Probing Ultralight Dark Matter with Gravitational Waves

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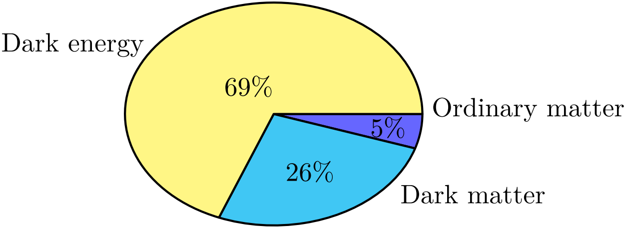
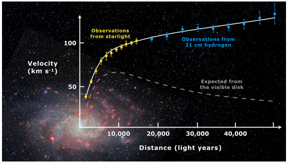
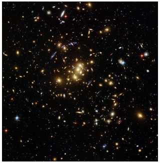
K. Kadota, J. H. Kim, P. Ko, **XYY** [2306.10828] PRD 109, 015022 (2024)
J. H. Kim, **XYY** [2407.14604]

Dark Matter

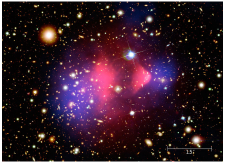




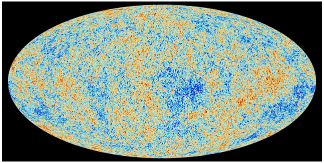
Galaxy Rotation Curves



Gravitational Lensing



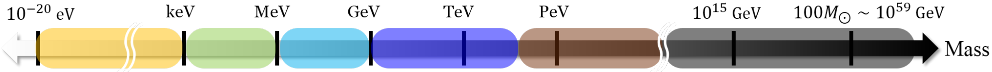
Bullet Cluster



Cosmic Microwave Background

What is dark matter ?

Dark Matter Candidates



Ultralight
(QCD) axion,
hidden photon,

Superlight
sterile ν,
warm DM

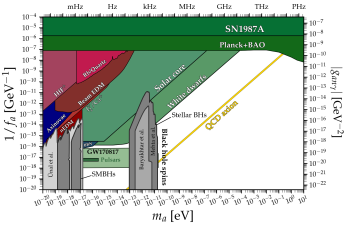
Light
SIMP,
ELDER

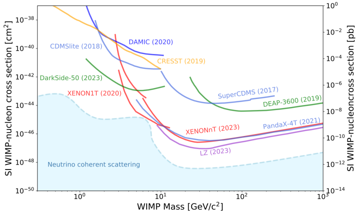
WIMP

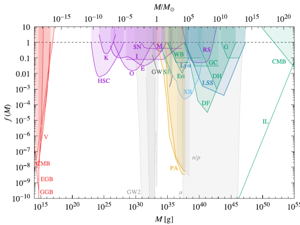
Superheavy
Composite DM

Cosmological Collider

PBH







- Tight bounds are imposed on WIMP
- Next decade: A paradigm shift ?
  - Ultralight Dark Matter ?
  - Primordial Black Holes ?

Ultralight Dark Matter

- $10^{-24}\,\mathrm{eV} \lesssim m \lesssim \mathrm{eV}$
  - Wave-like Nature: $\lambda_{\mathrm{dB}} = h/p$
    "wave" description rather than a "particle" description at macroscopic scales.
  - Forming a Bose–Einstein condensate (BEC) or a superfluid on galactic scales.

- ULDM classes
  - Fuzzy DM
    Ultra-light scalar field under the influence of gravitational potential.
  - Self-interacting ULDM
    Self-interacting scalar field with 2-body (or higher) interaction.
  - DM Superfluid
    Superfluid with specific EoS to reproduce MOND in galaxies.

Simplified Models for Self-Interacting ULDM

Consider a generic real scalar field ϕ

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\mathrm{pl}}^2}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

$$V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{1}{4!}\lambda\phi^4$$

- $\lambda > 0$: repulsive self-interaction
- $\lambda < 0$: attractive self-interaction

In the non-relativistic limit

$$\phi(t, \boldsymbol{x}) = \frac{1}{\sqrt{2m}}\left(e^{-imt}\psi(t, \boldsymbol{x}) + e^{imt}\psi^*(t, \boldsymbol{x})\right)$$

Fast-varying phase

Slowly varying complex scalar
(capturing the dynamics of ϕ)

Soliton

Metric

$$ds^2 = -[1 + 2\Phi(t, \boldsymbol{x})]dt^2 + [1 - 2\Psi(t, \boldsymbol{x})]\delta_{ij}dx^i dx^j$$

Schrödinger-Poisson equations

$$i\dot{\psi} = -\frac{\nabla^2\psi}{2m} + m\left(\Phi + \Phi_{\rm self}\right)\psi$$

