QIFT: Unruh-like and Bubble wall friction

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Contents

QFT vs Inhomogeneous field theory (IFT)

- (canonical, renormalizable, local) **QFT**

$$S = \int d^d x \, \mathcal{L}(\phi_a, \partial_\mu \phi_a; m, g_n)$$ Constant ➔ Poincare symmetric

- Classical and **quantum approaches**
- Preferred vacuum (Minkowski spacetime: global vacuum)
- Canonical quantization:

$$[\phi_a(t, \vec{x}), \pi_a(t, \vec{y})] = i\delta^{d-1}(\vec{x} - \vec{y}) \qquad \pi_a(x) \equiv \frac{\partial \mathcal{L}}{\partial \dot{\phi}_a}$$

- Inhomogeneous field theory (IFT)

$$S = \int d^d x \, \mathcal{L}(\phi_a, \partial_\mu \phi_a; m(x), g_n(x))$$

Inhomogeneous field theories (IFT)

- **High-Energy UV theory:**

  - introducing the Poincare symmetry is natural because most fields are dynamical

  - exampled: string theory, super Yang-Mills theory, ABJM theory, etc

- **Intermediate energy scale:**

  - some fields become non-dynamical

  - examples: field theories on curved spacetime (FTCS)

  - various non-dynamical field strengths in string or M-theory

- **Low-energy scale:**

  - **Inhomogeneity becomes prevalent!!**

  - examples: lattice structures, and experimental setups, specific phenomena like doping, superconductivity, and defects models in condensed matter physics

Questions related with Inhomogeneous field theories

- **What are classical effects for IFTs?**
  - Construction of Supersymmetric IFTs
  - BPS solutions in various IFTs
- **How to quantize IFTs?**
  - Renormalized Hadamard two-point functions and quantum EM tensor including leading quantum effects
- **To which physical systems can the quantum effects of IFT be applied?**
  - Unruh-like effects
  - Higgs condensate bubble expansion (assuming 1^{st} order phase transition)

- **What are classical effects for IFTs?**
  - Construction of Supersymmetric IFTs
  - BPS solutions in various IFTs

Supersymmetric IFT

- **N = 3 Inhomogeneous mABJM in 3d and N=1 inhomogeneous mSYM in 4d**

$$\mathcal{L}_{\text{ImABJM}} = \mathcal{L}_{\text{mABJM}} + m'(x) M_A{}^B Y^A Y_B^\dagger$$

ImABJM [Kim-OK,Kim-Kim-Kim-Kwon, 2018],
ImSYM [Arav-Gauntlett-Roberts-Rosen, Kim-OK-Tolla]

- Supersymmetric IFT models: Inhomogeneous CSH, AH in 3d and real scalar model in (1+1)-dim. [Kim-OK-Song-Kim, 2024, Kim-Jeon-OK-Song-Kim, 2024 Kwon-Kim-Kim, 2022]

$$\mathscr{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \overline{D_\mu \phi} D^\mu \phi - \frac{g^2}{2} [|\phi|^2 - v_0^2 - \sigma(\boldsymbol{x})]^2 + sg\sigma(\boldsymbol{x}) B$$

$$\mathcal{L} = -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi + i\bar{\psi}\gamma^\mu \partial_\mu \psi + i\frac{\partial^2 W}{\partial \phi^2} \bar{\psi}\psi - \frac{1}{2}\left(\frac{\partial W}{\partial \phi}\right)^2 \mp \frac{\partial W}{\partial x}$$

Inhomogeneous vacuum configuration

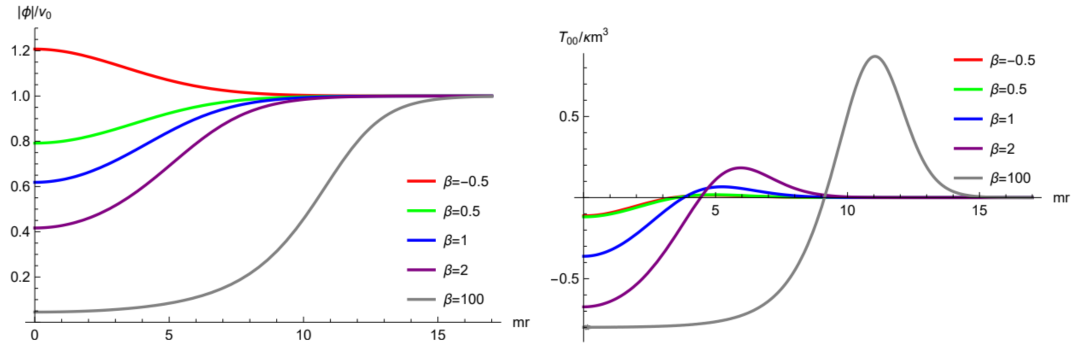
- Inhomogeneous Chern-Simons Higgs model $v^2(\mathbf{x}) = v_0^2 + \sigma(\mathbf{x})$

$$\mathscr{L} = \frac{\kappa}{2}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho - \overline{D_\mu\phi}D^\mu\phi - \frac{1}{\kappa^2}|\phi|^2(|\phi|^2 - v^2(\boldsymbol{x}))^2 + s\sigma(\boldsymbol{x})B$$

