Bremsstrahlung and Radiation Damping

An accelerated, electrically charged, classical particle radiates electromagnetic waves. The energy that comes out with this radiation is known to be proportional, in the nonrelativistic limit, to the square of the acceleration. Naturally, one is lead to believe this lost energy must be accounted for by the kinetic energy loss of the particle itself, and this leads to the Abraham–Lorentz–Dirac force, decelerating the particle. In the small velocity limit this force, Abraham–Lorentz–Dirac force, is proportional to the time-derivative of the acceleration. Review these two facts, which can be found in many graduate-level textbook on classical electrodynamics.

What happens if the charge particle is accelerated uniformly, which means that the acceleration has no time-derivative? In this case, radiation energy remain constant and nontrivial, yet the damping force is zero. Explain how this can be consistent with energy conservation? It is easy to see that the time-integrated version of the energy conservation works well if at the initial and the final time the acceleration is turned off, but the energy conservation should hold at all time.

This problem been addressed repeatedly by well known physicists over the last century in many different manners, and some aspects are available in text books. You might be able to solve the problem, or perhaps it may serve you better to review existing materials.

Bremsstrahlung vs. the Equivalence Principle

In general relativity, a massive (charged) particle under gravity and no other forces follows time-like geodesics which is as nearest to the inertial motion as one may define in general relativity. That is, there is no "force" term on the right hand side of the equation of motion for a freely falling particle. Does this mean that there is no "acceleration" so no electromagnetic radiation from a freely falling charged particle? An old paper by de Witt and Brehme, Annals of Physics 9, 220-259 (1960), appears to be the earliest serious attempt on the issue, where the authors extended the radiation damping to the context of curved spacetime.

Conversely, a massive object sitting on the surface of a static table against a uniform downward gravitational field is said to be, according to the equivalence principle, experiencing a uniform acceleration. Assuming the latter table is sitting on the surface of a non-rotating planet with no other gravitating bodies in its universe, should we expect Bremsstrahlung from this static particle as measured by inertial observers very far away from the planet? Bondi and Gold back in 1955 was one of the first to address the issue seriously, where they essentially evaded the problem by saying that a uniform gravitational field is impossible to realize.

A simpler and cleaner manifestation of this issue shows up, again, in the context of the Bremsstrahlung from a uniformly accelerated charged particle in a flat Minkowskii spacetime. If you take the equivalence principle seriously, one would arrive at the conclusion that there is no radiation, although you have already discussed the nontrivial energy budget in the Part 1. A definitive resolution, in my opinion, of this particular version of the problem was worked out by David Boulware decades ago: Annals of Physics Vol. 124, 169-188 (1980).

Go over these past works and draw your own conclusions.