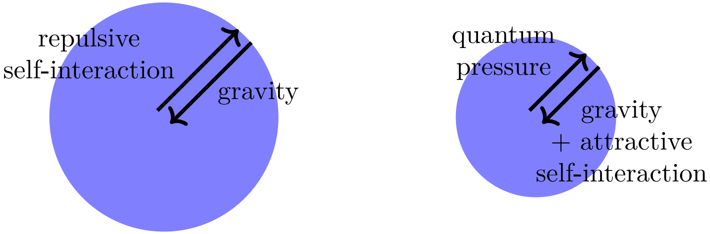
$$\nabla^2\Phi = 4\pi G\rho$$

$$\Phi_{\rm self} \equiv \frac{\lambda|\psi|^2}{8m^3}$$

$$\rho = m|\psi|^2$$

Soliton: equilibrium solution

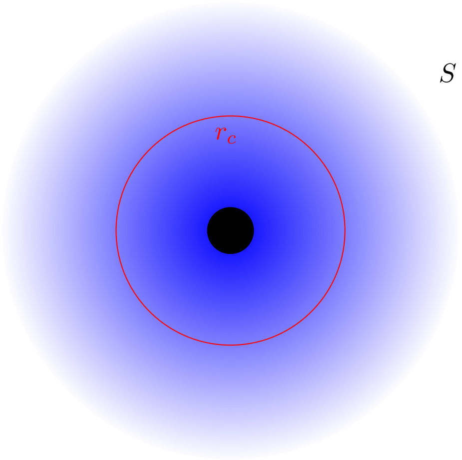
$\lambda > 0$ $\lambda < 0$



Larger soliton Smaller soliton

Gravitational wave probes on self-interacting dark matter surrounding an intermediate mass black hole
[2306.10828] PRD 109, 015022 (2024)

Repulsive Self-Interacting ULDM



Black Hole + DM Halo

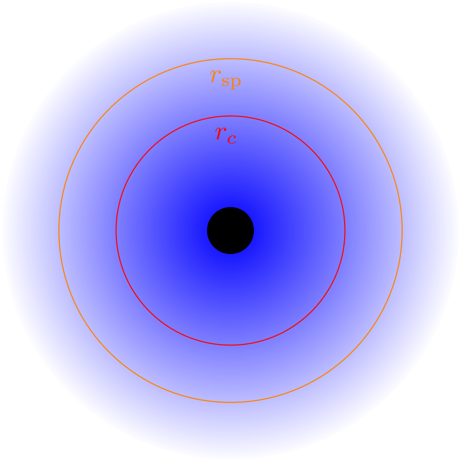
$$S = \int \mathrm{d}^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4}\phi^4\right]$$

$$\lambda > 0, \quad \text{repulsive interaction}$$

$$\rho_{\text{soliton}}(r) = \rho_{\sin}\frac{\sin(r/r_c)}{r/r_c} + \rho_{\cos}\frac{\cos(r/r_c)}{r/r_c}$$

$$r_c \equiv \sqrt{\frac{3\lambda}{16\pi G m^4}}$$

Spike DM Halo around Black Hole



Black Hole + DM Halo

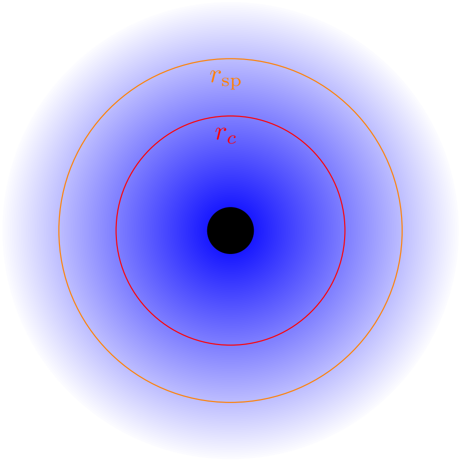
- Spike halo:
  the adiabatic growth of a black hole creates a high density dark matter region.

$$\rho_{\rm spike}(r) = \rho_{\rm sp}\left(\frac{r_{\rm sp}}{r}\right)^{\gamma_{\rm sp}}, \gamma_{\rm sp} = 7/3$$