- Rotationally symmetric configuration: $\sigma(\boldsymbol{x}) = -\beta v_0^2 e^{-\alpha^2 m^2 r^2}$

$$\frac{d^2\ln|\phi|^2}{dr^2} + \frac{1}{r}\frac{d\ln|\phi|^2}{dr} = \frac{4}{\kappa^2}|\phi|^2[|\phi|^2 - v_0^2(1 - \beta e^{-\alpha^2 m^2 r^2})]$$

- E=0 inhomogeneous vacuum configuration: ImCSH, ImAH, etc



(a) (b)

- **How to quantize IFTs? (one possible way)**
  - Renormalized Hadamard two-point functions and quantum EM tensor including leading quantum effects

Quantization of IFT

- In IFT, there is no preferred vacuum because Poincaré symmetry is absent.
- There is no guiding principle of quantization in IFT.
- On the other hand, in field theory on curved space(FTCS), a preferred vacuum does not exist, just as in IFT. Therefore, the canonical quantization method cannot be used, but the algebraic quantization method is employed instead. ➔ Hadamard two-point function
- Classical conversion relation between FTCS and IFT in (1+1)-dimensions.
- We propose to promote this classical conversion relation to the quantum level.

Quantization of IFT

- Conversion relation between FTCS and IFT in (1+1)-dimensions

Scalar field in 2D spacetime background (FTCS)

EPJP138(2023)202 JH, O-K. Kwon, S.A Park, S.-H. Yi

$$ds^2 = \frac{1}{(a+e^{-bx})^2}\left(-dt^2 + dx^2\right) \qquad ab = \frac{m_0}{2\xi}$$

$$S_{\mathrm{FTCS}} = \int_{\mathcal{M}} d^2x\sqrt{-g}\Big(-\frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}m_0^2\phi^2 - \frac{1}{2}\xi\mathcal{R}\phi^2\Big)$$

$$(-\Box + m_0^2 + \xi\mathcal{R})\phi = 0, \qquad \Box = \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\right)$$

Quantization of IFT

- Conversion relation between FTCS and IFT in (1+1)-dimensions

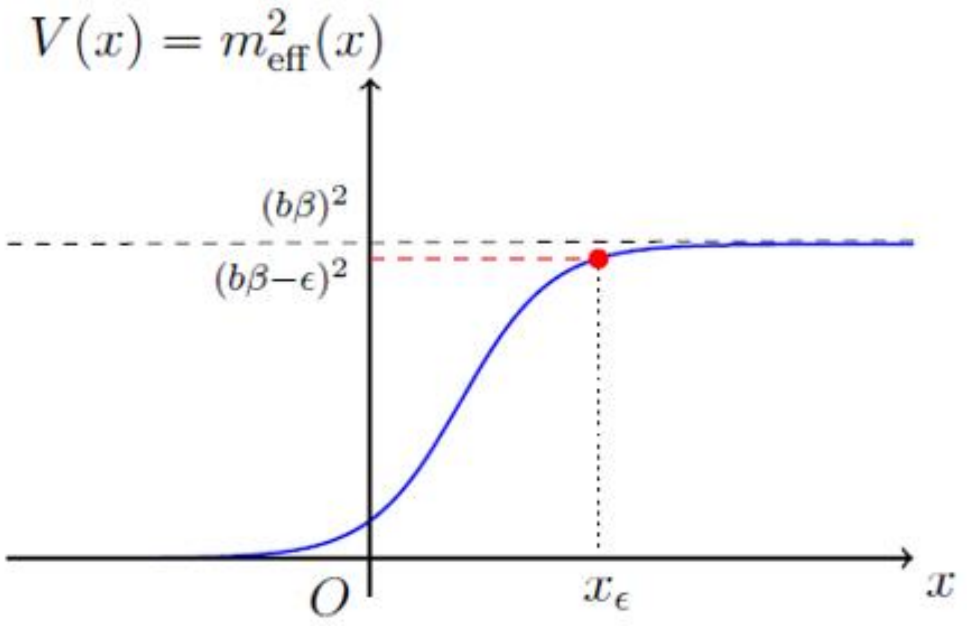
Scalar field in 2D Minkowski spacetime with position dependent mass (IFT)

$$S_{\text{IFT}} = \int d^2x \left[-\frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}m_{\text{eff}}^2(x)\phi^2 \right]$$

$$m_{\text{eff}}^2(x) = m^2(x) + m'(x), \quad m(x) = m_0 e^{\omega(x)}$$

$$e^{\omega(x)} = \frac{1}{a + e^{-bx}}, \qquad m_{\text{eff}}^2(x) = \frac{(m_0^2 e^{bx} + 2\xi a b^2)e^{bx}}{(ae^{bx}+1)^2}$$

$$\left(-\partial^2 + m_{\text{eff}}^2(x)\right)\phi = 0, \qquad \partial^2 = \eta^{\mu\nu}\partial_\mu\partial_\nu$$



