Quantum Gravity

§ Why ?

gravitational wave detected !

→ Need to quantize the theory of gravity in order to avoid the ultra-violet catastrophe.

§ Path-integral formulation

$$\int \underbrace{\mathcal{D}g_{\mu\nu}(\sigma)}_{\text{metric}} \; e^{-S_E[g_{\mu\nu}]}$$

Euclidean two-dim'l "<u>pure</u>" gravity.

curvature

$$S_E[g_{\mu\nu}] = \frac{1}{4\pi}\int d^2\sigma \sqrt{g}\cdot\left[\Phi_0 R + \Lambda\right]$$

const. $\sim \begin{bmatrix}\text{Newton}\\ \text{const}\end{bmatrix}^{-1}$ cosmological const.

① note that   "topological"

$$\frac{1}{4\pi}\int d^2\sigma\sqrt{g}\cdot R = \chi(M)$$ <u>Euler charateristic</u> !

$$g_s = e^{\Phi_0}$$

$$\int Dg_{\mu\nu}\, e^{-S_E[g_{\mu\nu}]} = \sum_{h} g_s^{-\chi(M_h)} \cdot \underbrace{\int [Dg_{\mu\nu}]_h \left(e^{-\frac{\Lambda}{4\pi}}\right)^{\int d^2\sigma \sqrt{g_h}}}$$

h : number of genus

M_h

$$\chi(M_h) = 2(1-h)$$

$$\int d^2\sigma \sqrt{g_h} = \mathrm{Area}\,(g^h_{\mu\nu})$$

$$= \sum_{h} g_s^{-\chi(M_h)} \underbrace{\int [Dg_{\mu\nu}]_h \left(e^{-\frac{\Lambda}{4\pi}}\right)^{\mathrm{Area}(g_{\mu\nu})}}$$

top'l ~~sum~~ expansion

$\boxed{Q}$: how to define this ?

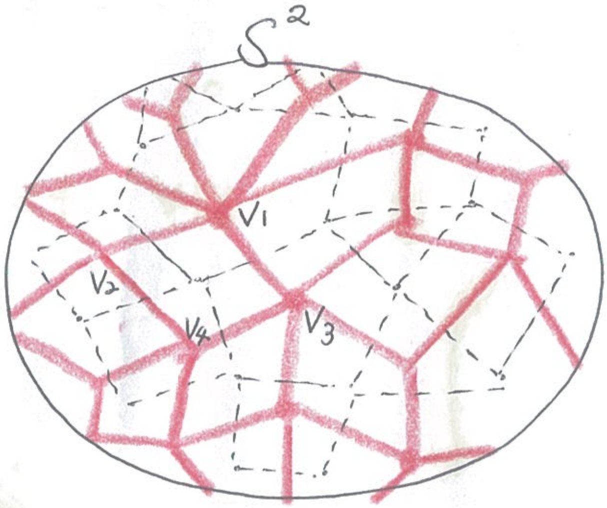
(A) continuum approach

discretization of 

followed by the continuum limit

let's take this approach ! ⟹

§ Discretization of surface



quadrangle lattice

(quadrangulation)

$$= \sum_{\substack{\text{all} \\ \text{quadrangulation}}} \left(e^{-\frac{\Lambda}{4\pi} \square} \right)^{V}$$

area of each quadrangle

number of quadrangles $\cong$ # of vertices of dual lattice

" random matrix model "

$\Uparrow$ ~~continuum limit~~

Use

① Feynman diagram & large N

② Saddle-point approximation

③ orthogonal polynomial method

$$\int [DM]\, e^{-V(M)}$$

$N \times N$ hermitian matrix

§ continuum limit ?

$$\left.\begin{array}{l} V \to \infty \\ ▨ \to 0 \\ \text{s.t. } V▨ \sim \text{const} \end{array}\right\} = \text{double scaling limit of matrix model}$$

[<u>remark</u> ~ rare-event analysis of statistical system]