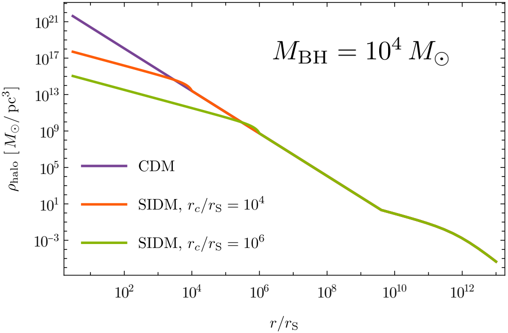
$$\int_{r_{\rm min}}^{5r_{\rm sp}} \rho_{\rm DM}(r) 4\pi r^2 \, \mathrm{d}r = 2M_{\rm BH}$$

$$r_{\rm min} = r_{\rm ISCO} = 3r_{\rm S}$$

Density Profile of DM Halo

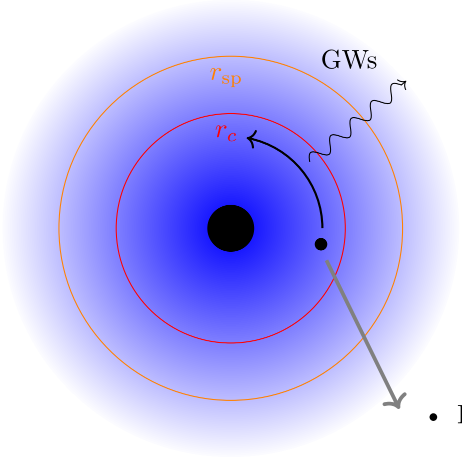


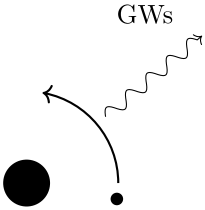
Black Hole + DM Halo



$$\rho_{\mathrm{halo}}(r) = \begin{cases} \rho_{\mathrm{soliton}}(r), & r_{\min} \le r < r_c \\ \rho_{\mathrm{spike}}(r), & r_c \le r < r_{\mathrm{sp}} \\ \rho_{\mathrm{NFW}}(r), & r_{\mathrm{sp}} \le r \le r_{\max} \end{cases}$$

Binary Black Holes





$$q \equiv m_2/m_1 \ll 1$$

- Dynamical friction

$$F_{\rm DF} = \frac{4\pi (G m_2)^2 \rho_{\rm halo}}{v^2} I(v/c_s, \Lambda)$$

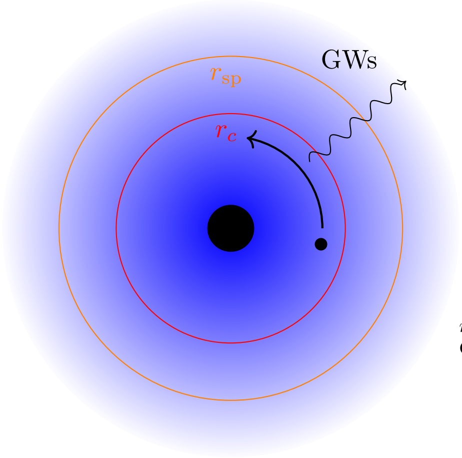
$$c_s^2 = \frac{3\lambda \rho_{\rm halo}}{4m^4}$$

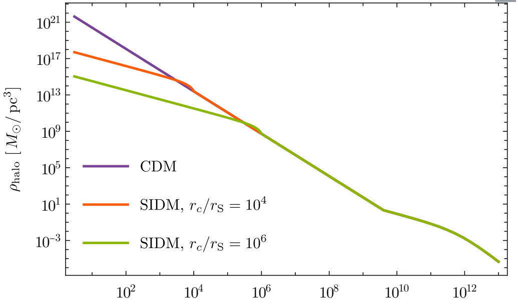
$$\Lambda = \sqrt{m_1/m_2}$$

- Accretion

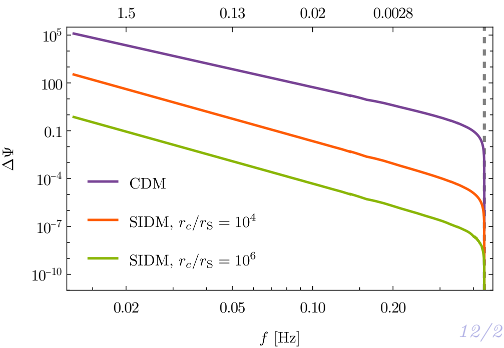
$$\dot{m}_2 = \frac{4\pi (G m_2)^2 \rho_{\rm halo}}{(c_s^2 + v^2)^{3/2}}$$

Dephasing of GWs





$m_1 = 10^4\,M_\odot, m_2 = 1\,M_\odot$
GW frequency is in LISA range



Dephasing of GWs

$$\Delta\Psi = \Psi(\text{vacuum}) - \Psi(\text{with DM halo})$$

Fisher Matrix Analysis

Predict how well the experiment will be able to
constrain the model parameters before doing the experiment.