Quantization of IFT

- ## Energy-Momentum Tensor in IFT

Non-existence of Poincare symmetry in IFT :
Defining the energy-momentum tensor in IFT as a conserved current is problematic.

Our IFT model possesses time translation symmetry ➔ the energy of the system is conserved. ➔ the (tt)-component of the energy-momentum tensor can be constructed canonically.

No explicit criteria exist for determining the remaining components of the energy-momentum tensor.

Quantization of IFT

- **In the Theory of Gravity (without Poincaré symmetry)**:

  - The energy-momentum tensor can be defined as the source of the gravitational field.

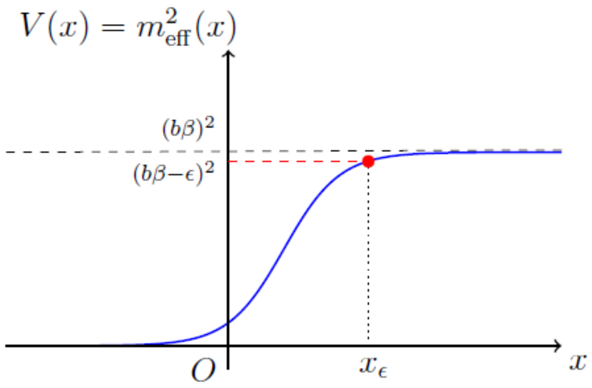
  - The energy-momentum tensor in IFT is directly read from the energy-momentum tensor in FTCS, using the **conversion relation** between FTCS and IFT.

  - This conversion relation provides a framework for **renormalization** of the energy-momentum tensor in QIFT.

Positive frequency Wightman function at a position of x_ϵ

Positive frequency Wightman function (R,ϵ - vacuum)

$$G^+_\epsilon(x,x') \simeq {}^{\epsilon}_{\rm R}\langle 0|\phi(x)\phi(x')|0\rangle^{\epsilon}_{\rm R} \simeq \int_0^\infty \frac{dk}{4\pi\omega_k}\sum_{i=\pm} v_k^{(i)}(x)\left(v_k^{(i)}(x')\right)^*$$



$$\left(-\partial^2 + m^2_{\rm eff}(x)\right)\phi = 0, \qquad \partial^2 = \eta^{\mu\nu}\partial_\mu\partial_\nu \qquad m^2_{\rm eff}(x) = \frac{(m_0^2 e^{bx} + 2\xi ab^2)e^{bx}}{(ae^{bx}+1)^2}$$

$$v_k^{(-)}(\boldsymbol{x}) = (1+e^{-bx})^{2\xi} F\left(A, A-C+1; A-B+1 \,\middle|\, -e^{-bx}\right) e^{-i(\omega t - kx)},$$

$$v_k^{(+)}(\boldsymbol{x}) = (1+e^{-bx})^{2\xi} F\left(B, B-C+1; B-A+1 \,\middle|\, -e^{-bx}\right) e^{-i(\omega t + kx)},$$

$$A = \frac{i}{b}(\omega - k) + \beta, \qquad B = \frac{i}{b}(\omega + k) + \beta, \qquad C = 1 + \frac{2i\omega}{b}, \qquad k^2 \equiv \omega^2 - (2b\xi)^2 \qquad \beta = 2\xi = \frac{m_0}{ab}$$

Singular structure of the Wightman function in 2D

$$G^+(\boldsymbol{x},\boldsymbol{x}') = \frac{1}{4\pi}\left(V(\boldsymbol{x},\boldsymbol{x}')\ln\left[\mu^2\sigma(\boldsymbol{x},\boldsymbol{x}')\right] + W(\boldsymbol{x},\boldsymbol{x}';\mu)\right)$$

Positive frequency Wightman function $G^{+}(x,x') = \langle 0|\phi(x)\phi(x')|0\rangle$

Hadamard two-point function: $G_H(x,x') = \langle 0|\{\phi(x),\phi(x')\}|0\rangle$

Hadamard functions

- $2\,\sigma(\boldsymbol{x},\boldsymbol{x}')$: the square of the geodesic distance between the points $\boldsymbol{x}$ and $\boldsymbol{x}'$, Synge function
- $V(x,x'), W(x,x')$: symmetric biscalar functions that remain regular as $\boldsymbol{x}'$ approaches $\boldsymbol{x}$, which are determined by the geometry and the field equation.
- $W(x,x')$ encodes **quantum effect of** $|0\rangle^{\epsilon}_{\mathrm{R}}$.

