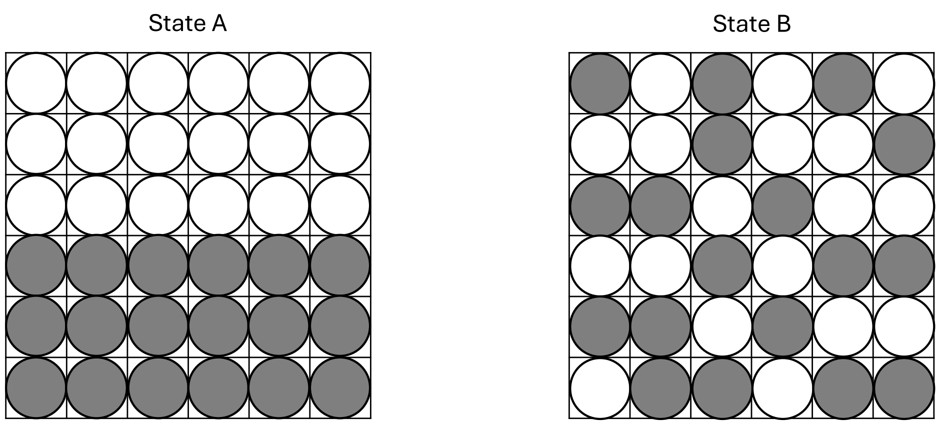
Entropy and order

1. Entropy and disorder

In statistical mechanics, **Boltzmann's equation** is a probability equation relating the entropy S to the number of microstates (Ω or W) corresponding to a particular macrostate:

$$S = k_B \ln \Omega$$

where $k_B$is the Boltzmann constant and $\ln$ is the natural logarithm function. In short, the Blotzmann formula shows the relationship between entropy and the number of ways the atoms or molecules of a certain kind of thermodynamic system can be arranged.



Assume that the two different types of 2D particles are approximately the same size, and they are placed on a space subdivided into a square lattice whose cells are the size of the particles. State A in above figure represent a microstate of a phase separation (macrostate). State B represents a microstate of mixing phase (macrostate).

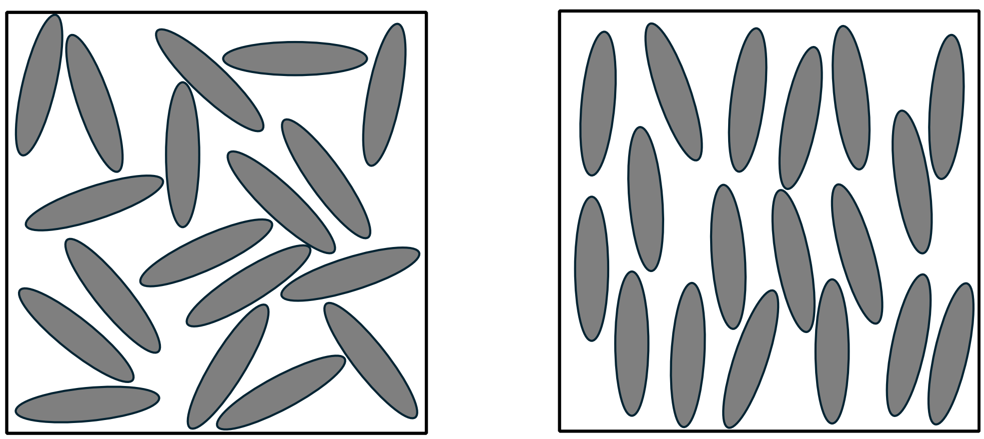
Q. Using the Blotzmann's equation, express the ΔS_{mix} (from the phase separation to the mixed phase) as function of the number of white particles (N_1) and the number of gray particles (N_2). If necessary, you can use Stirling's approximation.

Q. Why the mixed phase has a higher entropy than the separated phase?

2. Entropy and order

Typically, crystals have a unit cell that is periodically arranged to fill the space, and a crystal phase exhibits both positional order (with building blocks arranged in an ordered lattice) and orientational order (with building blocks mostly aligned in the same direction), unless the building blocks are isotropic in shape. In contrast, liquid crystals exhibit orientational order without positional order.

Isotropic (disordered) phase | Liquid crystal phase



Considering the Boltzmann's equation, which phase has higher entropy? Why?