Fisher information matrix

$$\Gamma_{ij} = \left(\frac{\partial \boldsymbol{d}(f)}{\partial \theta_i}, \frac{\partial \boldsymbol{d}(f)}{\partial \theta_j} \right)_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}}$$

$$\boldsymbol{d}(f) = \left[\frac{\tilde{h}_1(f)}{\sqrt{S_1(f)}}, \frac{\tilde{h}_2(f)}{\sqrt{S_2(f)}}, \ldots, \frac{\tilde{h}_N(f)}{\sqrt{S_N(f)}} \right]^{\mathrm{T}}$$

- $\boldsymbol{\theta}$ represents the vector of parameters with its true value denoted by $\hat{\boldsymbol{\theta}}$.
- $S_i(f)$ is the noise power spectral density for the ith detector.
- $\tilde{h}_i(f)$ is the Fourier transformation of the time domain signal.

Root-mean-squared errors for the parameters

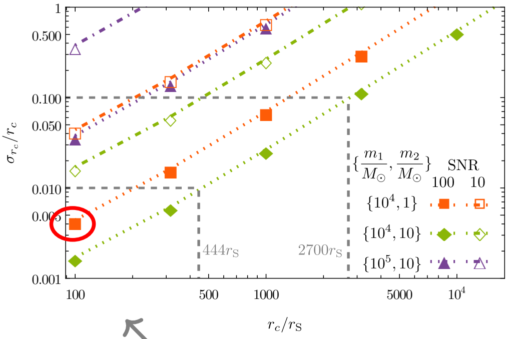
$$\sigma_{\theta_i} = \sqrt{(\Gamma^{-1})_{ii}}$$

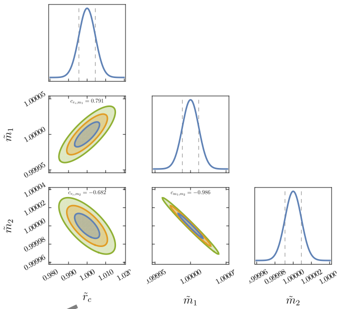
Detectable Regions of Parameter Space

Parameter set

$$\boldsymbol{\theta} = \{r_c; m_1, m_2, D_L, \iota, \chi, \vartheta, \varphi, \phi_{\rm ISCO}, t_{\rm ISCO}\}$$

Half year observation with LISA





Detectable Regions of Parameter Space

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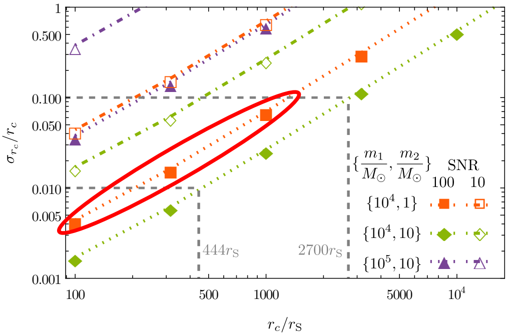
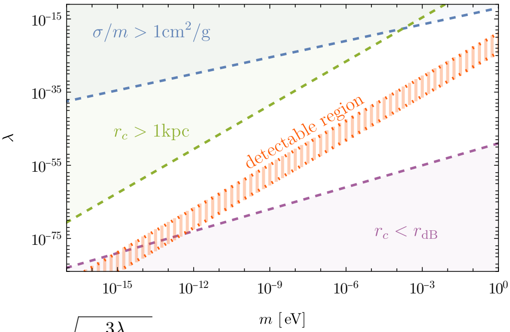




$$r_c \equiv \sqrt{\frac{3\lambda}{16\pi G m^4}}$$

Gravitational Wave Duet by Resonating Binary Black Holes with Axion-Like Particles
[2407.14604]

Axion-Like Particles

Potential of ALPs

$$V(\phi) = m^2 f_a^2 (1 - \cos\frac{\phi}{f_a}) \approx \frac{1}{2} m^2 \phi^2 - \frac{1}{4!}\frac{m^2}{f_a^2}\phi^4$$

- QCD axion $\rightarrow$ Strong CP problem.
- ALPs exhibit a broader spectrum of masses and coupling constants.
- Attractive self-interaction: $\lambda = -m^2/f_a^2 < 0$

General form of ALPs

$$\phi(t, \boldsymbol{x}) = \phi_0(\boldsymbol{x}) \cos(\omega_a t + \Upsilon(\boldsymbol{x}))$$