Key Properties of Energy-Momentum Tensor $\langle T_{\mu\nu} \rangle_H$ in Hadamard State

1. **Physical Validity**
   - The Hadamard state ensures that $\langle T_{\mu\nu} \rangle_H$ provides physically meaningful and finite values.
   - This avoids divergences and supports consistent physical interpretation in curved spacetimes.
2. **Covariant Conservation**
   - The energy-momentum tensor satisfies the covariant conservation law:
   $$\nabla^\mu \langle T_{\mu\nu} \rangle_H = 0$$
   - This is essential for maintaining energy conservation and consistency with general relativity.

Renormalized Positive frequency Wightman function

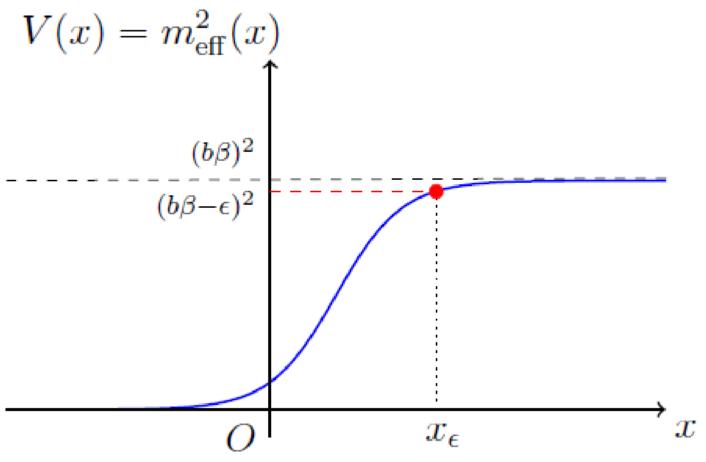
$$G^+_{\text{ren}}(x,x';\mu) = G^+_\epsilon(x,x') - \frac{1}{4\pi}V(x,x')\ln\left[\mu^2\sigma(x,x')\right] = \frac{1}{4\pi}W(x,x';\mu)$$

$$G^+_\epsilon(x,x') \simeq {}^{\epsilon}_{\mathrm{R}}\langle 0|\phi(x)\phi(x')|0\rangle^{\epsilon}_{\mathrm{R}} \simeq \int_0^\infty \frac{dk}{4\pi\omega_k}\sum_{i=\pm} v_k^{(i)}(x)\left(v_k^{(i)}(x')\right)^*$$

$$v_k^{(-)}(\boldsymbol{x}) = (1+e^{-bx})^{2\xi}F\left(A, A-C+1; A-B+1 \,\middle|\, -e^{-bx}\right)e^{-i(\omega t - kx)},$$

$$v_k^{(+)}(\boldsymbol{x}) = (1+e^{-bx})^{2\xi}F\left(B, B-C+1; B-A+1 \,\middle|\, -e^{-bx}\right)e^{-i(\omega t + kx)},$$

Calculation



$$G^+_{\rm ren}(\boldsymbol{x},\boldsymbol{x}';\mu) = \frac{1}{4\pi}W(\boldsymbol{x},\boldsymbol{x}';\mu) = G^+_\epsilon(\boldsymbol{x},\boldsymbol{x}') - \frac{1}{4\pi}V(\boldsymbol{x},\boldsymbol{x}')\ln\left[\mu^2\sigma(\boldsymbol{x},\boldsymbol{x}')\right]$$

Renormalized Wightman function

$$\begin{aligned} G^+_{\rm ren}(\boldsymbol{x},\boldsymbol{x}';\mu) &= \frac{1}{4\pi}\left(1+\frac{b^2}{16}\left(-(t-t')^2+(x-x')^2\right)\right)\ln\left(\frac{2\mu^2}{a^2b^2}\right) \\ &+ \frac{1}{2\pi}\left(-\gamma+\ln 2\right) + \frac{b^2}{32\pi}\left(-(t-t')^2+(x-x')^2\right)\left(1-\gamma+\ln 2\right) \\ &- \frac{1}{2\pi}\left(1+\frac{b^2}{24}(x-x')^2+\frac{b^2}{16}\left(-(t-t')^2+(x-x')^2\right)\right)e^{-b\frac{x+x'}{2}} + \cdots \end{aligned}$$

where the ellipsis $\cdots$ denotes higher-order terms in $(t-t')$, $(x-x')$, and $e^{-b\frac{x+x'}{2}}$.

