless $T_c \geq T_h/2$.

THANK YOU