- $\phi_0(\boldsymbol{x})$ and $\Upsilon(\boldsymbol{x})$ are functions that exhibit slow changes in positions
- Oscillation frequency of ALPs

$$\omega_a = m(1 + \frac{\lambda}{16m^2}\phi_0^2)$$

Metric Oscillation of ALP Soliton

Einstein equation

$$\phi \leftrightarrow T_{ab} \leftrightarrow G_{ab} \leftrightarrow \Psi$$

Metric oscillation

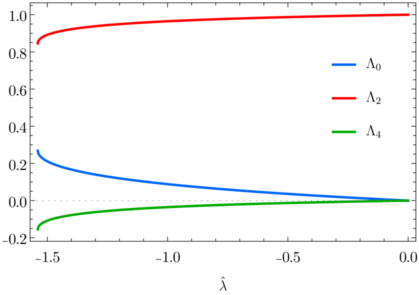
$$\ddot{\Psi} = -4\pi G \bar{\rho}_{\rm DM} \Big[\Lambda_2 \cos(2\omega_a t + 2\Upsilon) + \Lambda_4 \cos(4\omega_a t + 4\Upsilon) \Big]$$

Averaging density of ALPs over $2\pi/\omega_a$

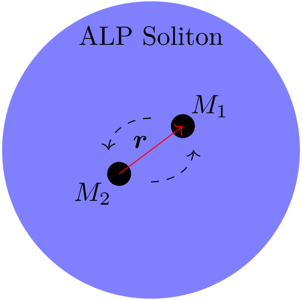
$$\bar{\rho}_{\rm DM} = \frac{1}{2} m^2 \phi_0^2 + \frac{3\lambda}{64} \phi_0^4 + \frac{\lambda^2}{1024 m^2} \phi_0^6$$

Dimensionless parameters
$\Lambda_{2,4}$ are functions of $\hat{\lambda}$
where $\hat{\lambda} \equiv \lambda \bar{\rho}_{\rm DM} / m^4$

$\Lambda_2 \cos(2\omega_a t + 2\Upsilon)$ is the dominant term



Oscillating Force on Binary Black Holes



Geodesic deviation equation for the binary black holes

$$\ddot{r}^i = -R^i{}_{0j0} r^j = -\ddot{\Psi} r^i$$

Force on binary black holes

$$\ddot{\boldsymbol{r}} = -F_{\rm DM} \hat{\boldsymbol{r}}$$

- Oscillating force on binary black holes induced by ALPs

$$F_{\rm DM} = \ddot{\Psi} r = -4\pi G \bar{\rho}_{\rm DM}\, r \Big[\Lambda_2 \cos(2\omega_a t + 2\Upsilon) + \Lambda_4 \cos(4\omega_a t + 4\Upsilon) \Big]$$

Orbital Evolution of Binary Black Holes

Orbital evolution due to oscillating force induced by ALPs

Orbital evolution due to emission of GWs

$$\frac{\mathrm{d}a}{\mathrm{d}t} = -2\sqrt{\frac{a^3}{GM}}\frac{e}{\sqrt{1-e^2}}\sin(\varphi-\varphi_\mathrm{p})F_\mathrm{DM} \quad + \quad \left\langle\frac{\mathrm{d}a}{\mathrm{d}t}\right\rangle = -\frac{64G^3\mu M^2}{5c^5a^3(1-e^2)^{7/2}}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)$$

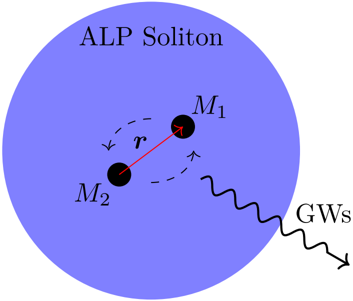
$$\frac{\mathrm{d}e}{\mathrm{d}t} = -\sqrt{\frac{a}{GM}}\sqrt{1-e^2}\sin(\varphi-\varphi_\mathrm{p})F_\mathrm{DM} \quad + \quad \left\langle\frac{\mathrm{d}e}{\mathrm{d}t}\right\rangle = -\frac{304G^3\mu M^2 e}{15c^5a^4(1-e^2)^{5/2}}\left(1+\frac{121}{304}e^2\right)$$

$$\frac{\mathrm{d}\varphi_\mathrm{p}}{\mathrm{d}t} = \sqrt{\frac{a}{GM}}\frac{\sqrt{1-e^2}}{e}\cos(\varphi-\varphi_\mathrm{p})F_\mathrm{DM}$$

$$\frac{\mathrm{d}\varphi}{\mathrm{d}t} = \sqrt{\frac{GM}{a^3}}\frac{[1+e\cos(\varphi-\varphi_\mathrm{p})]^2}{(1-e^2)^{3/2}}$$