Point-splitting method

$$S_{\mathrm{FTCS}} = \int d^2x\sqrt{-g}\Big[-\frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}m_0^2\phi^2 - \xi\mathcal{R}\phi^2\Big]$$

$$P_x \equiv -\Box_x + m_0^2 + \xi\mathcal{R}$$

$$T_{\mu\nu} = \nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}g_{\mu\nu}\Big[(\nabla\phi)^2 + m_0^2\,\phi^2\Big] + \xi\Big(-\nabla_\mu\nabla_\nu + g_{\mu\nu}\nabla^2\Big)\phi^2 \quad - 2\xi\delta_\mu^{\mu'}\partial_{\mu'}\partial_{\nu'} + \frac{1}{4}g_{\mu\nu'}P_x$$

$$\langle T_{\mu\nu}(\boldsymbol{x}\,;\mu)\rangle_{\mathrm{R}}^{\epsilon} = \lim_{\boldsymbol{x}'\to\boldsymbol{x}} \mathcal{T}_{\mu\nu'}G^{+}_{\mathrm{ren}}(\boldsymbol{x},\boldsymbol{x}';\mu)$$

Calculation

$$\langle T_{tt}\rangle_{\mathrm{R}}^{\epsilon} = \frac{b^2}{16\pi}\left[-\gamma+\ln 2+\frac{1}{2}\ln\left(\frac{2\mu^2}{a^2b^2}\right)\right] + \frac{b^2}{8\pi}\left[-\frac{1}{6}+\gamma-\ln 2-\frac{1}{2}\ln\left(\frac{2\mu^2}{a^2b^2}\right)\right]e^{-bx}+\mathcal{O}\left(e^{-2bx}\right),$$

$$\langle T_{xx}\rangle_{\mathrm{R}}^{\epsilon} = \frac{b^2}{16\pi}\left[\gamma-\ln 2-\frac{1}{2}\ln\left(\frac{2\mu^2}{a^2b^2}\right)\right] + \frac{b^2}{8\pi}\left[\frac{1}{2}-\gamma+\ln 2+\frac{1}{2}\ln\left(\frac{2\mu^2}{a^2b^2}\right)\right]e^{-bx}+\mathcal{O}\left(e^{-2bx}\right),$$

$$\langle T_{tx}\rangle_{\mathrm{R}}^{\epsilon} = \langle T_{xt}\rangle_{\mathrm{R}}^{\epsilon} = \mathcal{O}\left(e^{-2bx}\right).$$

Conservation of VEV of EM tensor $\nabla_\mu \langle T^{\mu\nu}\rangle_{\mathrm{R}}^{\epsilon} = 0,$ up to the order of $\mathcal{O}(e^{-2bx})$

Fixing μ in the VEV of Minkowski vacuum

Requiring ${}_{\mathrm{M}}\langle 0|T_{\mu\nu}|0\rangle_{\mathrm{M}} = 0.$ in this Minkowski metric, $ds^2 = \frac{1}{a^2}\left(-dt^2 + dx^2\right).$

$$\langle T_{tt}\rangle_{\mathrm{M}} = \frac{m_0^2}{4\pi a^2}\left[-\gamma + \ln 2 + \frac{1}{2}\ln\left(\frac{\mu^2}{2m_0^2}\right)\right],$$

$$\langle T_{xx}\rangle_{\mathrm{M}} = \frac{m_0^2}{4\pi a^2}\left[\gamma - \ln 2 - \frac{1}{2}\ln\left(\frac{\mu^2}{2m_0^2}\right)\right],$$

$$\langle T_{tx}\rangle_{\mathrm{M}} = \langle T_{xt}\rangle_{\mathrm{M}} = 0.$$

$$\mu = \frac{m_0}{\sqrt{2}}e^{\gamma}$$

Result of the VEV of Energy-Momentum Tensor

$$\langle T_{tt}\rangle^{\epsilon}_{\mathrm{R}} = -\frac{b^2}{48\pi}e^{-bx} + \mathcal{O}\left(e^{-2bx}\right),$$
$$\langle T_{xx}\rangle^{\epsilon}_{\mathrm{R}} = \frac{b^2}{16\pi}e^{-bx} + \mathcal{O}\left(e^{-2bx}\right),$$
$$\langle T_{tx}\rangle^{\epsilon}_{\mathrm{R}} = \langle T_{xt}\rangle^{\epsilon}_{\mathrm{R}} = \mathcal{O}\left(e^{-2bx}\right).$$

The non-vanishing quantities in the above result represent the quantum effects of the state $|0\rangle^{\epsilon}_{\mathrm{R}}$

- **Which physical systems can the quantum effects of IFT be applied to?**
  - Interpretation for the quantum effect of EM tensor
  - Higgs condensate bubble expansion (assuming 1^{st} order phase transition)

$$\langle T_{tt}\rangle_{\mathrm{R}}^{\epsilon} = -\frac{b^2}{48\pi}e^{-bx} + \mathcal{O}(e^{-2bx}),$$
$$\langle T_{xx}\rangle_{\mathrm{R}}^{\epsilon} = \frac{b^2}{16\pi}e^{-bx} + \mathcal{O}(e^{-2bx}),$$
$$\langle T_{tx}\rangle_{\mathrm{R}}^{\epsilon} = \langle T_{xt}\rangle_{\mathrm{R}}^{\epsilon} = \mathcal{O}(e^{-2bx}).$$

Negative quantity : $\langle T_{tt}\rangle_{\mathrm{R}}^{\epsilon} < 0$

(Review) Unruh effect :

R. M. Wald, ΓQuantum Field Theory in Curved Space-Time and Black Hole Thermodynamics,Γ University of Chicago, (1994);
S. Hollands and R. M. Wald, ΓQuantum fields in curved spacetime,Γ Phys. Rept. 574, 1-35 (2015) [arXiv:1401.2026 [gr-qc]].

If $\langle T_{tt}\rangle_{\mathrm{M}}$ the VEV of T_{tt} for the Minkowski vacuum $|0\rangle_{\mathrm{M}}$, is set to zero by the renormalization condition,

$\langle T_{tt}\rangle_{\mathrm{Rindler}}$ the VEV of T_{tt} for the Rindler vacuum $|0\rangle_{\mathrm{Rindler}}$, becomes negative.

➔The energy density of the Rindler vacuum state is lower than that of the Minkowski vacuum.

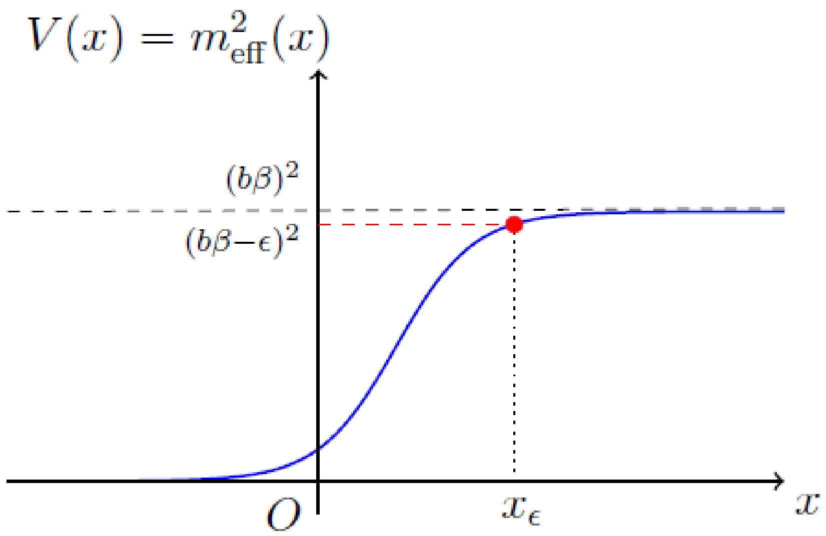
➔Minkowski observers : NO particles

Negative quantity : $\langle T_{tt}\rangle^{\epsilon}_{\mathrm{R}} < 0$

$$\langle T_{tt}\rangle^{\epsilon}_{\mathrm{R}} = -\frac{b^2}{48\pi}e^{-bx} + \mathcal{O}(e^{-2bx}),$$
$$\langle T_{xx}\rangle^{\epsilon}_{\mathrm{R}} = \frac{b^2}{16\pi}e^{-bx} + \mathcal{O}(e^{-2bx}),$$
$$\langle T_{tx}\rangle^{\epsilon}_{\mathrm{R}} = \langle T_{xt}\rangle^{\epsilon}_{\mathrm{R}} = \mathcal{O}(e^{-2bx}).$$

$\langle T_{tt}\rangle_{\mathrm{R(M)}}$, the VEV of T_{tt} for the Minkowski vacuum $|0\rangle_{\mathrm{R(M)}}$, has been **set to zero** by the renormalization condition,

then, $\langle T_{tt}\rangle^{\epsilon}_{\mathrm{R}}$, the VEV of T_{tt} for the vacuum $|0\rangle^{\epsilon}_{\mathrm{R}}$, becomes **negative.**



In the analogy with the Unruh effect,

Unruh-like Effect An observer located slightly out of the right asymptotic region would detect a thermal-like particle distribution for the field in the right Minkowski vacuum.