$$M = M_1 + M_2$$
$$\mu = M_1M_2/M$$

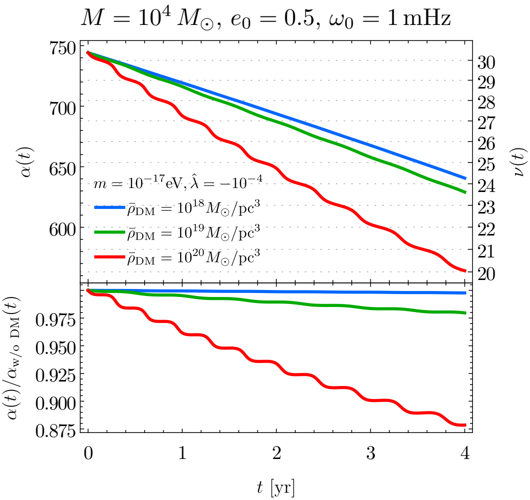
Non-zero eccentricity



Evolution of Semi-major Axis

Dimensionless semi-major axis
$\alpha \equiv a/R_*$
$R_* \equiv GM/c^2$

Raito with respect to vacuum case



Frequency ratio
$\nu \equiv 2\omega_a/\omega$

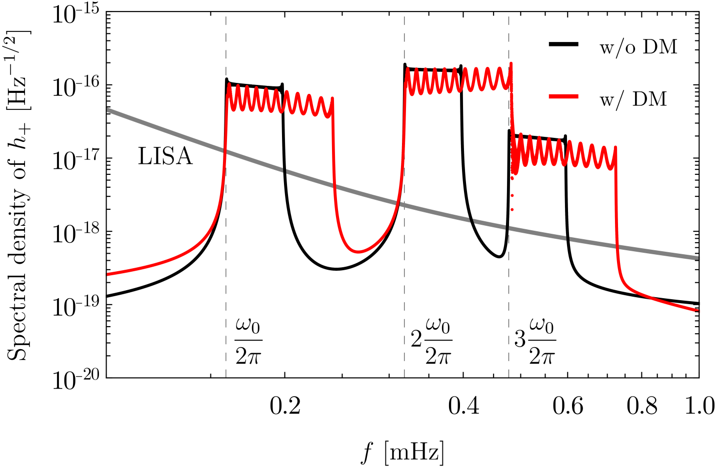
ALPs frequency
$\omega_a = m(1 + \frac{\lambda}{16m^2}\phi_0^2)$

Binary frequency
$\omega = \sqrt{GM/a^3}$

- Distinctive oscillatory features in α, characterized by periodic dips occurring at specific intervals of ν.
- This behavior highlights the dynamic interaction between the gravitational effects of the binary system and the surrounding ALPs environment.

Gravitational Waves

$$h_+(t) =_{\rm ret} \frac{1}{1-e^2}\frac{4(GM_c)^{5/3}\omega^{2/3}}{d_L c^4}\left\{\frac{1+\cos^2\iota}{2}\cos(2\varphi-2\beta)+\frac{e}{4}\sin^2\iota\left[\cos(\varphi-\varphi_{\rm p})+e\right]\right.$$
$$\left.+\frac{e}{8}(1+\cos^2\iota)\left[5\cos(\varphi-2\beta+\varphi_{\rm p})+\cos(3\varphi-2\beta-\varphi_{\rm p})+2e\cos(2\beta-2\varphi_{\rm p})\right]\right\}$$



Each broad peak corresponds to the n-th harmonic in Fourier decomposition of Keplerian motion.

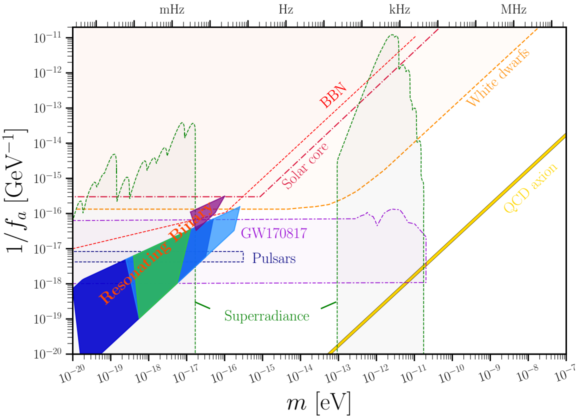
$m = 10^{-17}\,\mathrm{eV}$
$\hat{\lambda} = -10^{-4}$
$\bar{\rho}_{\rm DM} = 10^{20}\, M_\odot/\,\mathrm{pc}^3$

$d_L = 0.1\,\mathrm{Gpc}$
$\{\iota, \beta\} = \pi/4$

- Identifying oscillatory patterns in GWs may indicate the existence of ALPs.