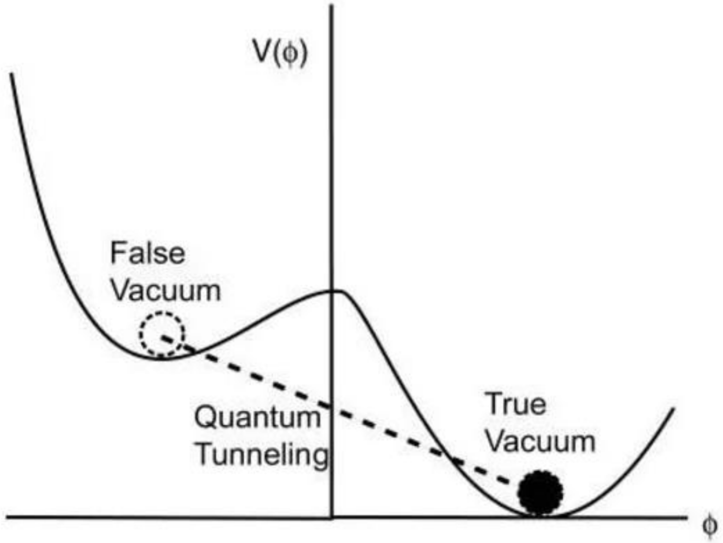
Positive quantity : $\langle T_{xx} \rangle_{\mathrm{R}}^{\epsilon} > 0$

Higgs condensate bubble expansion

Bubble nucleation, expansion, and plasma friction

$$\langle T_{tt} \rangle_{\mathrm{R}}^{\epsilon} = -\frac{b^2}{48\pi} e^{-bx} + \mathcal{O}(e^{-2bx}),$$
$$\langle T_{xx} \rangle_{\mathrm{R}}^{\epsilon} = \frac{b^2}{16\pi} e^{-bx} + \mathcal{O}(e^{-2bx}),$$
$$\langle T_{tx} \rangle_{\mathrm{R}}^{\epsilon} = \langle T_{xt} \rangle_{\mathrm{R}}^{\epsilon} = \mathcal{O}(e^{-2bx}).$$

Nucleation of true vacuum bubble within the false vacuum phase during the first-order electroweak phase transition in the early universe S.R. Coleman (1977) ; C.G. Callan, Jr. and S.R. Coleman (1977), A.D. Linde (1983)

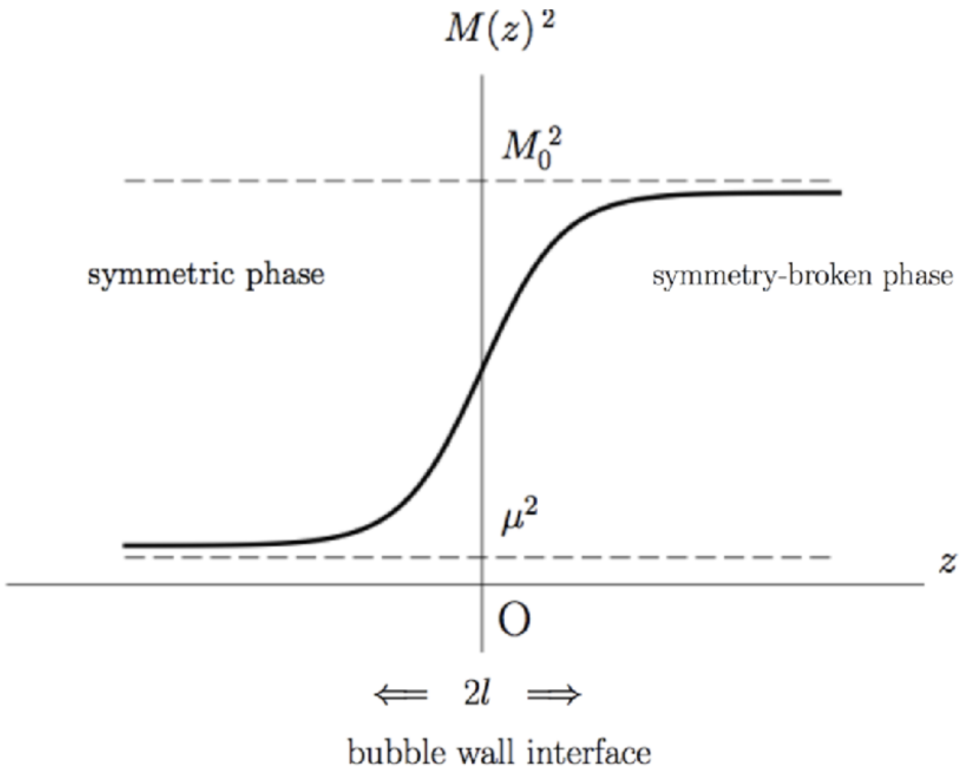


Bubble expansion outward driven by the energy difference between the true and false vacua.

Bubble nucleation, expansion, and plasma friction

Plasma friction **against bubble wall expansion**

through interaction with surrounding plasma of particles



Terminal vel. of bubble walls and gravitational waves

Fictional effect ➔ Terminal vel.

Ultra-relativistic or much slower than the speed of light

Study of **speed of the bubble wall expansion** is crucial for detecting **the gravitational wave as the evidence for the electroweak phase transition.**

Expansion speed ⬆ ➔ Collision Energy ⬆

➔ Gravitational Wave Amplitude, Peak frequency of gravitational waves ⬆

Quantum effect to Bubble wall expansion

A. D. Linde, Nucl. Phys. B 216, 421 (1983) [erratum: Nucl. Phys. B 223, 544 (1983)]

The total pressure exerted on the bubble wall

$$P_{\rm tot} = -\Delta V + \Delta P,$$

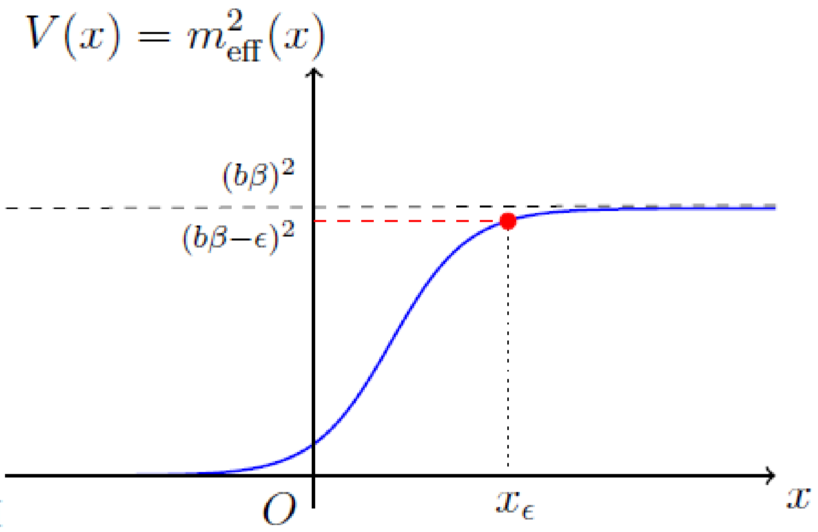
In the context of our effective IFT model for bubble wall expansion (the bubble wall rest frame), the driving force ΔV becomes irrelevant, while ΔP needs to be taken into account.

ΔP includes both classical and quantum effects

$$\Delta P = \Delta P_{\rm classical} + \Delta P_{\rm quantum}$$

$$\Delta P_{\rm classical} = 0 \quad \phi_{\rm classical} = 0$$

Quantum effect to Bubble wall expansion



Vacuum $|0\rangle_{\mathrm{R}}^{\epsilon}$ Effect to the bubble wall to the observer at x_ϵ

: $\Delta P_{\text{quantum}} = \Delta P_\epsilon$

$$\Delta P_\epsilon = P_\epsilon - P_{\mathrm{H}} = \langle T_{xx}\rangle_{\mathrm{IFT}}^{\epsilon} - \langle T_{xx}\rangle_{\mathrm{IFT}}^{\mathrm{H}},$$

the pressure difference ➔ Frictional force

$$\Delta P_\epsilon = \frac{b^2}{16\pi} e^{-bx} + \mathcal{O}\left(e^{-2bx}\right)$$

$$\langle T_{tt}\rangle_{\mathrm{R}}^{\epsilon} = -\frac{b^2}{48\pi} e^{-bx} + \mathcal{O}\left(e^{-2bx}\right),$$
$$\langle T_{xx}\rangle_{\mathrm{R}}^{\epsilon} = \frac{b^2}{16\pi} e^{-bx} + \mathcal{O}\left(e^{-2bx}\right),$$
$$\langle T_{tx}\rangle_{\mathrm{R}}^{\epsilon} = \langle T_{xt}\rangle_{\mathrm{R}}^{\epsilon} = \mathcal{O}\left(e^{-2bx}\right).$$

$\Delta P_\epsilon > 0$: Quantum effect of the vacuum contributes to the friction opposing the bubble wall expansion.

Conclusions

- Developed a supersymmetric inhomogeneous field theory for various models
- Obtained inhomogeneous BPS vacuum solutions for models like CSH, AH, and others.
- Calculated the renormalized Hadamard two-point function and the VEV of the EM tensor, showing quantum effects in IFTs
- Investigated the quantum frictional effect on bubble wall expansion during the electroweak first-order phase transition
- Trials: Extend the framework to higher-dimensional models and quantum effects including fermion fields

Thank you for attention!!