Detectable Regions of ALPs Parameter Space

4 years of observation time (LISA), SNR = 100



$M = 10^2\, M_\odot$
$e_0 = 0.5$
$\bar{\rho}_{\rm DM} = 10^{18}\, M_\odot/\,{\rm pc}^3$

$M = 10^4\, M_\odot$

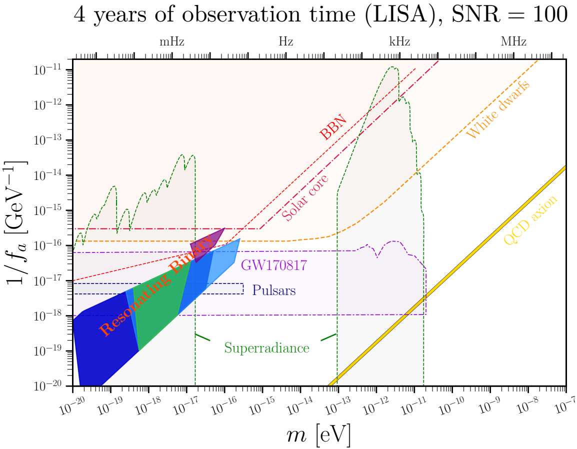
$e_0 = 0.3$

$\bar{\rho}_{\rm DM} = 10^{16}\, M_\odot/\,{\rm pc}^3$

Fisher Matrix Analysis

$$\boldsymbol{\theta} = \{m, \hat{\lambda}, \bar{\rho}_{\rm DM}; M, \eta, \omega_0, e_0, \varphi_0, d_L, \iota, \beta, \theta_{\rm s}, \phi_{\rm s}, \chi\}$$

Detectable Regions of ALPs Parameter Space



$M = 10^2\, M_\odot$
$e_0 = 0.5$
$\bar{\rho}_{\rm DM} = 10^{18}\, M_\odot/\,\mathrm{pc}^3$

$M = 10^4\, M_\odot$

$e_0 = 0.3$

$\bar{\rho}_{\rm DM} = 10^{16}\, M_\odot/\,\mathrm{pc}^3$

- This result does not rely on presupposed ALPs interaction with photons or nucleons, highlighting potential of GWs to detect ALPs solely through their gravitational effects.
- This method stands as one of solutions in the “nightmare scenario” for dark matter detection, when ALPs have no couplings to the Standard Model particles.

Summary

- Ultralight DM has been receiving a lot of attention in the past few years given its interesting property of forming a Bose–Einstein condensate or a superfluid on galactic scales.

- The accretion of DM around black holes could lead to the formation of surrounding halo. The gravitational waves from intermediate mass ratio inspiral with surrounding halo can be probes on the self-interacting ultralight DM.

- The resonant interactions between binary black holes and axion-like particles can generate distinct oscillatory patterns in gravitational waves, which could be detected by upcoming experiments such as LISA, highlighting the potential of gravitational waves to detect axion-like particles solely through their gravitational effects.

Backup

Orbital Evolution Equations

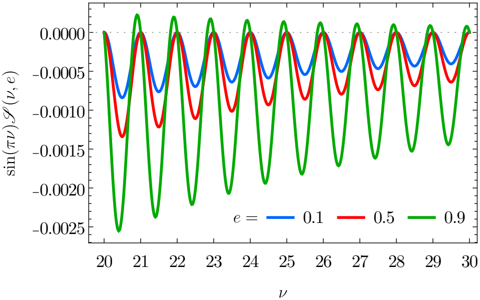
$$\left\langle \frac{\mathrm{d}\alpha}{\mathrm{d}\tau} \right\rangle = \zeta \alpha^{5/2} \frac{2e}{\sqrt{1-e^2}} \Big[\Lambda_2 \sin(\pi\nu + \gamma) \mathscr{S}(\nu, e) + \Lambda_4 \sin(2\pi\nu + 2\gamma) \mathscr{S}(2\nu, e) \Big] - \frac{64\eta}{5\alpha^3 (1-e^2)^{7/2}} \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right),$$

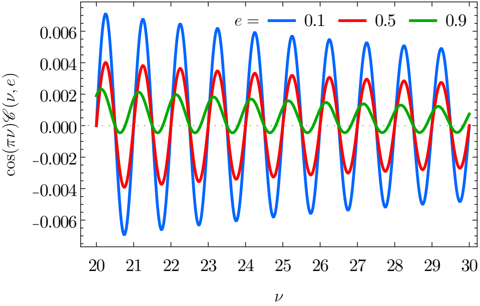
$$\left\langle \frac{\mathrm{d}e}{\mathrm{d}\tau} \right\rangle = \zeta \alpha^{3/2} \sqrt{1-e^2} \Big[\Lambda_2 \sin(\pi\nu + \gamma) \mathscr{S}(\nu, e) + \Lambda_4 \sin(2\pi\nu + 2\gamma) \mathscr{S}(2\nu, e) \Big] - \frac{304\eta e}{15\alpha^4 (1-e^2)^{5/2}} \left(1 + \frac{121}{304} e^2 \right),$$

$$\left\langle \frac{\mathrm{d}\varphi_{\mathrm{p}}}{\mathrm{d}\tau} \right\rangle = -\zeta \alpha^{3/2} \frac{\sqrt{1-e^2}}{e} \Big[\Lambda_2 \cos(\pi\nu + \gamma) \mathscr{C}(\nu, e) + \Lambda_4 \cos(2\pi\nu + 2\gamma) \mathscr{C}(2\nu, e) \Big],$$

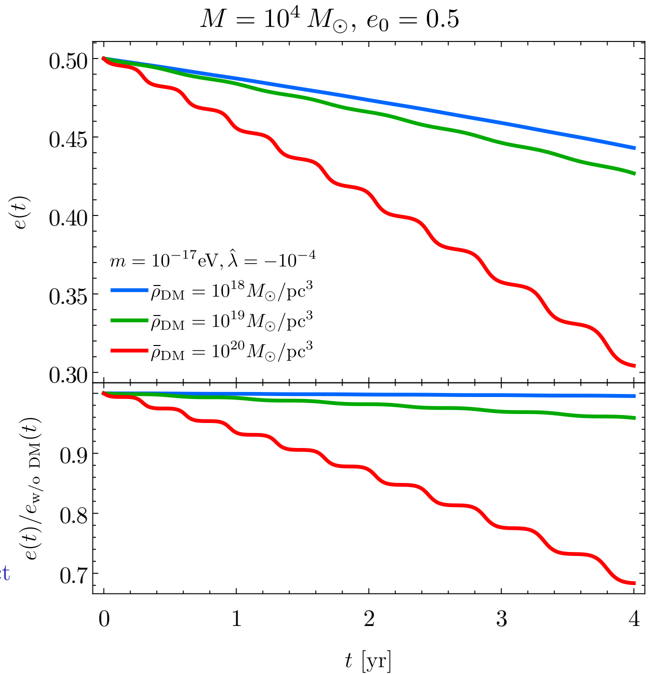
$$\left\langle \frac{\mathrm{d}\varphi}{\mathrm{d}\tau} \right\rangle = \alpha^{-3/2}.$$

$$\tau \equiv \frac{tc}{R_*}, \quad \eta \equiv \frac{\mu}{M}, \quad \zeta \equiv \frac{4\pi G \bar{\rho}_{\mathrm{DM}} R_*^2}{c^2},$$



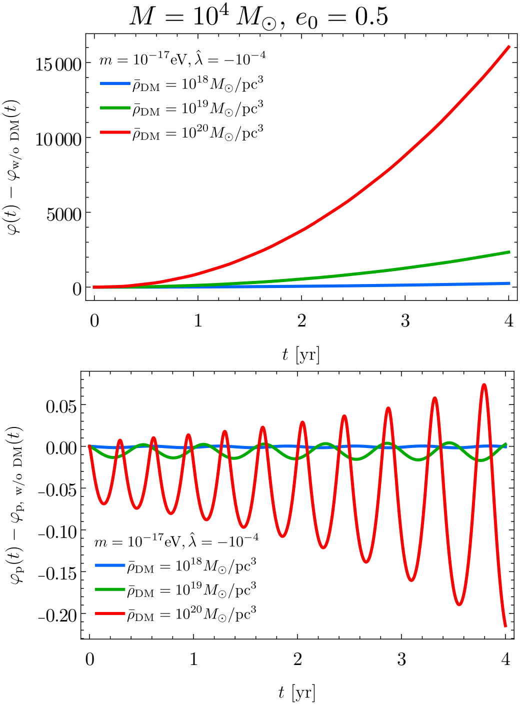


Evolution of Eccentricity



Eccentricity decreases in time

Raito with respect to vacuum case


$M = 10^4\, M_\odot,\ e_0 = 0.5$
$m = 10^{-17}\mathrm{eV}, \hat{\lambda} = -10^{-4}$
$\bar{\rho}_{\mathrm{DM}} = 10^{18} M_\odot/\mathrm{pc}^3$
$\bar{\rho}_{\mathrm{DM}} = 10^{19} M_\odot/\mathrm{pc}^3$
$\bar{\rho}_{\mathrm{DM}} = 10^{20} M_\odot/\mathrm{pc}^3$
$\varphi(t) - \varphi_{\mathrm{w/o\ DM}}(t)$
t [yr]
$\varphi_{\mathrm{p}}(t) - \varphi_{\mathrm{p,\ w/o\ DM}}(t)$
t [yr]